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TOLERANCE ANALYSIS OF A WIPING SYSTEM USING THE PROBABILISTIC APPROACH

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Abstract. *A method for tolerance analysis using the probabilistic approach in described in this manuscript. It is applied with success to an industrial problem. The distributions of the random variables are identified using measurements performed on parts collected from the production lines. Finite element analyses are used to model the mechanical behavior and meta-models are subsequently calibrated to reduce the numerical efforts. A reliability analysis is performed in order to compute the defect probability and estimate the quality of the products.*

1 INTRODUCTION

Engineers are aware that uncertainties in the dimension of manufactured products cannot be avoided, i.e. mechanical components manufactured using the same tools and the same raw materials have slightly different shapes; and their dimensions are also different from the designers request. Tolerance analysis offers a rational framework to study such uncertainties, and allows guaranteeing that the quality associated with the production remains acceptable. This quality is quantified by estimating the defect probability, which is often expressed in parts per million. In this contribution, the probabilistic approach is used, and the dimensions of the components are modeled using random variables.

In previous works on tolerance analysis, it is generally assumed that the components are infinitively rigid, and there is no strain (see e.g. [11, 4]). In this contribution, the strain can not be neglected and has to be fully accounted for. Hence, finite elements simulations are used to model the mechanical behavior.

The objective of this study is the application of the procedure described previously to an industrial problem; the method is developed in collaboration with Valeo Wiper Systems.

This manuscript is structured as follows: the problem is described in Section 2, the proposed methology is discussed in the third section. The paper closes with some conclusions and outlooks.

2 STATEMENT OF THE PROBLEM

Industrial manufacturing processes are affected by uncertainties, and products manufactured in the same production line exhibit slight differences in their dimensions, in their material properties, *etc*. The probabilistic approach offers a rational framework to deal with such problems, and each uncertain parameter is modeled with a random variable.

This contribution is focused on the investigation of the consequences of such uncertainties on the performance of a mechanical system. A wiping system, as shown in Figure 2a is studied. Such devices are used in the automotive industry to remove the rain water and debris from the windshield.

(a) (b)

Figure 1: (a) Aspect of the blade of a common wiper, reproduced from Valeo's website [1]. (b) Simplified mechanical representation.

The different components are [7] :

- the heel blocking vertebrae (metallic parts of the brushes which come to fit into the blade for fixation) and realizes the connection between the blade and the structure of the wiper applying stress;

- the hinge that contributes to turning the blade (when the wiper reaches the end of the windshield and must turn back);
- And the fir whose lip ensures wiping and windshield cleaning.

In this study, only the uncertainties associated with the rubber blade are considered. A simplified mechanical model is subsequently created using a finite element software. A schematic representation of this model is depicted in Figure 2b.

The performance of the wiping system is determined using the contact angle (*i.e.* the angle between the lips of the blade and the windshield), the locking angle (measured at the contact point petween the fir and the heel) and the maximum strain in the blade.

3 METHODS OF ANALYSIS

3.1 Identification of the joint probability density function

Components of the wiping systems have been periodically collected from the production lines and their dimensions have been measured. In total, 44 different quantities are determined for each blade (length and width at various locations, fillet radii, *etc*).

These results are then used to identify the distributions of the random variables associated with the geometry of the components. It is assumed that the uncertainties can be fully characterized using the linear correlation matrix and the marginal distributions. The correlation matrix is directly computed from the measurements.

It is assumed that the random variables follow either a uniform, exponential, normal or lognormal distribution. In each case, it is necessary to identify the parameters of the distribution leading to the best match with the data obtained from the factory. These parameters are obtained by maximizing the likelihood function, which is expressed as:

$$L(x_i^{(1)}, x_i^{(2)}, ..., x_i^{(N)}, \boldsymbol{\theta}) = \prod_{j=1}^{N} f_i(x_i^{(j)}, \boldsymbol{\theta}) \quad (1)$$

where $x_i^{(j)}$ denotes the measurements of a specific wiper blade dimension, which are used to identify the distribution of the corresponding random variable, $\boldsymbol{\theta}$ is a vector regrouping all the parameters of the distribution (*e.g.* the mean, the standard deviation, the bounds) and f_i denotes the probability density function of the random variable x_i. The value of the terms of $\boldsymbol{\theta}$ is selected such that L maximized.

The most suitable distribution is then selected using the Akaike Information Criterion (AIC) [2]:

$$AIC = -2\ln L(\hat{\boldsymbol{\theta}}) + 2q \quad (2)$$

where $\hat{\boldsymbol{\theta}}$ denotes the optimal value of the distribution parameters (which minimize Equation (1)), and q is number of parameters associated with the distribution. For an exponential distribution, q is equal to one (and in this case $\hat{\boldsymbol{\theta}}$ is a scalar), otherwise q is equal to two (and $\hat{\boldsymbol{\theta}}$ is a vector with two rows and one column).

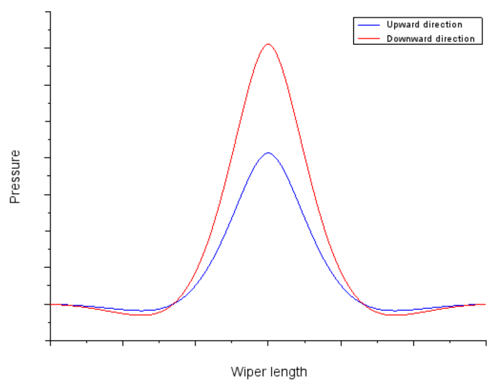
This operation is repeated for all the dimensions considered and, in total, 44 distributions are identified using the procedure described above.

The Young moduli are also modeled as a random variable, their distributions have been determined in-house at Valeo. Two additional variables are introduced to account for the material uncertainties, and the uncertain model includes 46 random variables in total.

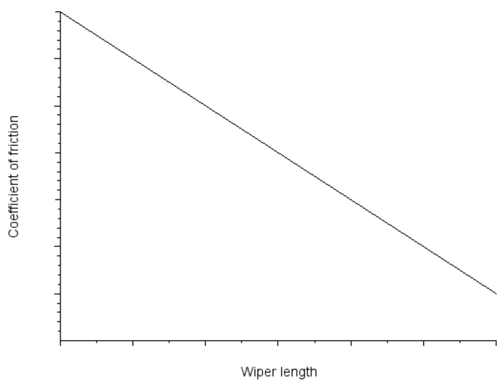
3.2 Mechanical model

The line load F_n applied at the top edge of the heel blocking vertebrae varies along the wiper length and is expressed in N.mm^{-1}. This variation is caused by the geometry of the wiping system and the curvature of the windshield. Figure 2a represents the distribution of the load in terms of the position among the wiper blade. Two distinct types of forces are presented. The first curve (in blue) corresponds to the efforts in the upward direction and the second (in red) to the downward direction. Indeed, the inclination of the wiper on the windshield causes aerodynamic effects on movement. The upward movement when wiping has positive effects of air, and hence the applied load is lower. However, the downward movement is adversely affected by air, which causes a higher load.

The friction coefficient between the blade and the windshield varies among the wiper length as well, it is influenced by the operating conditions (lubrication, velocity of the blade, *etc*). The friction coefficient μ follows a linear curve along the length of the wiper as shown in Figure 2b. Indeed, the difference of values is explained by the fact that the speed is not the same along the wiper. During wiping, the outer portion covers a greater distance and therefore has a higher speed than the inner part.



(a)



(b)

Figure 2: (a) Evolution of the force with respect to the position along the wiper blade. (b) Evolution of the coefficient of friction between the blade and the windshield, expressed with respect to the position.

A simplified two dimensional mechanical model is used in this study. It is assumed that the total length of the blade is considerably greater than its height and width, and that 3D effects can be neglected. The behavior of a cross-section of the wiper blade is modeled using a two dimensional finite element model; the load and the coefficient of friction are selected using the curves shown in Figure 2.

3.3 Meta-modeling

Meta-models may be calibrated and used to reduce the numerical efforts associated with a reliability analysis [3]. Design of experiments is used and the corresponding finite element simulations are performed. The design variables used in this analysis are the dimensions of the wiper blade (*i.e.* the random variables associated with the geometry of the blade, the strategy used to identify their distribution is described in Section 3.1), the Young moduli, the coefficient of friction and the applied load. In total, 3003 calibration points are available.

Response surfaces [10] are used in this contribution in order to approximate the outcome of the finite element model. Three quantities are used to characterize the performance of a wiping system (the contact angle, the locking angle and the maximum strain). It is hence necessary to calibrate three independent meta-models. Fully quadratic polynomials are used, and 1225 coefficient need to be identified to calibrate such meta-models.

The quality of the fit may be assessed using leave one out cross validation (see e.g. [6]). A sample is removed from the calibration set and the response surface is calibrated using all the other samples. The omitted sample is then used to evaluated the error introduced by the meta-model. The coefficient of determination for prediction R_p^2 provides an indicator of the quality of the fit obtained by a meta-model (see e.g. [9]). It is expressed as:

$$R_p^2 = 1 - \frac{\sum_{i=1}^{n} \left(y_i - \hat{y}_i^{(\sim i)} \right)^2}{\sum_{i=1}^{n} \left(y_i - \bar{y} \right)^2} \tag{3}$$

where y_i denotes the values obtained using the finite element simulations, $\bar{y}$ denotes their mean value, $\hat{y}_i^{(\sim i)}$ is the prediction of y_i obtained with a meta-model calibrated using all the samples but the one associated with y_i and n is the total number of samples used to compute the coefficient of determination. The closer to one R_p^2 is, the better the meta-model prediction, considering the samples used to compute the coefficient of determination only.

Table 1 shows the coefficients of determination obtained in this study. The quadratic response surfaces lead to a fair fit for the three functional requirements considered in this study.

Quantity of interest	R_p^2
Contact angle	0.99
Locking angle	0.97
maximum strain	0.97

Table 1: Coefficients of determination obtained using leave-one out.

3.4 Definition of the performance functions

The wiper blade is assumed to be functional at a given position with d as a coordinate if the maximum strain is below a predefined value; the contact angle and the locking angle are within a predefined range. Hence, five normalized performance functions are introduced, they are defined as:

$$
\begin{aligned}
g_1(\boldsymbol{x}, d) &= \frac{\varepsilon_{max}^u - \varepsilon_{max}(\boldsymbol{x}, d)}{\varepsilon_{max}^{th}} \\
g_2(\boldsymbol{x}, d) &= \frac{\alpha_l(\boldsymbol{x}, d) - \alpha_l^l}{\alpha_l^l} \\
g_3(\boldsymbol{x}, d) &= \frac{\alpha_l^u - \alpha_l(\boldsymbol{x}, d)}{\alpha_l^u} \\
g_4(\boldsymbol{x}, d) &= \frac{\alpha_c(\boldsymbol{x}, d) - \alpha_c^l}{\alpha_c^l} \\
g_5(\boldsymbol{x}, d) &= \frac{\alpha_c^u - \alpha_c(\boldsymbol{x}, d)}{\alpha_c^u}
\end{aligned} \tag{4}
$$

where $\boldsymbol{x}$ denotes the vector of the random variables; ε_{max}, α_c and α_l denote the functional requirements, *i.e.* the maximum strain, the contact angle and the locking angle, respectively; ε_{max}^u is the maximum admissible strain; α_c^u and α_l^u are the maximum admissible contact and locking angle, respectively; α_c^l and α_l^l are the minimum admissible contact and locking angle, respectively.

The functions described in Equation (4) can be used to describe the behavior of the wiper blade locally, in the vicinity of the point with the coordinate d, whereas the functionality of the wiper needs to be determined at the level of the system. For a set of random variables x (geometric and material), the profile is functional if it leaves no visible defects on the windshield during wiping. A defect is visible in case the functional requirements are not respected continuously on a small given length (*i.e.* if one of the performance functions stated in Equation (4) is less than zeros on the given length). Therefore, a non-compliance of functional requirements is tolerated on these short lengths of the wiper (integration lengths).

Thus, the coordinate among the wiper blade is discretized using a step of 1 mm, and the performance functions expressed in Equation (4) are evaluated for each point. The reliability problem is subsequently formulated using system reliability.

3.5 Monte-Carlo simulation

The Monte-Carlo simulation is performed in the so-called *standard normal space*, where all the random variables have a standard normal distribution, with a zero mean and a unit standard deviation. An isoprobabilistic transformation is applied, et is expressed as:

$$z_i = \Phi^{-1}\left(F_i\left(x_i\right)\right) \text{ for } i = 1, ..., 46 \tag{5}$$

where Φ^{-1} denotes the inverse of the standard normal cumulative density function, F_i is the cumulative distribution function associated with the variable x_i and z_i denotes the random variables in the standard normal space. In case the variables x_i and x_j are correlated before the transformation described in Equation (5), the variables z_i and z_j are correlated as well, and ρ'_{ij} denotes the correlation coefficient. The approximation of ρ'_{ij} available in [8] are used here.

The correlation matrix associated with the random variables after such transformation is defined as:

$$\Sigma' = \left[\rho'_{ij}\right]_{1 \leqslant i \leqslant 46, 1 \leqslant j \leqslant 46} \tag{6}$$

The Karhunen-Loève transform is used to generate samples of correlated variables with a Gaussian distribution:

$$\boldsymbol{z} = \sum_{i=1}^{46} \theta_i \sqrt{\lambda_i} \boldsymbol{\phi}_i \tag{7}$$

where $\boldsymbol{z} = [z_1, ..., z_{46}]$, $\theta_i,\ i = 1...46$ denotes samples generated from independent variables with a standard normal distribution, λ_i and $\boldsymbol{\phi}_i$ denote the eigenvalues and the eigenvectors associated with the matrix Σ', respectively.

3.6 Results

The procedure described in the previous section allows to determine the failure probability associated with the wiper blade model. The numerical value are not discussed herein. These results are compatible with the quality requirement at Valeo.

4 CONCLUSIONS

In this contribution, the probabilistic approach is used to deal with an industrial problem of tolerance analysis. The joint probability density function is identified using factory data. A finite element model is used to model the mechanical behavior of the wiper blade, and meta-models are calibrated in order to reduce the numerical efforts. The defect probability is subsequently determined using a reliability analysis, and allows to quantify the product quality.

Future work is geared towards the application of advanced reliability procedures. Kriging-based methods (see *e.g.* [5]) look promising in this regards, as they allow to determine low failure probabilities with reduced computational time. Reliability-based optimization may as well be used in order to maximize the quality associated with the wiper system, i.e. identifying the geometry of this structure leading to the lowest probability of defect.

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