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RESPONSE STATISTICS OF STRUCTURAL SYSTEMS SUBJECTED TO FULLY NON-STATIONARY SPECTRUM COMPATIBLE EXCITATION

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Abstract. *The probabilistic analysis of structural systems subjected to seismic excitations requires the spectral characterization of both the input excitation and the structural response; in order to reproduce the typical characteristics of real earthquakes ground-motion time history, the seismic excitation should be modelled as a non-stationary stochastic process. Once the characterization of the ground motion acceleration is done, it is possible to analyze the safety of structural systems subjected to those excitations; in particular, the largest absolute value peak of stochastic response is a useful design information. In order to define the statistics of the maximum of the stochastic response of structures, it is necessary to evaluate the Non-Geometric Spectral Moments (NGSMs).*

In this paper the fully non-stationary spectrum compatible model has been adopted to define the ground motion acceleration process. Then the evaluation of the NGSMs is performed. The main steps of the proposed approach are: i) the study of a set of real earthquakes to catch their most important features; ii) the introduction of the fully non-stationary spectrum compatible process; iii) the definition of the NGSMs, in the time domain; iv) the evaluation of the mean frequency; v) the validation with the Monte Carlo Simulation.

1. INTRODUCTION

For earthquake-resistant design of structures, the earthquake-induced ground motion is generally represented in the form of a response spectrum of pseudo-acceleration or displacement. The spectrum used as input is usually obtained by scaling an elastic spectrum by factors that account for, amongst other phenomena, the influence of inelastic structural response [1]. There are, however, situations in which the scaled response spectrum is not considered appropriate, and fully dynamic analysis is required. Consequently the engineer will generally have to employ time-history analysis, which can be: a) synthetic accelerograms generated from seismological source models and accounting for path and site effects [2]; b) real accelerograms recorded during earthquakes earthquakes [3,4]; c) artificial spectrum-compatible accelerograms by generating a Power Spectral Density (PSD) function from the smoothed response spectrum, and then to derive sinusoidal signals having random phase angles and amplitudes [5,6,7].

The attraction of the third approach is obvious because it is possible to obtain acceleration time-series that are almost completely compatible with the elastic design spectrum, which in some cases will be the only information available to the design engineer regarding the nature of the ground motions to be considered. Moreover, they can be adopted in the framework of stochastic dynamics, so avoiding long time consuming deterministic analyses. However, it is now widely accepted that the generation of the PSD leads to stationary artificial accelerograms which generally have an excessive number of cycles of strong motion and consequently they possess unreasonably high energy content [8].

Furthermore in the framework of non-stationary analysis of structures, one time-dependent parameter, very useful in describing the time-variant spectral properties of the stochastic process, is the mean frequency, $\nu_X^+(t)$, which evaluates the variation in time of the mean up-crossing rate of the time axis; from the analysis of real accelerograms records a decrease in time of this parameter is observed, on the contrary of the stationary or quasi-stationary process models where the mean value of zero crossing versus the time is constant.

It follows that a fully non-stationary model is more straightforward, being the one and only that is able to catch the time varying mean frequency.

In this paper the fully non-stationary spectrum compatible model has been adopted to define the ground motion acceleration process [7]; to take into account both the simultaneous amplitude and frequency non-stationarity the Spanos and Solomos [9] process model has been selected. The main steps of the proposed approach are: i) the study of a set of real earthquakes to catch the most important features; ii) the introduction of the fully non-stationary spectrum compatible process; iii) the definition of the Non-Geometric Spectral Moments (NGSMs), in the time domain, as element of the pre-envelope covariance matrix; iv) the evaluation of the mean frequency; v) the validation with the Monte Carlo Simulation.

2. REAL EARTHQUAKES AND SPECTRUM COMPATIBILITY

One of the most important problem in seismic engineering is the correct characterization of the ground motion acceleration; in code-based seismic design and assessment it is often allowed the use of real records as an input for nonlinear dynamic analysis. On the other hand, international seismic guidelines, concerning this issue, have been found hardly applicable by practitioners. This is related to both the difficulty in rationally relating the ground motions to the hazard at the site and the required selection criteria. Consequently the use of artificial spectrum-compatible accelerograms, obtained by generating a PSD function from the smoothed response spectrum, is more spread; the main problem of this approach is the excessive number of cycles of strong motion and consequently the unreasonably high energy content [8] of the stationary artificial accelerograms. It follows that a fully non-stationary model is more straightforward.

The first aim of this paper is to obtain fully non-stationary artificial earthquakes that are spectrum compatible and able to reproduce the main features of real recorded time histories; firstly it is necessary to study and analyze a set of real accelerograms associated to seismic events and recorded in a determinate site in order to define the trend of the mean value of zero-crossing versus the time. It is important to notice that the comparison with only one result cannot be relevant, so it is necessary to select a set of real recorded earthquakes in order to perform a statistical analysis.

The chosen database is the “PEER: Pacific Earthquake Engineering Research Center: NGA Database”; 50 recorded accelerograms have been selected to perform the statistical analysis. In particular, all of them are time histories of seismic events that happened in the Imperial Valley (California). For each of the recorded accelerograms the mean frequency has been evaluated, as shown in Fig. 1.

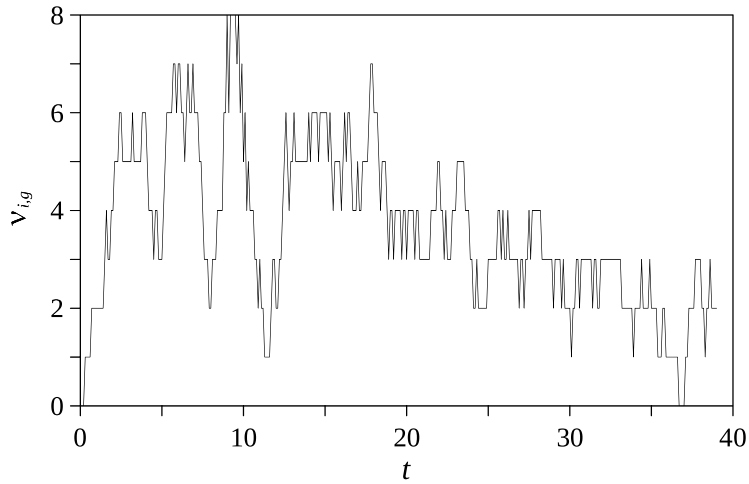

Figure 1. Mean frequency of the *i*-th accelerogram

Then, in order to perform a statistical analysis, all the time histories longer than 35 seconds have been selected, so it is possible to define the average of the mean frequency (see Fig. 2).

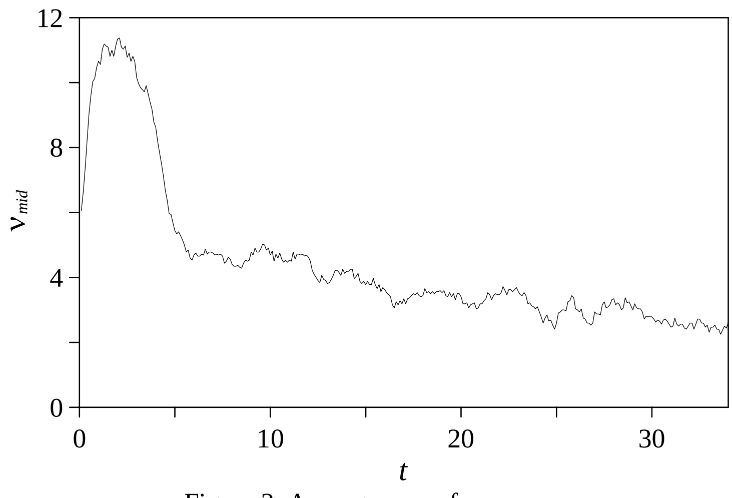

Figure 2. Average mean frequency

It is possible to notice that the average mean frequency, sensitively different from the one of the general time history (see Figg. 1 vs 2), is not constant but it decreases in time.

An efficient method to generate stationary artificial spectrum compatible accelerograms was established by Cacciola-Colajanni-Muscolino [6]; the one-sided Power Spectral Density (PSD) can be written as:

$$\begin{aligned} &G_0(\omega) = 0, \quad 0 \le \omega \le \omega_l; \\ &G_0(\omega_k) = \frac{4\zeta_0}{\omega_k \pi - 4\zeta_0 \omega_{k-1}} \left(\frac{S_e^2(\omega_k,\zeta_0)}{\eta^2(\omega_k,\zeta_0)} - \Delta\omega \sum_{j=1}^{k-1} G_0(\omega_j) \right), \quad \omega > \omega_l; \end{aligned} \tag{1}$$

where $\zeta_0 = 0.05$ is the damping ratio, $S_e(\omega_k,\zeta_0)$ is the target spectrum (see Fig. 3) and $\eta(\omega_k,\zeta_0)$ is the peak factor

$$\eta(\omega_k,\zeta_0) = \sqrt{2\ln\left\{ \frac{T_s\,\omega_k}{\pi} \left(-\ln 0.5\right)^{-1} \left[1 - \exp\left[-\delta^{1.2} \sqrt{\pi \ln\left(\frac{T_s\,\omega_k}{\pi} \left(-\ln 0.5\right)^{-1} \right)} \right] \right] \right\}} \tag{2}$$

in which

$$\delta = \left[1 - \frac{1}{1-\zeta_0^2} \left(1 - \frac{2}{\pi} \arctan \frac{\zeta_0}{\sqrt{1-\zeta_0^2}} \right)^2 \right]^{1/2} \tag{3}$$

is the bandwidth factor.

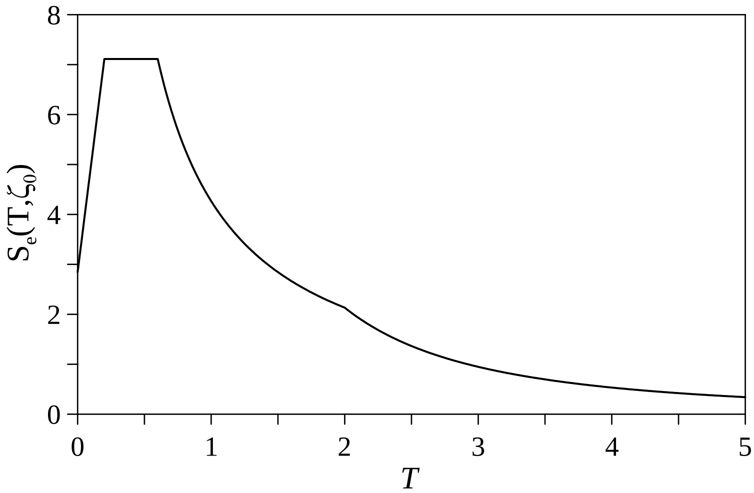

Figure 3. Target spectrum

The N spectrum compatible artificial earthquakes are obtained thanks to the formula proposed by Shinozuka and Sato [10]:

$$\ddot{u}_{i,g}(t) = \sum_{r=1}^{m_c} \sqrt{2G_0(\omega_r)\Delta\omega}\,\sin\left(\omega_r t + \theta_r^{(i)}\right); \quad 0 \le t \le t_f \tag{4}$$

where $\omega_r = r\,\Delta\omega \;\left(r = 1,2,...,m_c\right)$, $\Delta\omega = \omega_c / m_c$ is the frequency increment, ω_c is the upper cut-off circular frequency and $\theta_r^{(i)}$ are random phase angles uniformly distributed over the interval $[0-2\pi)$.

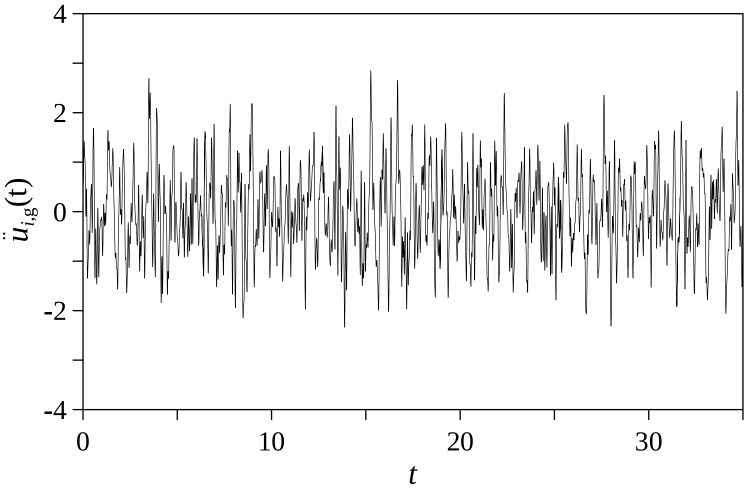

Figure 4. *i*-th artificial stationary time-history

The main problem is that the stationary model of the ground motion acceleration process is unable to catch the characteristics of real earthquakes, such as the amplitude and frequency modulation of the signal (see Fig. 4); then the energy of the artificial time history is proportional to the duration of the process itself. So, mathematically speaking, stationary samples have infinite energy. A modified method, that is able to generate artificial spectrum compatible earthquakes, for fully non-stationary (when both time and frequency content change) random processes, is herein proposed, by applying the procedure proposed by Cacciola [7]. In order to take into account the main features of seismic ground motion, that is the “build-up” and the “die off” segments as well as a decreasing dominant frequency, the time-frequency modulating function $a(\omega,t)$ proposed by Spanos and Solomos [9] is chosen:

$$a(\omega,t) = \varepsilon\left(\omega\right) t \exp\left(-\alpha\left(\omega\right) t\right) \mathcal{U}\left(t\right) \tag{5}$$

where $\mathcal{U}\left(t\right)$ is the Unit Step function.

The N fully non-stationary artificial earthquakes are obtained by introducing in Eq. (4) the time-frequency modulating function $a(\omega,t)$

$$\ddot{u}_{i,g}(t) = \sum_{r=1}^{m_c} \left|a(\omega,t)\right| \sqrt{2\,G_0\left(\omega_r\right)\Delta\omega}\,\sin\left(\omega_r\,t + \theta_r^{(i)}\right); \quad 0 \le t \le t_f \tag{6}$$

The set of artificial fully non-stationary time-histories will not result spectrum-compatible; so this method defines a new spectrum compatible PSD, modifying the previous one, $G_0(\omega_k)$, with the term $\Delta G_0(\omega_k)$, that depends on the gap between the target spectrum and the average one $\tilde{S}_e(\omega_k,\zeta_0)$:

$$\begin{aligned} &\tilde{G}_0(\omega) = 0, \quad 0 \le \omega \le \omega_l; \\ &\tilde{G}_0(\omega_k) = G_0(\omega_k) + \Delta G_0(\omega_k), \quad \omega > \omega_l; \end{aligned} \tag{7}$$

where

$$\Delta G_0(\omega_k)=\frac{4\zeta_0}{\omega_i\pi-4\zeta_0\omega_{k-1}}\left(\frac{S_e^2(\omega_k,\zeta_0)-\left(\beta\,\tilde{S}_e(\omega_k,\zeta_0)\right)^2}{\eta_s^2(\omega_k,\zeta_0)}-\Delta\omega\sum_{j=1}^{i-1}\Delta G_0(\omega_j)\right)$$
$$\times\mathcal{U}\left(\frac{S_e^2(\omega_k,\zeta_0)-\left(\beta\,\tilde{S}_e(\omega_k,\zeta_0)\right)^2}{\eta_s^2(\omega_k,\zeta_0)}-\Delta\omega\sum_{j=1}^{i-1}\Delta G_0(\omega_j)\right) \tag{8}$$

with $\eta_s(\omega_k,\zeta_0)$ approximately posed equal to $\eta(\omega_k,\zeta_0)$ and

$$\beta=1 \quad \text{if} \quad S_e(\omega_k,\zeta_0)\geq\tilde{S}_e(\omega_k,\zeta_0) \quad \text{or} \quad \beta=\min\left\{\frac{S_e(\omega_k,\zeta_0)}{\tilde{S}_e(\omega_k,\zeta_0)}\right\} \tag{9}$$

If the difference between the target PSD and the modified one is more than 10%, depending on the recorded earthquake time history, iterative improvement of the power spectral density of the corrective term could be necessary for satisfying code provisions. To this aim the following iterative scheme is proposed

$$\tilde{G}_0^j(\omega)=\tilde{G}_0^{j-1}(\omega)\,\frac{S_e(\omega,\zeta_0)^2}{S_e^{j-1}(\omega,\zeta_0)^2} \tag{10}$$

in which $S_e^j(\omega,\zeta_0)^2$ represents the mean response spectrum determined at the j-th iteration. In Eq. (10) $\tilde{G}_0^j(\omega)$ is the one-sided PSD function of the so-called "embedded" stationary counterpart process; the fully non-stationary random process $F(t)$ is defined by the one-sided Evolutionary Power Spectral Density (EPSD) that can be expressed, in the Priestley [11] representation, as the product between the modulating function and the PSD of the "embedded" stationary counterpart:

$$\begin{aligned} &G_{FF}(t,\omega)=\left|a(\omega,t)\right|^2\tilde{G}_0^j(\omega), && \omega\geq 0;\\ &G_{FF}(t,\omega)=0, && \omega<0. \end{aligned} \tag{11}$$

Remarkably, via the proposed procedure spectrum compatible fully non-stationary earthquakes in agreement with code provisions can be simulated. The non-stationary behaviour relies on the recorded signal that would be chosen so as to reflect local geotechnical and seismological characteristics. The spectrum compatible criteria are satisfied by the superposition of the corrective quasi-stationary Gaussian random process whose power spectral density have been determined through a handy recursive formula.

3. EXPLICIT SOLUTIONS FOR THE NGSMs

In many cases of engineering interest the probabilistic assessment of structural failure is derived as a function of barrier crossing rates, distribution of peaks and extreme values. These quantities can be evaluated as a function of the so-called Non-Geometric Spectral Moments (NGSMs) [12,13,14] for non-stationary stochastic processes.

Let's assume a linear quiescent n-degree-of-freedom (n-DOF) classically damped structural system subjected to a seismic excitation whose dynamic behavior is governed by the following equation of motion:

$$\mathbf{M}\ddot{\mathbf{u}}(t)+\mathbf{C}\dot{\mathbf{u}}(t)+\mathbf{K}\mathbf{u}(t)=-\mathbf{M}\,\boldsymbol{\tau}\,\ddot{u}_g\left(t\right) \tag{12}$$

where **M**, **C**, and **K** are the $n\times n$ mass, damping, and stiffness matrices of the structure; $\mathbf{u}(t)$ is the $n\times 1$ vector of displacements, having for *i*-th element $u_i(t)$, $\boldsymbol{\tau}$ is the $n\times 1$ vector of spatial distribution of loads, $\ddot{u}_g\left(t\right)$ is the zero-mean Gaussian spectrum-compatible fully non-stationary ground motion acceleration process, and a dot over a variable denotes differentiation with respect to time. The equation of motion can be uncoupled by applying the modal analysis because of the hypothesis of classically damped system; in order to achieve this aim is necessary to introduce the modal coordinate transformation:

$$\mathbf{u}(t)=\boldsymbol{\Phi}\,\mathbf{q}(t)=\sum_{j=1}^{m}\boldsymbol{\phi}_j q_j(t)\;\;\Rightarrow\;\; u_i(t)=\sum_{j=1}^{m}\phi_{ij}q_j(t) \tag{13}$$

where $\boldsymbol{\Phi}=\left[\boldsymbol{\phi}_1\quad\boldsymbol{\phi}_2\quad\ldots\quad\boldsymbol{\phi}_m\right]$ is the modal matrix, of order $n\times m$, collecting the *m* eigenvectors $\boldsymbol{\phi}_j$, normalized with respect to the mass matrix $\mathbf{M}$, solutions of the eigenproblem:

$$\mathbf{K}^{-1}\mathbf{M}\;\boldsymbol{\Phi}=\boldsymbol{\Phi}\,\boldsymbol{\Omega}^{-2};\quad \boldsymbol{\Phi}^{T}\mathbf{M}\;\boldsymbol{\Phi}=\mathbf{I}_m \tag{14}$$

In the previous equation $\boldsymbol{\Omega}$ is a diagonal matrix collecting the undamped natural circular frequency ω_j and $\mathbf{I}_m$ is the identity matrix of order *m*.

Once the modal matrix $\boldsymbol{\Phi}$ is evaluated, by applying the modal coordinate transformations to Eq.(12), the *j*-th modal differential equation can be written as:

$$\ddot{q}_j(t)+2\zeta_j\omega_j\;\dot{q}_j(t)+\omega_j^2 q_j(t)=p_j\,\ddot{u}_g(t);\; j=k,\ell;\; k=1,\cdots,m;\ell=1,\cdots,m \tag{15}$$

in which ζ_j is the modal damping ratio of the *j*-th modal oscillator and

$$p_j(t)=\boldsymbol{\phi}_j^T\,\mathbf{M}\,\boldsymbol{\tau} \tag{16}$$

is the so-called *j*-th participation factor. The structural systems are conceived and designed to survive to natural actions. If the excitations are modelled as random processes, the dynamic responses are random processes too, and the structural safety needs to be evaluated in a probabilistic sense. Among the models of failure, the simplest one, which is also the most widely used in practical analyses, is based on the assumption that a structure fails as soon as the response at a critical location exits a prescribed safe domain for the first time. The probability of failure, in this case, coincides with the first passage probability. In random vibration theory, the problem of probabilistically predicting this event is termed *first passage problem*. Unfortunately, the solution of this problem has not been derived in exact form, even in the simplest case of the stationary response of a Single-Degree-of-Freedom (SDoF) linear oscillator under zero mean Gaussian white noise. In the framework of approximate methods, for zero-mean Gaussian non-stationary input, the evaluation of the so-called Non-Geometric Spectral Moments (NGSMs) is required [12-16]. Therefore the *j*-th NGSMs, $\lambda_{j,u_iu_i}(t)$ ($i=0,1,2$), of the *i*-th nodal response, $u_i(t)$, are given as a function of modal NGSMs, $\lambda_{j,k\ell}(t)$ ($\ell,k=1...,m$), "purged" by participation factors, by the following relationships:

$$\lambda_{0,u_iu_i}(t) = \mathrm{E}\left\langle u_i\left(t\right)u_i^*\left(t\right)\right\rangle = \sum_{k=1}^{m}\sum_{\ell=1}^{m} p_k\; p_\ell\, \phi_{ik}\, \phi_{i\ell}\, \lambda_{0,k\ell}(t);$$
$$\lambda_{1,u_iu_i}(t) = \sum_{k=1}^{m}\sum_{\ell=1}^{m} p_k\; p_\ell\, \phi_{ik}\, \phi_{i\ell}\, \lambda_{1,k\ell}(t); \tag{17}$$
$$\lambda_{2,u_iu_i}(t) = \mathrm{E}\left\langle \dot{u}_i\left(t\right)\dot{u}_i^*\left(t\right)\right\rangle = \sum_{k=1}^{m}\sum_{\ell=1}^{m} p_k\; p_\ell\, \phi_{ik}\, \phi_{i\ell}\, \lambda_{2,k\ell}(t).$$

Notice that, since the introduction of the one-sided PSD $\tilde{G}_0^j(\omega)$ (see Eq. (10)) the ground motion acceleration process $\ddot{u}_g\left(t\right)$ is defined by the EPSD function obtained in Eq. (11). It follows that the response processes $u_i(t)$ is a complex function. Moreover, it has to be emphasized that the zero-th NGSM, $\lambda_{0,u_iu_i}(t)$, and the second order NGSM, $\lambda_{2,u_iu_i}(t)$, are real functions that coincide with the covariance of the response in terms of displacement and velocity, respectively; while the first order NGSM, $\lambda_{1,u_iu_i}(t)$, is a complex quantity whose real part can be evaluated as the cross-covariance between the response process and the response velocity process of the same linear system subjected to a non-stationary input whose stationary counterpart is proportional to its Hilbert transform.

It is possible to evaluate the modal "purged" NGSMs in compact form as elements of the following *pre-envelope covariance* (PEC) matrix [15]:

$$\boldsymbol{\Sigma}_{k\ell}(t) = \int_0^\infty \mathbf{Y}_k\left(\omega,t\right)\mathbf{Y}_\ell^{*\,T}\left(\omega,t\right)\tilde{G}_0^j(\omega)\mathrm{d}\omega \equiv \begin{bmatrix} \lambda_{0,k\ell}(t) & \mathrm{i}\lambda_{1,k\ell}(t) \\ -\mathrm{i}\lambda_{1,k\ell}^*(t) & \lambda_{2,k\ell}(t) \end{bmatrix} \tag{18}$$

where, for $j=k,\ell$, the vector $\mathbf{Y}_j\left(\omega,t\right)$ represents the *modal evolutionary frequency response vector function* in terms of state variables and can be evaluated in closed form solutions as recently proposed by the authors [16]. It has to be emphasized that, thanks' to the proposed approach, the computation of any Hilbert transform is avoided.

4. NUMERICAL APPLICATION

In this section in order to verify the accuracy of the proposed procedure the benchmark quiescent classically damped linear MDOFs [16] is analysed; this frame has a uniform story height $H = 320$ cm and a bay width $L = 600$ cm, as shown in Figure 7. The steel columns are made of European HE340A wide flange beams with moment of inertia along the strong axis $I = 27690\ \mathrm{cm}^4$. The steel material is modelled as linear elastic with Young's modulus $E =$ 200 GPa. The beams are considered rigid to enforce a typical shear building behaviour. Under this assumptions, the shear-frame is modelled as a three DOF linear system. The frame described above is assumed to be part of a building structure with a distance between frames $L_0 = 600\,\mathrm{cm}$. The tributary mass per story, M, is obtained assuming a distributed gravity load of $q = 8\ \mathrm{kN/m}^2$, accounting for the structure's own weight, as well as for permanent and live loads, and is equal to $M = 28800\,\mathrm{kg}$. The modal periods of the linear elastic undamped shear-frame are $T_1 = 0.376$ s, $T_2 = 0.134$ s and $T_3 = 0.093$ s, with corresponding effective modal mass ratios of 91.41%, 7.45% and 1.10% respectively. The damping ratio $\zeta = 0.05$ is assumed equal for the three modes of vibration.

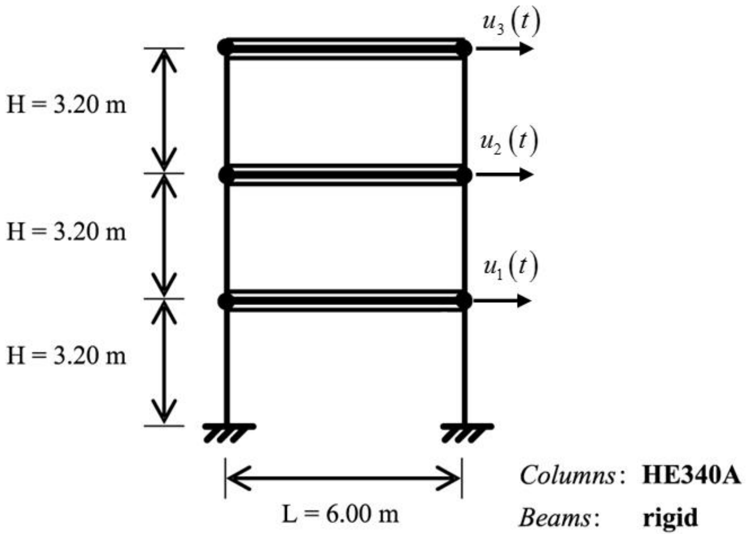

Figure 5. Geometric configuration of benchmark three-storey one-bay shear-type frame.

The benchmark structural model undergoes to a stochastic earthquake base excitation, modelled by a zero mean Gaussian spectrum-compatible fully non-stationary process. The target spectrum (see Fig. 3) is obtained following the EC8 instructions [17] selecting $a_g = 2.474\ \mathrm{m/sec^2}$ as peak ground acceleration and for the type "C" of soil and a spectrum of type 1 the parameters $S = 1{,}15$, $T_B = 0{,}2$ sec, $T_C = 0{,}6$ sec and $T_D = 2{,}0$ sec.

The parameters selected for Spanos and Solomos [9] time- modulating function are:

$$\alpha(\omega) = \frac{1}{2}\left(0.15 + \frac{\omega^2}{225\pi^2}\right); \quad \varepsilon(\omega) = \frac{\omega}{15\pi}\sqrt{2} \tag{19}$$

In Figure 6 the comparison between the stationary spectrum compatible PSD (see Eq. (1)) and the modified one, obtained through the iterative procedure (see Eqs. (7)-(10)) is shown; in Figure 7 the *i*-th artificial fully non-stationary time history (see Eq.(6)) is depicted.

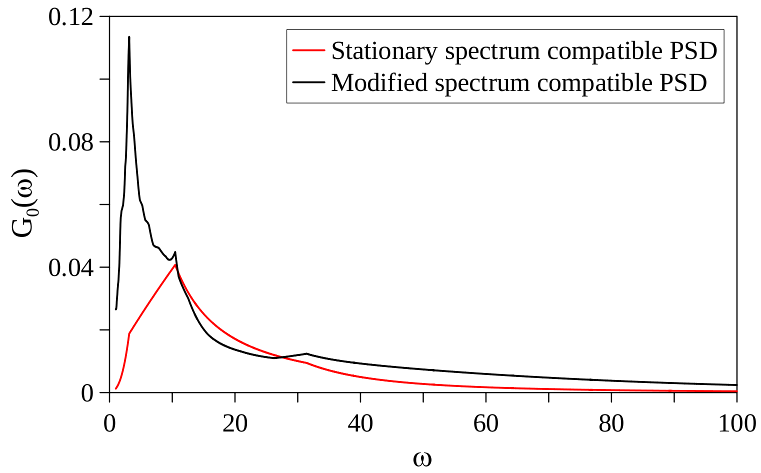

Figure 6. Spectrum compatible PSD, stationary (red line) and modified (black line)

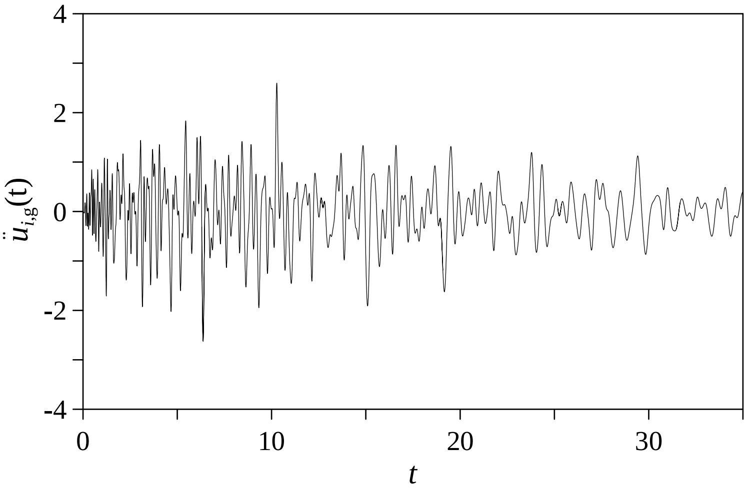

Figure 7. i-th artificial fully non-stationary time-history

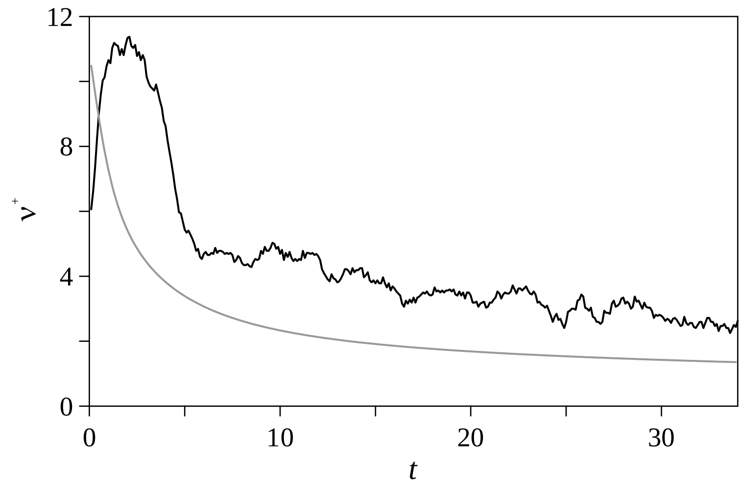

Figure 8. Comparison between the average mean frequency (black line) and the mean frequency of the proposed model (gray line)

As shown in Figure 8 the proposed model is able to catch the time varying mean frequency, that follows the trend of the average real one.

In Figure 9-11 the time histories of the relative to ground displacement NGSMs of the three floors are depicted; in order to verify the proposed procedure the time-variant NGSMs evaluated by the proposed analytical approach are compared with the ones obtained by Monte-Carlo Simulation (MCS). To obtain the MCS results, $N = 1000$ samples of input the random process have been generated. Notice that since the samples of the input stochastic process, $F^{(i)}(t)$, are chosen in such a way that the input process, $F(t)$, possess one-sided EPSD function (see Eq.(11)), the input non-stationary process has to be a complex process having stationary counterpart of the imaginary part proportional to Hilbert transform of the real stationary counterpart part itself. It follows the "purged" NGSMs of the complex output process have to be evaluated by means of the following relationships:

$$\begin{aligned}\operatorname{Re}\left\{\lambda_{0,k\ell}(t)\right\} &= \frac{1}{N}\sum_{i=1}^{N}\left[z_k^{(i)}(t)z_\ell^{(i)}(t) + x_k^{(i)}(t)x_\ell^{(i)}(t)\right];\\ \operatorname{Re}\left\{\lambda_{1,k\ell}(t)\right\} &= \frac{1}{N}\sum_{i=1}^{N}\left[x_k^{(i)}(t)\dot{z}_\ell^{(i)}(t) - z_k^{(i)}(t)\dot{x}_\ell^{(i)}(t)\right];\\ \operatorname{Re}\left\{\lambda_{2,k\ell}(t)\right\} &= \frac{1}{N}\sum_{i=1}^{N}\left[\dot{z}_k^{(i)}(t)\dot{z}_\ell^{(i)}(t) + \dot{x}_k^{(i)}(t)\dot{x}_\ell^{(i)}(t)\right]\end{aligned} \tag{20}$$

where $z_k^{(i)}(t)$ and $x_\ell^{(i)}(t)$ are the responses of the dummy oscillators, whose motion is governed by the differential equation (15), with the position $p_j = 1$, subjected to the forcing function $\operatorname{Re}\left\{F^{(i)}(t)\right\}$ and $\operatorname{Im}\left\{F^{(i)}(t)\right\}$, respectively. The generic sample of the forcing function can be evaluated as:

$$\begin{aligned}\operatorname{Re}\left\{F^{(i)}(t)\right\} &= \frac{\sqrt{2}}{2}\sum_{r=1}^{m_c}\sqrt{2G_{FF}\left(t,\omega_r\right)\Delta\omega}\,\sin\left(\omega_r t + \theta_r^{(i)}\right);\\ \operatorname{Im}\left\{F^{(i)}(t)\right\} &= \frac{\sqrt{2}}{2}\sum_{r=1}^{m_c}\sqrt{2G_{FF}\left(t,r\,\Delta\omega\right)\Delta\omega}\,\cos\left(r\,\Delta\omega t + \theta_r^{(i)}\right).\end{aligned} \tag{21}$$

In Eq. (21) $\Delta\omega = \omega_c/m_c = 0.1$ is the frequency increment; $\omega_c = 100\ \mathrm{rad/s}$ is the adopted upper cut-off circular frequency; $m_c = 1000$ and $\theta_r^{(i)}$ are random phase angles uniformly distributed over the interval $[0 - 2\pi)$. Notice that, in order to rightly apply Eq. (20), the samples of random phase angles have to be generated at the same time in Eq. (21).

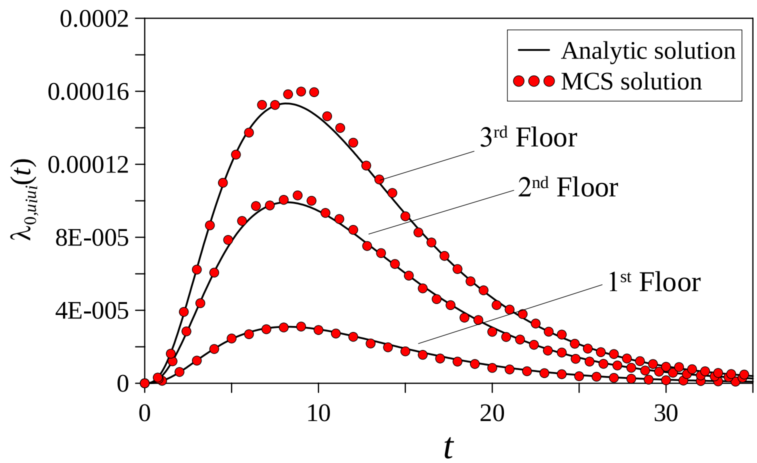

Figure 9. Comparison between the time-variant histories of the $\lambda_{0,u_iu_i}(t)$ NGSMs $\left[\mathrm{m}^2\right]$, of the three relative to ground floor displacements, by applying the proposed analytical solution and the Monte Carlo Simulation (MCS).

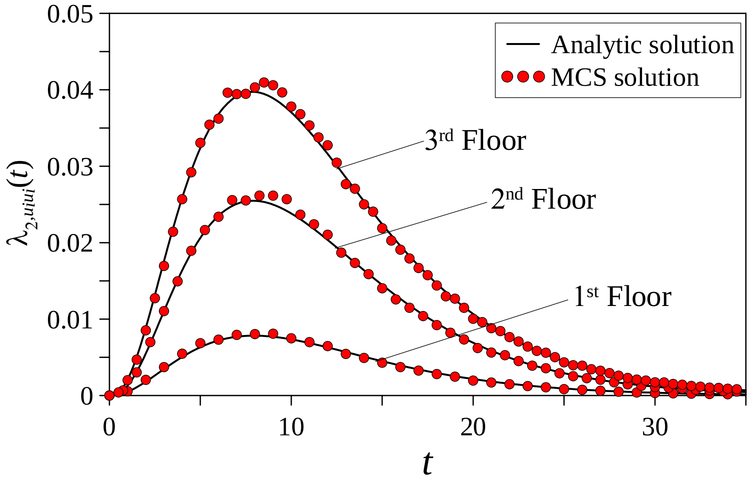

Figure 10. Comparison between the time-variant histories of the $\lambda_{2,u_iu_i}(t)$ NGSMs $\left[\mathrm{m}^2/\mathrm{s}^2\right]$, of the three relative to ground floor displacements, by applying the proposed analytical solution and the Monte Carlo Simulation (MCS).

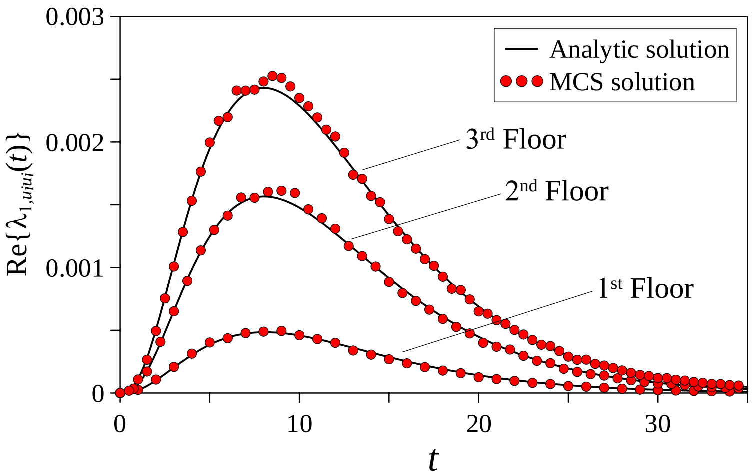

Figure 11. Comparison between the time-variant histories of the $\mathrm{Re}\left\{\lambda_{1,u_iu_i}(t)\right\}$ NGSMs $\left[\mathrm{m}^2/\mathrm{s}\right]$, of the three relative to ground floor displacements, by applying the proposed analytical solution and the Monte Carlo Simulation (MCS).

5. CONCLUSIONS

The probabilistic analysis of structural systems subjected to seismic excitations requires the spectral characterization of both the input excitation and the structural response; in this paper the fully non-stationary spectrum compatible model has been adopted to define the ground motion acceleration process. Via this procedure, through a handy recursive formula, is it possible to simulate fully non-stationary earthquakes in agreement with code provisions. As results from the comparison with real recorded accelerograms, the proposed model is able to catch the time varying mean frequency together with the variation in time of the frequency content.

Once the characterization of the ground motion acceleration is done, it is possible to analyze the safety of structural systems subjected to those excitations; in particular, the largest absolute value peak of stochastic response is a useful design information. In order to define the statistics of the maximum of the stochastic response of structures, it is necessary to evaluate the Non-Geometric Spectral Moments (NGSMs).

In this paper, the validation of the proposed model is done by the comparison of the statistics of structural response obtained by the Monte Carlo Simulation of spectrum compatible non-stationary accelerograms with the NGSMs evaluated in time domain by very handy formulas, recently proposed by the authors.

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