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RELIABILITY ASSESSMENT OF INTERVAL UNCERTAIN STRUCTURAL SYSTEMS SUBJECTED TO SPECTRUM COMPATIBLE SEISMIC EXCITATIONS

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Abstract. The present paper deals with reliability assessment of linear structures with uncertain parameters subjected to seismic excitations modeled as stationary spectrum compatible random Gaussian processes. Structural uncertainties are described by applying the interval model, stemming from the *interval analysis*. Under the Vanmarcke assumption that the up-crossings of a specified threshold occur in clumps, an efficient procedure for the evaluation of the bounds of the interval *reliability function* of the generic response process is presented. The key idea is to consider the interval *reliability function* as depending on the zero-, first- and second-order interval spectral moments of the stationary response rather than on the interval structural parameters. This allows to determine the bounds of the interval *reliability function* for a given barrier level as the minimum and maximum among the values pertaining to the eight combinations of the bounds of the interval spectral moments of the response. The effectiveness of the proposed approach lies in the evaluation of the bounds of the interval spectral moments of the response in approximate explicit form. To this aim, the so-called *Interval Rational Series Expansion* is applied in conjunction with the *improved interval analysis*.

For validation purposes, numerical results concerning the region of the interval *reliability function* of a spatial structure with interval stiffness properties subjected to stationary spectrum compatible seismic excitation are presented.

1 INTRODUCTION

Real structures are design to fulfill prescribed safety requirements under environmental loads, such as earthquake ground motion, sea waves or gusty winds, commonly modeled as random processes. Uncertainties affecting structural parameters play a crucial role in reliability assessment of randomly excited structures. Over the past decades, well-established probabilistic methods have been developed to analyze the effects of uncertainties on structural performance. The main drawback of traditional probabilistic approaches is the high sensitivity of the probabilities of failure to small variations of the assumed probabilistic models [1,2]. This implies that the credibility of reliability estimates becomes questionable when available data are insufficient to build reliable probabilistic distributions of the uncertain parameters.

After the pioneering study by Ben-Haim [1], who first introduced a non-probabilistic concept of reliability, the application of non-traditional uncertainty models to structural safety assessment has increasingly spread (see e.g. [3-6]). Among these approaches, the interval model, originally developed on the basis of the *interval analysis* [7], turns out to be very useful for handling non-deterministic properties described by range information only. To the best of the authors' knowledge, available contributions in the field of non-probabilistic reliability analysis mainly focus on static problems, whereas only a few papers deal with the reliability assessment of randomly excited structures (see e.g. [8-10]).

This study presents an efficient non-probabilistic procedure for reliability analysis of linear structures with uncertain parameters subjected to seismic excitations modeled as stationary Gaussian spectrum compatible random processes. Structural uncertainties are represented as interval variables according to the interval model. The probability of failure is identified with the first passage probability, under the Vanmarcke assumption that up-crossings of a specified threshold occur in clumps [11]. The *reliability function* of a selected response process turns out to be an interval function. In particular, it is conveniently considered as depending on three interval quantities, say the interval spectral moments of the response of zero-, first- and second-order. Then, for a given barrier level, the bounds of the interval *reliability function* are determined as the minimum and maximum among the values of the *reliability function* corresponding to all possible combinations of the endpoints of the interval spectral moments, say 2^3. The bounds of the interval spectral moments are evaluated in approximate explicit form by applying the *improved interval analysis* [12] combined with the *Interval Rational Series Expansion* (*IRSE*) for deriving the explicit approximate inverse of an interval matrix with small rank r modifications. The proposed approach thus allows a drastic reduction of the computational effort required by the classical combinatorial procedure, known as *vertex method*. Indeed, for a structure with r interval parameters, the *vertex method* involves 2^r reliability analyses, as many as are the combinations of the endpoints of uncertainties. Conversely, the proposed approach requires: *i*) to determine the bounds of the interval spectral moments of the response; *ii*) to evaluate the *reliability function* corresponding to the 2^3 combinations of the bounds of the interval spectral moments of the response and then seek the minimum and maximum for a given barrier level.

For validation purpose, a spatial frame with interval Young's moduli under spectrum compatible seismic excitation is analyzed. The proposed bounds of the interval *reliability function* are compared with the ones provided by the *vertex method*.

2 FORMULATION OF THE PROBLEM

Let us consider an idealized multi-storey building with rigid floor diaphragms where the floor masses are lumped. The lateral resistance is provided by resisting frames in the x and y

directions. The structure has three degrees of freedom (DoF) for each floor: two translations along the *x* and *y* axes and a rotation around the vertical axis placed in the origin *O* of the reference system, which is assumed the same for each floor.

Let the building be subjected to a horizontal ground acceleration $\ddot{u}_{\mathrm{g}}(t)$ modelled as a stationary spectrum compatible zero-mean Gaussian stochastic process, fully characterized from a probabilistic point of view by the one-sided *power spectral density* (*PSD*) function $G_{\ddot{u}_{\mathrm{g}}\ddot{u}_{\mathrm{g}}}(\omega)$. Practically, it is assumed that the input *PSD* function is compatible with the elastic target pseudo-acceleration spectrum $S_e(T)$. The line of action of ground acceleration is defined by the angle β_{g} between the epicentral direction and the x axis. Furthermore, the stiffness properties of the structure are assumed uncertain and are represented as interval variables according to the interval model of uncertainty.

Under the previous assumptions, the equations of motion of the linear quiescent *n*-floor building read:

$$\mathbf{M}\ddot{\mathbf{u}}(\boldsymbol{\alpha}^I,t)+\mathbf{C}(\boldsymbol{\alpha}^I)\dot{\mathbf{u}}(\boldsymbol{\alpha}^I,t)+\mathbf{K}(\boldsymbol{\alpha}^I)\mathbf{u}(\boldsymbol{\alpha}^I,t)=-\mathbf{M}\boldsymbol{\tau}\ddot{u}_g(t) \tag{1}$$

where the apex *I* characterizes the interval variables; $\boldsymbol{\alpha}^I$ is the interval vector collecting the r dimensionless interval parameters modelling the fluctuations of the stiffness properties around their nominal values; $\mathbf{M}$, $\mathbf{C}(\boldsymbol{\alpha}^I)$, and $\mathbf{K}(\boldsymbol{\alpha}^I)$ are the inertia, damping, and stiffness matrices of the structure; $\mathbf{u}(\boldsymbol{\alpha}^I,t)=\left\{\mathbf{u}_x^{\mathrm{T}}(\boldsymbol{\alpha}^I,t)\quad \mathbf{u}_y^{\mathrm{T}}(\boldsymbol{\alpha}^I,t)\quad \mathbf{u}_\theta^{\mathrm{T}}(\boldsymbol{\alpha}^I,t)\right\}^{\mathrm{T}}$ is the vector of floor displacements relative to the ground, collecting the *n* translational components in the *x* and *y* directions, $u_{x,j}$ and $u_{y,j}$, respectively, and the *n* rotational components around the vertical axis, $u_{\theta,j}$, $(j=1,2,\dots,n)$; a dot over a variable denotes differentiation with respect to time *t* and the apex T means transpose matrix; finally, $\boldsymbol{\tau}$ is the $3n$ array listing the influence coefficients of the ground shaking. The Rayleigh model is herein adopted for the interval damping matrix, i.e.:

$$\mathbf{C}(\boldsymbol{\alpha}^I)=c_0\mathbf{M}+c_1\mathbf{K}(\boldsymbol{\alpha}^I) \tag{2}$$

where c_0 and c_1 are the Rayleigh damping constants having units s^{-1} and s, respectively.

Denoting by $\mathbb{IR}$ the set of all closed real interval numbers, the dimensionless uncertain parameter vector is a bounded set-interval vector of real numbers, $\boldsymbol{\alpha}^I \triangleq [\underline{\boldsymbol{\alpha}},\overline{\boldsymbol{\alpha}}]\in \mathbb{IR}^r$, such that $\underline{\boldsymbol{\alpha}}\le\boldsymbol{\alpha}\le\overline{\boldsymbol{\alpha}}$. The symbols $\underline{\boldsymbol{\alpha}}$ and $\overline{\boldsymbol{\alpha}}$ denote the *lower bound* (*LB*) and *upper bound* (*UB*) vectors. By applying the interval algebra formalism, the *i*-th element of the interval vector $\boldsymbol{\alpha}^I$ can be defined as $\alpha_i^I \triangleq [\underline{\alpha}_i,\overline{\alpha}_i]$, where $\alpha_i^I\in\mathbb{IR}$, $\underline{\alpha}_i$ and $\overline{\alpha}_i$ are the *LB* and *UB* of the *i*-th fluctuation, respectively. According to the so-called *improved interval analysis* [12], the dimensionless fluctuation of the *i*-th uncertain parameter $d_i^I=d_{0,i}(1+\alpha_i^I)$ around its nominal value $d_{0,i}$ can be represented by the following symmetric interval variable:

$$\alpha_i^I=\alpha_{0,i}+\Delta\alpha_i\,\hat{e}_i^I=\Delta\alpha_i\,\hat{e}_i^I,\quad (i=1,2,\dots,r) \tag{3}$$

where $\hat{e}_i^I\triangleq[-1,+1]$ is the *EUI* associated to α_i^I and

$$\alpha_{0,i} = \frac{1}{2}\left(\underline{\alpha}_i + \overline{\alpha}_i\right) = 0; \quad \Delta\alpha_i = \frac{1}{2}\left(\overline{\alpha}_i - \underline{\alpha}_i\right) > 0 \tag{4a,b}$$

denote the midpoint value (or mean) and the deviation amplitude (or radius), respectively. To assure physically meaningful values of the uncertain structural properties, the deviation amplitudes $\Delta\alpha_i$ must satisfy the conditions $\Delta\alpha_i < 1$.

According to the interval symbolism, a generic interval-valued function f and a generic interval-valued matrix function $\mathbf{A}$ of the interval vector $\boldsymbol{\alpha}^I$ will be denoted in equivalent form, respectively, as:

$$\begin{aligned} f^I &\equiv f(\boldsymbol{\alpha}^I) \Leftrightarrow f(\boldsymbol{\alpha}), \quad \boldsymbol{\alpha} \in \boldsymbol{\alpha}^I = \left[\underline{\boldsymbol{\alpha}}, \overline{\boldsymbol{\alpha}}\right]; \\ \mathbf{A}^I &\equiv \mathbf{A}(\boldsymbol{\alpha}^I) \Leftrightarrow \mathbf{A}(\boldsymbol{\alpha}), \quad \boldsymbol{\alpha} \in \boldsymbol{\alpha}^I = \left[\underline{\boldsymbol{\alpha}}, \overline{\boldsymbol{\alpha}}\right]. \end{aligned} \tag{5a,b}$$

3 FREQUENCY DOMAIN ANALYSIS

The interval random response process of linear structures with uncertain-but-bounded parameters under zero-mean stationary stochastic Gaussian excitation is completely characterized in the frequency domain by the one-sided interval *PSD* function matrix, $\mathbf{G}_{\mathbf{uu}}(\boldsymbol{\alpha}^I, \omega)$. By applying interval extension, such matrix is given by the following relationship [13]:

$$\mathbf{G}_{\mathbf{uu}}(\boldsymbol{\alpha}, \omega) = G_{\ddot{u}_g \ddot{u}_g}(\omega)\ \mathbf{H}^*(\boldsymbol{\alpha}, \omega)\mathbf{p}\,\mathbf{p}^{\mathrm{T}}\,\mathbf{H}^{\mathrm{T}}(\boldsymbol{\alpha}, \omega), \quad \boldsymbol{\alpha} \in \boldsymbol{\alpha}^I = \left[\underline{\boldsymbol{\alpha}}, \overline{\boldsymbol{\alpha}}\right] \tag{6}$$

where $\mathbf{p} = -\mathbf{M}\,\boldsymbol{\tau}$, the asterisk means complex conjugate and $\mathbf{H}(\boldsymbol{\alpha}^I, \omega)$ is the *interval frequency response function* (*FRF*) matrix (also referred to as *transfer function matrix*), defined as follows:

$$\mathbf{H}^I(\omega) \equiv \mathbf{H}(\boldsymbol{\alpha}^I, \omega) = \left[-\omega^2\mathbf{M} + \mathrm{j}\omega\mathbf{C}^I + \mathbf{K}^I\right]^{-1} \tag{7}$$

where $\mathrm{j} = \sqrt{-1}$ denotes the imaginary unit. Recently, the authors [14, 15] proposed a procedure for evaluating the interval *FRF* matrix and the associated statistics of the interval stationary response in approximate explicit form. This procedure relies on the use of a novel series expansion, called *Interval Rational Series Expansion* (*IRSE*), which provides an approximate explicit expression of the inverse of an invertible interval matrix with a rank-*r* modification. In order to derive the *IRSE*, it is observed that the interval stiffness matrix $\mathbf{K}^I$ of a linear elastic structure can always be expressed as a linear function of the uncertain parameters. Based on this observation, the interval stiffness and damping matrices can be expressed as:

$$\mathbf{K}^I = \mathbf{K}_0 + \sum_{i=1}^{r}\mathbf{K}_i\,\Delta\alpha_i\,\hat{e}_i^I; \quad \mathbf{C}^I = \mathbf{C}_0 + c_1\sum_{i=1}^{r}\mathbf{K}_i\,\Delta\alpha_i\,\hat{e}_i^I \tag{8a,b}$$

where

$$\mathbf{K}_0 = \mathbf{K}(\boldsymbol{\alpha})\big|_{\boldsymbol{\alpha}=\mathbf{0}}; \quad \mathbf{K}_i = \left.\frac{\partial\mathbf{K}(\boldsymbol{\alpha})}{\partial\alpha_i}\right|_{\boldsymbol{\alpha}=\mathbf{0}}; \quad \mathbf{C}_0 = c_0\mathbf{M} + c_1\mathbf{K}_0. \tag{9a-c}$$

In the previous equations, $\mathbf{K}_0$ and $\mathbf{C}_0$ denote the stiffness and damping matrices of the nominal structural system, which are positive definite symmetric matrices of order $3n \times 3n$; $\mathbf{K}_i$ are positive semi-definite symmetric matrices of order $3n\times 3n$ and rank p_i; $\Delta\alpha_i$ is the dimensionless deviation amplitude of the i-th uncertain parameter satisfying the condition $\Delta\alpha_i < 1$.

The starting point to derive the *IRSE* is the decomposition of the $3n\times 3n$ matrix $\mathbf{K}_i$ in Eq. (9b) as sum of rank-one matrices, i.e.:

$$\mathbf{K}_i = \sum_{\ell=1}^{p_i} \lambda_i^{(\ell)} \mathbf{v}_i^{(\ell)} \mathbf{v}_i^{(\ell)T}; \qquad \mathbf{v}_i^{(\ell)} = \mathbf{K}_0 \boldsymbol{\psi}_i^{(\ell)} \tag{10a,b}$$

$\boldsymbol{\psi}_i^{(\ell)}$ and $\lambda_i^{(\ell)}$ being the ℓ-th eigenvector and the associated eigenvalue, solutions of the following eigenproblem:

$$\mathbf{K}_i \boldsymbol{\psi}_i^{(\ell)} = \lambda_i^{(\ell)} \mathbf{K}_0 \boldsymbol{\psi}_i^{(\ell)}, \qquad (i=1,\dots,r;\ \ell=1,\dots,p_i). \tag{11}$$

The eigenvalues $\lambda_i^{(\ell)}$ are real positive numbers. Furthermore, the eigenvectors $\boldsymbol{\psi}_i^{(\ell)}$ are assumed to satisfy the orthonormalization condition:

$$\boldsymbol{\Psi}_i^{\mathrm{T}} \mathbf{K}_0 \boldsymbol{\Psi}_i = \mathbf{I}_{p_i}; \quad \boldsymbol{\Psi}_i = \left[\boldsymbol{\psi}_i^{(1)} \quad \boldsymbol{\psi}_i^{(2)} \quad \cdots \quad \boldsymbol{\psi}_i^{(p_i)} \right], \tag{12}$$

so that the following relationship holds:

$$\boldsymbol{\Psi}_i^{\mathrm{T}} \mathbf{K}_i \boldsymbol{\Psi}_i = \boldsymbol{\Lambda}_i; \quad \boldsymbol{\Lambda}_i = \mathrm{Diag}\left[\lambda_i^{(1)}, \quad \lambda_i^{(2)}, \quad \dots \quad \lambda_i^{(p_i)} \right]. \tag{13}$$

Notice that only $p_i < 3n$ eigenvalues are different from zero and the generic term $\lambda_i^{(\ell)} \mathbf{v}_i^{(\ell)} \mathbf{v}_i^{(\ell)T}$ of the summation in Eq. (10a) provides a rank-one matrix. Based on the above decomposition of the matrix $\mathbf{K}_i$, the interval *FRF* matrix (7) takes the following form:

$$\mathbf{H}^I(\omega) = \left[\mathbf{H}_0^{-1}(\omega) + p(\omega) \sum_{i=1}^{r} \sum_{\ell=1}^{p_i} \lambda_i^{(\ell)} \mathbf{v}_i^{(\ell)} \mathbf{v}_i^{(\ell)T} \Delta\alpha_i\, \hat{e}_i^I \right]^{-1} \tag{14}$$

where $\mathbf{H}_0(\omega)$ is the *FRF* matrix of the nominal structural system and $p(\omega) = 1 + \mathrm{j}\omega c_1$. Notice that the deviation with respect to the inverse of the nominal *FRF* matrix, $\mathbf{H}_0^{-1}(\omega)$, is expressed as sum of $r \times p_i$ matrices of rank one. Then, the *IRSE* truncated to first-order terms gives [15]:

$$\mathbf{H}^I(\omega) \approx \mathbf{H}_0(\omega) - \sum_{i=1}^{r} \sum_{\ell=1}^{p_i} \frac{p(\omega) \lambda_i^{(\ell)} \Delta\alpha_i\, \hat{e}_i^I}{1 + p(\omega) \Delta\alpha_i\, \hat{e}_i^I \lambda_i^{(\ell)}\, b_{i\ell}(\omega)} \mathbf{B}_{i\ell}(\omega) = \mathbf{H}_{\mathrm{mid}}(\omega) + \mathbf{H}_{\mathrm{dev}}^I(\omega) \tag{15}$$

with

$$b_{i\ell}(\omega) = \mathbf{v}_i^{(\ell)T} \mathbf{H}_0(\omega)\, \mathbf{v}_i^{(\ell)}; \quad \mathbf{B}_{i\ell}(\omega) = \mathbf{H}_0(\omega)\, \mathbf{v}_i^{(\ell)} \mathbf{v}_i^{(\ell)T} \mathbf{H}_0(\omega). \tag{16a,b}$$

Equation (15) provides the interval *FRF* matrix as sum of the midpoint, $\mathbf{H}_{\mathrm{mid}}(\omega)$, plus the interval deviation, $\mathbf{H}_{\mathrm{dev}}^I(\omega)$, matrices given, respectively, by:

$$\begin{aligned} \mathbf{H}_{\mathrm{mid}}(\boldsymbol{\alpha}, \omega) &= \mathbf{H}_0(\omega) + \sum_{i=1}^{r} \sum_{\ell=1}^{p_i} a_{0,i\ell}(\omega)\, \mathbf{B}_{i\ell}(\omega); \\ \mathbf{H}_{\mathrm{dev}}^I(\omega) &= \sum_{i=1}^{r} \sum_{\ell=1}^{p_i} \Delta a_{i\ell}(\omega)\, \mathbf{B}_{i\ell}(\omega)\, \hat{e}_i^I = \sum_{i=1}^{r} \mathbf{R}_i(\omega)\, \hat{e}_i^I \end{aligned} \tag{17a,b}$$

with

$$\mathbf{R}_i(\omega) = \sum_{\ell=1}^{p_i} \Delta a_{i\ell}(\omega)\, \mathbf{B}_{i\ell}(\omega) \tag{18}$$

and

$$a_{0,i\ell}(\omega) = \frac{\left[p(\omega)\lambda_i^{(\ell)}\Delta\alpha_i\right]^2 b_{i\ell}(\omega)}{1-\left[p(\omega)\lambda_i^{(\ell)}\Delta\alpha_i\, b_{i\ell}(\omega)\right]^2};\quad \Delta a_{i\ell}(\omega) = \frac{p(\omega)\lambda_i^{(\ell)}\Delta\alpha_i}{1-\left[p(\omega)\lambda_i^{(\ell)}\Delta\alpha_i\, b_{i\ell}(\omega)\right]^2} \tag{19a,b}$$

where the argument $\Delta\alpha_i$ of the functions $a_{0,i\ell}(\omega)$ and $\Delta a_{i\ell}(\omega)$ is omitted for the sake of conciseness. The *IRSE* in Eq. (15) holds if and only if the conditions $\left\| p(\omega)\Delta\alpha_i\, \lambda_i^{(\ell)}\, b_{i\ell}(\omega)\right\| < 1$ are satisfied, where the symbol $\|\bullet\|$ means modulus of $\bullet$.

As known, the generic response quantity of practical interest, $Y(\boldsymbol{\alpha}, t)$, can be determined from the displacement vector $\mathbf{u}(\boldsymbol{\alpha}, t)$ by means of the following relationship:

$$Y(\boldsymbol{\alpha}, t) = \mathbf{q}^{\mathrm{T}}\, \mathbf{u}(\boldsymbol{\alpha}, t) \Rightarrow Y(\boldsymbol{\alpha}, \omega) = \mathbf{q}^{\mathrm{T}}\, \mathbf{U}(\boldsymbol{\alpha}, \omega) = \mathbf{q}^{\mathrm{T}}\mathbf{H}(\boldsymbol{\alpha}, \omega)\,\mathbf{p}\,\mathcal{F}\left\langle \ddot{u}_{\mathrm{g}}(t)\right\rangle, \quad \boldsymbol{\alpha}\in\boldsymbol{\alpha}^I = [\underline{\boldsymbol{\alpha}}, \overline{\boldsymbol{\alpha}}] \tag{20}$$

where $\mathbf{q}$ is a vector collecting the combination coefficients relating the response process $Y(\boldsymbol{\alpha}, t)$ to $\mathbf{u}(\boldsymbol{\alpha}, t)$; $\mathbf{U}(\boldsymbol{\alpha}, \omega)$ and $\mathcal{F}\left\langle \ddot{u}_{\mathrm{g}}(t)\right\rangle$ are the Fourier Transform of the displacement vector $\mathbf{u}(\boldsymbol{\alpha}, t)$ and ground acceleration $\ddot{u}_g(t)$, respectively. Depending on the selected response quantity, $Y(\boldsymbol{\alpha}, t)$, the vector $\mathbf{q}$ may depend on the uncertain parameters; such dependency is here omitted for the sake of brevity.

The interval spectral moments of order ℓ of the interval random response process $Y(\boldsymbol{\alpha}^I, t)$, useful for structural reliability evaluation, can be computed as [10]:

$$\begin{aligned} \lambda_{\ell,Y}(\boldsymbol{\alpha}) &= \int_0^\infty \omega^\ell\, G_{YY}(\boldsymbol{\alpha}, \omega)\,\mathrm{d}\omega = \mathbf{q}^{\mathrm{T}} \int_0^\infty \omega^\ell\, \mathbf{G}_{\mathbf{uu}}(\boldsymbol{\alpha}, \omega)\,\mathrm{d}\omega\,\mathbf{q} \\ &= \mathrm{mid}\left\{\lambda_{\ell,Y}(\boldsymbol{\alpha})\right\} + \mathrm{dev}\left\{\hat{\lambda}_{\ell,Y}(\boldsymbol{\alpha})\right\}, \quad \boldsymbol{\alpha}\in\boldsymbol{\alpha}^I = [\underline{\boldsymbol{\alpha}}, \overline{\boldsymbol{\alpha}}]; \quad \ell = 0, 1, 2 \end{aligned} \tag{21}$$

where $G_{YY}^I(\omega) \equiv G_{YY}(\boldsymbol{\alpha}^I, \omega) = \mathbf{q}^{\mathrm{T}}\mathbf{G}_{\mathbf{uu}}^I(\omega)\mathbf{q}$ is the interval *PSD* function of $Y(\boldsymbol{\alpha}^I, t)$. Substituting the approximate interval *FRF* matrix, $\mathbf{H}^I(\omega)$, given by Eq. (15) into Eq.(6) and then the resulting *PSD* function matrix $\mathbf{G}_{\mathbf{uu}}^I(\omega)$ into Eq. (21), the following expressions of the *LB* and *UB* of the interval spectral moments of the random process $Y(\boldsymbol{\alpha}^I, t)$ are obtained:

$$\underline{\lambda}_{\ell,Y}(\boldsymbol{\alpha}) = \mathrm{mid}\left\{\lambda_{\ell,Y}^I\right\} - \Delta\hat{\lambda}_{\ell,Y}(\boldsymbol{\alpha}); \qquad \overline{\lambda}_{\ell,Y}(\boldsymbol{\alpha}) = \mathrm{mid}\left\{\lambda_{\ell,Y}^I\right\} + \Delta\hat{\lambda}_{\ell,Y}(\boldsymbol{\alpha}), \quad \ell = 0, 1, 2 \tag{22a,b}$$

where

$$\Delta\hat{\lambda}_{\ell,Y}(\boldsymbol{\alpha}) = \sum_{i=1}^{r}\left| \int_0^\infty \omega^\ell\, G_{\ddot{u}_{\mathrm{g}}\ddot{u}_{\mathrm{g}}}(\omega)\, \mathbf{q}^{\mathrm{T}}\left[\mathbf{H}_{\mathrm{mid}}^*(\boldsymbol{\alpha}, \omega)\,\mathbf{p}\,\mathbf{p}^{\mathrm{T}}\mathbf{R}_i^{\mathrm{T}}(\omega) + \mathbf{R}_i^*(\omega)\,\mathbf{p}\,\mathbf{p}^{\mathrm{T}}\,\mathbf{H}_{\mathrm{mid}}^{\mathrm{T}}(\boldsymbol{\alpha}, \omega)\right]\mathbf{q}\,\mathrm{d}\omega\right|. \tag{23}$$

The over hat in Eqs. (21) - (23) means that, in order to simplify interval computations, terms associated with powers of $\Delta\alpha_i$ greater than one are neglected [10].

4 INTERVAL RELIABILITY FUNCTION

The probability of failure, for structural systems subjected to stochastic excitations, is commonly identified with the first passage probability, i.e. the probability that the *extreme value* random process, $Y_{\max}(\boldsymbol{\alpha},T)$, for the generic structural response process of interest, $Y(\boldsymbol{\alpha},t)$, (e.g. displacement, strain or stress at a critical point), firstly exceeds the safety bounds within a specified time interval $[0,T]$. For a structure with uncertain-but-bounded parameters, the *extreme value* random process, over a time interval $[0,T]$, is mathematically defined as:

$$Y_{\max}(\boldsymbol{\alpha},T) = \max_{0\le t\le T}\left|Y(\boldsymbol{\alpha},t)\right|, \;\; \boldsymbol{\alpha}\in\boldsymbol{\alpha}^I = [\underline{\boldsymbol{\alpha}},\overline{\boldsymbol{\alpha}}] \tag{24}$$

where the symbol $|\bullet|$ denotes absolute value. The *cumulative distribution function* (*CDF*) $L_{Y_{\max}}(\boldsymbol{\alpha},b,T)$ of the *extreme value* random process, $Y_{\max}(\boldsymbol{\alpha},T)$, also called *reliability function*, represents the probability that $Y_{\max}(\boldsymbol{\alpha},T)$ is equal to or less than the barrier level b within the time interval $[0,T]$. By applying interval extension to the *reliability function* obtained by Vanmarcke [11] for zero-mean stationary narrow band processes, the following expression of the interval *reliability function* $L^I_{Y_{\max}}(b,T)$ is obtained:

$$\begin{aligned} L^I_{Y_{\max}}(b,T) &= \mathcal{P}\left[Y^I_{\max}(T)\le b\right] \approx \exp\left[-T\,\frac{1}{\pi}\sqrt{\frac{\lambda^I_{2,Y}}{\lambda^I_{0,Y}}}\exp\left(-b\left(\delta^I_Y\right)^{1.2}\sqrt{\frac{\pi}{2\lambda^I_{0,Y}}}\right)\right] \\ &\equiv L^I_{Y_{\max}}\left(b,T;\lambda^I_{0,Y},\lambda^I_{1,Y},\lambda^I_{2,Y}\right) \end{aligned} \tag{25}$$

where unitary initial interval probability is assumed and δ^I_Y denotes the bandwidth interval parameter defined as [11]:

$$\delta^I_Y = \sqrt{1-\frac{\left(\lambda^I_{1,Y}\right)^2}{\lambda^I_{0,Y}\,\lambda^I_{2,Y}}}. \tag{26}$$

Within the interval framework, the aim of reliability analysis is to evaluate the *LB* and *UB* of the interval *reliability function*, $L^I_{Y_{\max}}(b,T)$, for the selected *extreme value* random process, $L^I_{Y_{\max}}(b,T)$. Since the interval *CDF*, $L^I_{Y_{\max}}(b,T)$, is a monotonic function of the generic uncertain parameter α^I_i, its exact bounds can be determined by applying a combinatorial procedure, known as *vertex method*, which requires to evaluate the *reliability function* of the selected *extreme value* process, $Y_{\max}(\boldsymbol{\alpha},T)$, for all the combinations of the bounds of the r uncertain parameters α^I_i, say 2^r, and then take, for a fixed barrier level b, the maximum and minimum value among all the *CDFs* so obtained. Unfortunately, this method becomes prohibitive for real-sized structures involving a large number of uncertainties.

The key idea of the proposed approach is to consider the interval *reliability function* as depending on three interval parameters, say $\lambda^I_{0,Y}$, $\lambda^I_{1,Y}$ and $\lambda^I_{2,Y}$ rather than on the r interval variables α^I_i. This provides substantial computational savings since the bounds of the interval *CDF* can be evaluated by a combinatorial approach performing only 2^3 evaluations of the *CDF* corresponding to all possible combinations of the bounds of the three intervals $\lambda^I_{0,Y}$, $\lambda^I_{1,Y}$

and $\lambda^I_{2,Y}$, defined in explicit form by Eqs. (22a,b). Specifically, the bounds of the interval *reliability function*, $L^I_{Y_{\max}}(b,T)$, can be evaluated as follows:

$$\underline{L}_{Y_{\max}}(b,T) = \min L_{Y_{\max}}\left(b,T;\lambda_{0,Y},\lambda_{1,Y},\lambda_{2,Y}\right); \quad \overline{L}_{Y_{\max}}(b,T) = \max L_{Y_{\max}}\left(b,T;\lambda_{0,Y},\lambda_{1,Y},\lambda_{2,Y}\right)$$
$$\text{subject to} \tag{27}$$
$$\lambda_{0,Y} \in \lambda^I_{0,Y} = [\underline{\lambda}_{0,Y},\overline{\lambda}_{0,Y}],\ \lambda_{1,Y} \in \lambda^I_{1,Y} = [\underline{\lambda}_{1,Y},\overline{\lambda}_{1,Y}],\ \lambda_{2,Y} \in \lambda^I_{2,Y} = [\underline{\lambda}_{2,Y},\overline{\lambda}_{2,Y}].$$

It is worth emphasizing that the described procedure requires only knowledge of the bounds of the interval spectral moments of the response process.

5 NUMERICAL APPLICATION

The effectiveness of the proposed procedure is assessed by analyzing the ten-storey spatial frame structure depicted in Fig.1 subjected to seismic excitation.

The seismic excitation is modelled here as a zero-mean stationary spectrum compatible Gaussian random process fully characterized from a probabilistic point of view by the one-sided *PSD* function $G_{\ddot{u}_g\ddot{u}_g}(\omega)$. The geometrical properties of the structure are reported in Fig. 1 and Table 1. The mass of each floor is $M = 360000$ Kg. Ten uncertain parameters are considered, namely, the Young's moduli of columns *a* and *b* on the first, second, third, ninth and tenth floor, which exhibit interval fluctuations α^I_i around the nominal value $E_0 = 2.5\times10^{10}\ \mathrm{N/m^2}$, i.e. $E^I_i = E_0(1+\Delta\alpha_i\,\hat{e}^I_i),\quad (i=1,2,\ldots,10)$. The values $c_0 = 0.22015\ \mathrm{s^{-1}}$ and $c_1 = 0.01048\ \mathrm{s}$ of the Rayleigh damping constants have been assumed in such a way that the modal damping ratio for the first and second modes of the nominal structure is $\zeta_0 = 0.05$.

The one-sided *PSD* of ground motion acceleration can be written in discretized form as:

$$G_{\ddot{u}_g\ddot{u}_g}(\omega_k) = \frac{4\zeta_0}{\omega_k\pi - 4\zeta_0\omega_{k-1}}\left(\frac{S^2_e(2\pi/\omega_k)}{\eta^2_U(\omega_k,\zeta_0)} - \Delta\omega\sum_{j=1}^{k-1}G_{\ddot{u}_g\ddot{u}_g}(\omega_j)\right) \tag{28}$$

where $\zeta_0 = 0.05$ is the damping ratio, $S_e\left(2\pi/\omega_k\right)$ is the target spectrum with $\omega_k = k\Delta\omega$ ($\Delta\omega = 0.01$) and cut-off frequency $\omega_f = 100\,\mathrm{rad/s}$; $\eta_U(\omega_k,\zeta_0)$ and δ_U are the peak factor and bandwidth factor, respectively expressed as:

$$\eta_U(\omega_k,\zeta_0) = \sqrt{2\ln\left\{\frac{T_s\,\omega_k}{\pi}\left(-\ln 0.5\right)^{-1}\left[1-\exp\left[-\delta_U^{1.2}\sqrt{\pi\ln\left(\frac{T_s\,\omega_k}{\pi}\left(-\ln 0.5\right)^{-1}\right)}\right]\right]\right\}};$$
$$\delta_U = \left[1-\frac{1}{1-\zeta_0^2}\left(1-\frac{2}{\pi}\arctan\frac{\zeta_0}{\sqrt{1-\zeta_0^2}}\right)^2\right]^{1/2} \tag{29a,b}$$

with time window $T_s = 20$ s. The target spectrum $S_e(2\pi/\omega_k)$ follows the Italian building code [16] and the parameters representing characteristics of the soil condition and of the structure are selected as $S = 1$, $\eta = 1$, $a_g = 0.237g$, $F_0 = 2.411$, $T_B = 0.1203$s, $T_C = 0.361$s and $T_D = 4a_g/(g+1)$.

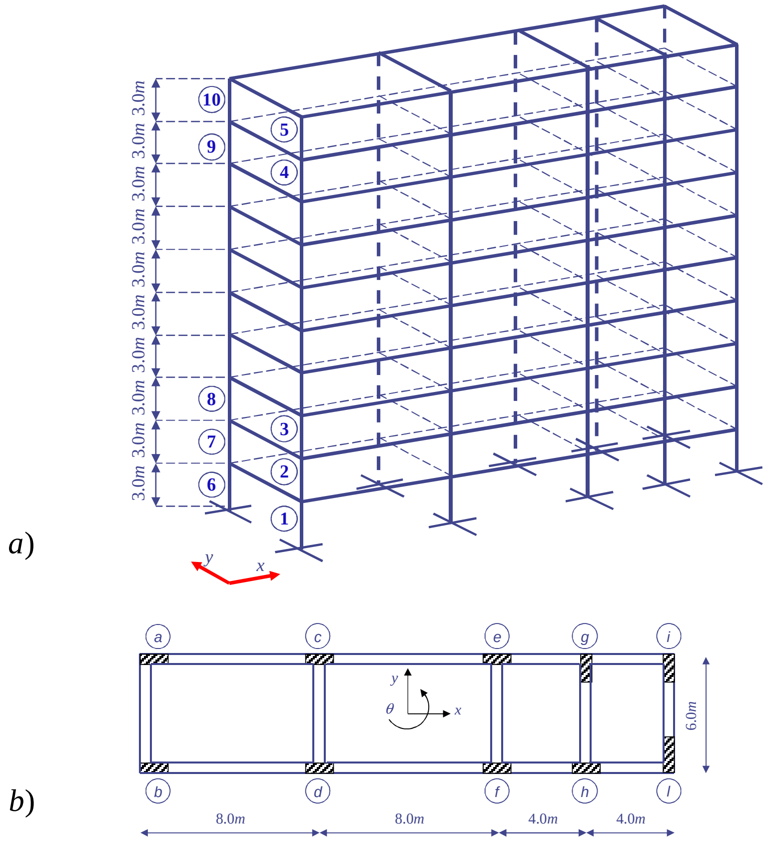

Figure 1: Ten-storey frame structure: a) 3D model and b) plan view.

Floor	Columns:a,b,c,d,e,f,g,h,i	Column:l
1-2	30 x 80 cm	30 x 100 cm
3-4	30 x 70 cm	30 x 90 cm
5-6	30 x 60 cm	30 x 80 cm
7-10	30 x 50 cm	30 x 70 cm

Table 1: Cross section of the columns for each floor.

The accuracy of the proposed approximate explicit expression of the *CDF*, $L_{Y_{\max}}(b;T)$, of the *extreme value* random process, $Y_{\max}(T)$ (see Eq.25) obtained by applying the *IRSE* is first demonstrated. Figure 2 displays the comparison between the exact and approximate *CDF* of the *extreme value* process, $Y_{\max} = U_{1x\max}$, of the horizontal displacement of the centre of gravity of the first floor evaluated setting the interval Young's moduli $E_i^I = E_0(1+\Delta\alpha_i\,\hat{e}_i^I)$ of the involved columns a and b at their upper bounds, that is $\hat{e}_i^I = +1$, with $\Delta\alpha_i = 0.15$

$(i=1,2,\ldots,10)$. In the same figure, the *CDF* pertaining to the nominal structure ($\Delta\alpha_i=0$) is also depicted. It is worth noting that the proposed analytical expression in Eq.(25) provides very accurate estimates of the *extreme value CDF* even for relatively large uncertainty levels, at least for the selected case study.

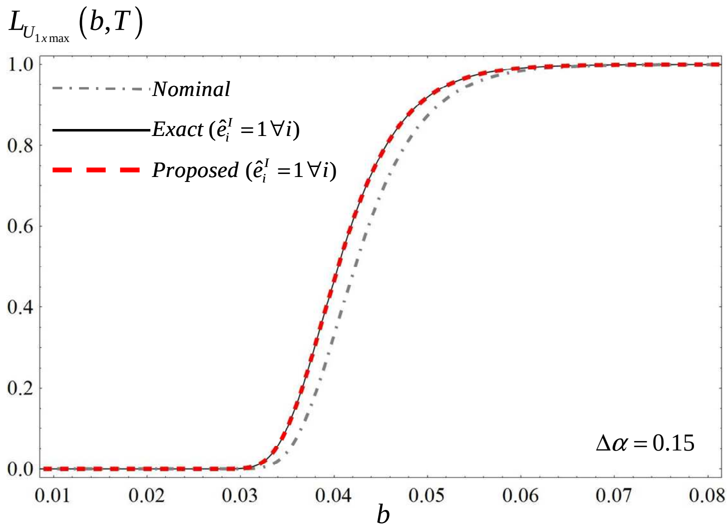

Figure 2: Comparison between the proposed and exact *CDF* of the *extreme value* process $Y_{\max}^I(T)=U_{1x\max}^I(T)$ of the horizontal displacement of the centre of gravity of the first floor evaluated setting the interval Young's moduli of the involved columns *a* and *b* at their upper bounds, i.e., $\hat{e}_i^I=+1, i=1,2,\ldots,10$ $(T=1000T_0)$

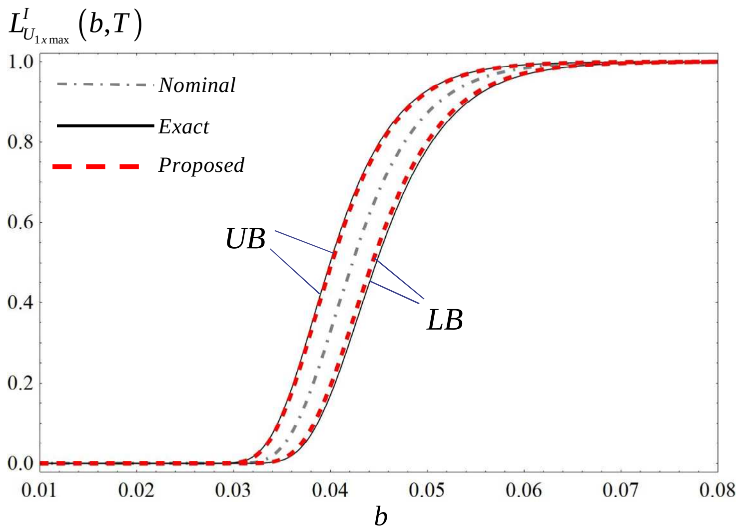

Figure 3: Comparison between the exact and proposed *UB* and *LB* of the *CDF* of the *extreme value* process $Y_{\max}^I(T)=U_{1x\max}^I(T)$ of the horizontal displacement of the centre of gravity of the first floor with interval Young's moduli $E_i^I=E_0(1+\Delta\alpha_i\,\hat{e}_i^I)$ for $\Delta\alpha_i=0.15$ for $i=1,2,\ldots,10$ $(T=1000T_0)$

Assuming that the uncertain parameters are bounded by intervals i.e. $E_i^I=E_0(1+\Delta\alpha_i\,\hat{e}_i^I)$, $(i=1,2,\ldots,10)$, with deviation amplitudes $\Delta\alpha_i=\Delta\alpha=0.15$, the accuracy of the proposed estimates of the bounds of the *CDF* of the *extreme value* process $Y_{\max}^I(T)$ is now analyzed.

The attention is again focused on the displacement component along the x-direction of the centre of gravity of the first floor, i.e. $Y^I_{\max}(T) = U^I_{1x\max}(T)$. In Fig. 3, the *UB* and *LB* of the *reliability function*, $L^I_{U_{1x\max}}(b,T)$, obtained by applying Eqs.(27a,b) are compared with the exact bounds evaluated following the philosophy of the *vertex method*. As shown in the figure, the excellent agreement between the estimates of the *UB* and *LB* confirms the validity of the proposed procedure.

6 CONCLUSIONS

Reliability analysis of linear structures with interval structural parameters subjected to seismic excitations modelled as spectrum compatible stationary Gaussian random processes has been addressed. Under the Vanmarcke assumption that up-crossing of a specified threshold occur in clumps, an efficient procedure for evaluating the bounds of the interval *reliability function* of the generic response process has been presented. The main feature of the proposed approach is that only knowledge of the bounds of the first three interval spectral moments of the response is required. Furthermore, such bounds are evaluated in approximate explicit form by applying the so-called *Interval Rational Series Expansion* in conjunction with the *Improved Interval Analysis*. Numerical results concerning a spatial frame with uncertain Young's moduli subjected to spectrum compatible seismic excitation have demonstrated the accuracy of the proposed procedure.

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