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UNCERTAINTY QUANTIFICATION OF DYNAMIC CHARACTERISTICS OF COMPOSITES – A FUZZY APPROACH

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Abstract. *The quantification of uncertainty in composite structures has intuitively significant threat to ensure structural reliability. Due to inherent complexities, composite structures are difficult to manufacture accurately according to its exact design specifications resulting in unavoidable uncertainties. Typical uncertainties are inadvertently induced due to intra-laminate voids, incomplete curing of resin, excess resin between plies, excess matrix voids, porosity, variations in material properties and fibre parameters. In general, random field models are extensively used to represent a spatially varying function. Different probabilistic approaches (Monte Carlo simulation, perturbation methods, random matrix, and generalized polynomial chaos with Karhunen-Loève expansion) are employed for composites. In a probabilistic setting, uncertainty associated with the system parameters can be modelled as random variables or stochastic processes using the so-called parametric approach. But in real-life situation due to the availability of limited sample data (crisp inputs), it will be more practical or realistic to follow non-probabilistic approach rather than probabilistic approach. In the present study, fuzzy approach is introduced to carry out the uncertainty propagation in natural frequencies of laminated composite plates using Gram-Schmidt Polynomial Chaos (PC). The proposed PC fuzzy model is integrated with finite element to predict the possible two extreme bound of responses for different degree of fuzziness. The fuzzy variable is represented as a set of interval variables via membership function. The most significant input parameters are identified and then fuzzified. Fuzzy analysis of the first three natural frequencies for typical laminate configuration is presented to illustrate the results and its performance.*

1 INTRODUCTION

On the basis of extent and nature of uncertainties exists in composite structures can be computationally analyzed in technical and scientific research. Most of the practical engineering problems related to composite materials are very complex and ill-defined to be modelled by conventional deterministic approach. Such problems contain fuzzy information of the system which is qualitative, linguistic, imprecise, vague or incomplete in nature. The predictive judgment of variation of inputs or boundary conditions significantly influences the interpretation and results of the analysis. In deterministic approach, all the system parameters are assumed to be precisely known but in reality, it is subjected to large amount of variability due to uncertainty of system input parameters such as geometric properties and material properties of the composites. In case of dynamic analysis, such variability plays a very significant extent on uncertainty in natural frequencies. This paper considers only the low frequency regime (first three natural frequencies).

In real-life problems, original Monte Carlo simulation is expensive due to high computational time. Therefore, the aim of the majority of current research is to reduce the computational cost. Under the possibilistic interpretation of fuzzy sets [1] and uncertainty environment [2], fuzzy variables would become a generalized interval variables. Consequently techniques employed in interval analysis such as classical interval arithmetic [3], affine analysis [4] or vertex theorems [5] can be used. The response surface based method [6] is also proposed for fuzzy analysis. In this context, recently fuzzy analysis is employed to deal with uncertainties in engineering problems using only available data [7]. Traditionally, Monte Carlo simulation approach is employed to obtain the uncertain random natural frequency wherein a large number of samples are taken into account. Although the uncertainty can be quantified by conventional Monte Carlo approach, it incurs high cost of computation for stochastic analysis. Gram-Schmidt polynomial chaos is employed in conjunction to the fuzzy analysis of composite structures to reduce the iteration time. This present algorithm allows derivation of polynomial chaos terms for random variables with arbitrary probability distribution functions. The obtained polynomial chaos expansion [8] acts as a surrogate model for the full finite element model of composite structure. The regression coefficients of the PCE are determined by first sampling in the space of input parameters (D-optimal design) and then by using a least square technique. In the present study, four layered graphite-epoxy composite laminated cantilever plate is considered as furnished in Figure 1. This paper numerically investigates the demonstration of application of the aforesaid approach for graphite epoxy cross-ply composite cantilever plate. Ply orientation angle, elastic modulus, mass density and shear modulus as random input variables and first three natural frequencies as fuzzy output are considered in the present study.

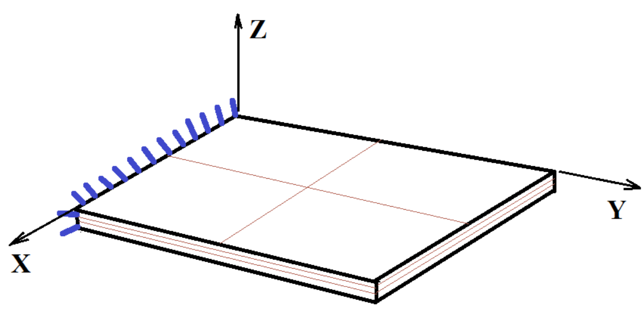


Fig. 1 Composite cantilever plate

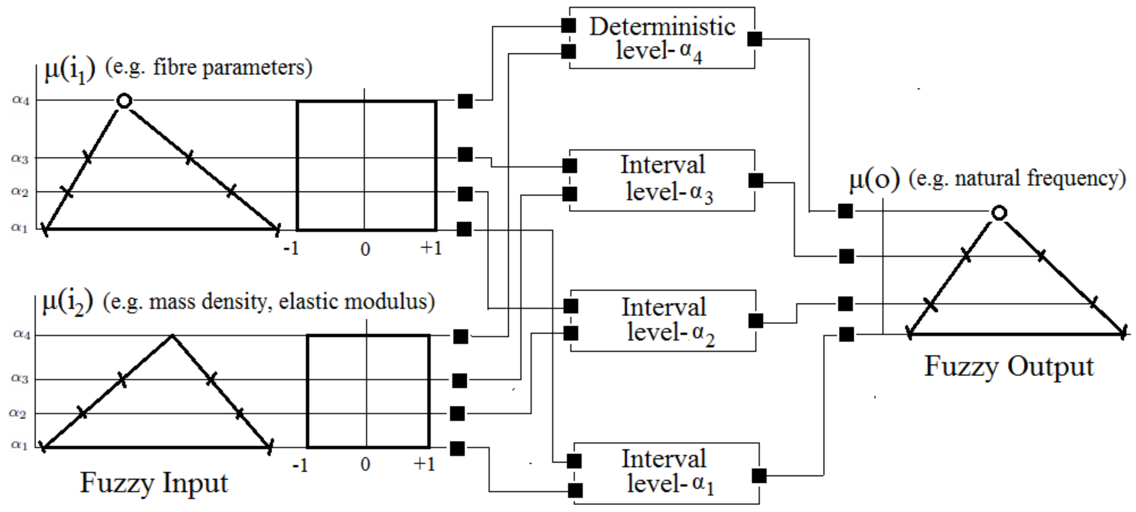


Fig. 2 Scheme for fuzzy FE analysis

2 FUZZY FORMULATION

The concept of fuzzy [9] introduces a set of transitional states between the members and non-members which are represented by membership function [$\mu_{f(i)}$] that indicates the degree to which each element in the domain belongs to the fuzzy set. The fuzzy number [$\hat{f}_i(\omega_\alpha)$] considering triangular membership function can be expressed as,

$$\hat{f}_i(\omega_\alpha) = [f_i^U, f_i^M, f_i^L] \quad (1)$$

where f_i^M, f_i^U and f_i^L denote the mean value, the upper bound and lower bounds, respectively. ω_α indicates the fuzziness corresponding to α-cut where α is known as membership grade or degree of fuzziness ranging from 0 to 1.

In this paper, the triangular shaped membership function is employed. The fuzzy input number f_i can be expressed into the set F_i of (n+1) intervals $f_i^{(j)}$ using the α-cut method

$$F_i(\omega_\alpha) = [f_i^{(0)}, f_i^{(1)}, f_i^{(2)}, f_i^{(3)} \ldots\ldots f_i^{(k)} \ldots\ldots f_i^{(n)}] \quad (2)$$

where n denotes the number of α-cut levels. The interval of the k-th level of the i-th fuzzy number is given by

$$f_i^{(k)} = [f_i^{(k,L)}, f_i^{(k,U)}] \quad (3)$$

where $f_i^{(k,L)}$ and $f_i^{(k,U)}$ denote the lower and upper bounds of the interval at the j-th level, respectively. At k=n, $f_i^{(n,L)} = f_i^{(n,U)} = f_i^N$. Here L and U denote the lower and upper bounds, respectively. In order to propagate uncertainty in a system where uncertain model parameters are represented by fuzzy input numbers, one may apply a numerical procedure of interval analysis at a number of α-levels [10]. In the present analysis, the orthogonal polynomial chaos basis functions, derived from Gram-Schmidt algorithm is employed for uncertainty propagation. The solution to fuzzy generalised equation at each α-level may be expanded into a polynomial chaos expansion as follows:

$$Y = G\ \boldsymbol{\psi}(\chi^{(\alpha)}) \qquad \text{for } \alpha=0,\ldots\ldots 1 \tag{4}$$

where Y denotes the assembled vector of output data, $\boldsymbol{\psi}(\chi^{(\alpha)})$ denotes the assembled vector of polynomial chaos basis functions and G indicates the matrices for the coefficients of polynomial expansion. Gram-Schmidt algorithm provides the opportunity to derive the polynomial chaos basis functions for arbitrary probability distribution on ‘ $\chi^{(\alpha)}$ ’. The interval variable ‘ $\chi^{(\alpha)}$ ’ is denoted by normalised random variable ‘ χ ’ with the following uniform probability distribution function

$$f(\chi) = \begin{Bmatrix} \frac{1}{2} & -1 < \chi < 1 \\ 0 & elsewhere \end{Bmatrix} \tag{5}$$

and equation (5) can be written as

$$Y = G\ \boldsymbol{\psi}(\chi) \tag{6}$$

The method of Gram-Schmidt algorithm is used to determine the polynomial chaos functions $\psi_k(\chi)$. The D-optimal design approach [11] is employed to evaluate the input design points for the respective samples at each α-cut and subsequently those values are called for finite element iteration. From Hamilton’s principle [12], the governing equations for the composite plate are derived based on Mindlin’s theory incorporating rotary inertia, transverse shear deformation to find the effect of propagation of uncertainty due to variation of input parameters (within tolerance limit) using eigenvalue problem [13] towards output natural frequencies from equation (6) for each α-cut

$$\lambda^{(k,l)}(\omega_\alpha) = \frac{1}{\{\omega_n(\omega_\alpha)\}^2} \tag{7}$$

3 FUZZY REPRESENTATION OF INPUT PARAMETERS

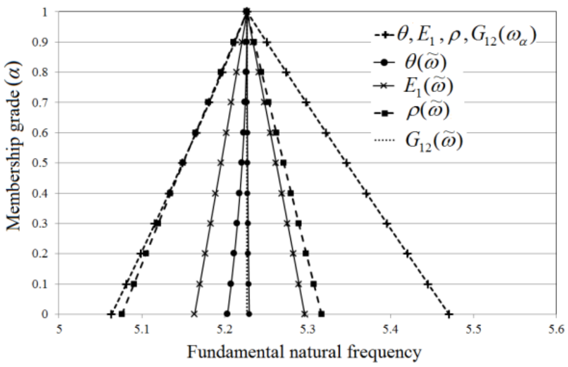
The fuzzy input parameters are considered at each layer of laminated composite cantilever plate. It is assumed that the distribution of fuzzy input parameters exists within a certain tolerance zone with their crisp values. The fuzzy input variables considered in each layer of laminate are for only variation of ply-orientation angle, longitudinal elastic modulus, mass density, longitudinal shear modulus and the combined variation of ply orientation angle, longitudinal elastic modulus, mass density and shear modulus (longitudinal). In present study, ± 5º for ply orientation angle and ± 10% tolerance for material properties respectively from fuzzy crisp values are considered. The membership grades are considered as 0 to 1 in step of 0.1. Figure 4 presents the flowchart of present fuzzy approach.

Ply orientation angle (θ)	Present FEM (6 x 6)	Present FEM (8 x 8)	Qatu and Leissa [14]
0°	1.0133	1.0107	1.0175
90°	0.2567	0.2547	0.2590

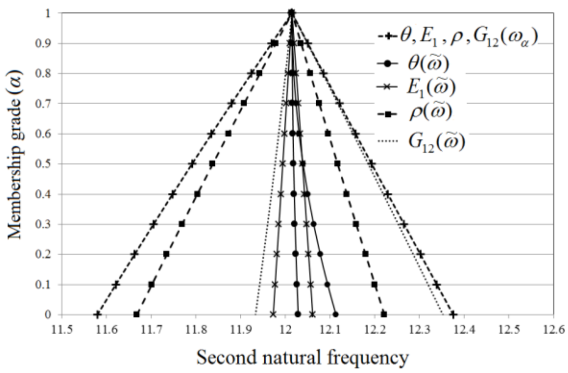
Table 1: Non-dimensional fundamental natural frequencies of three layered (θ°/-θ°/θ°) graphite-epoxy untwisted composite plates, a/b=1, b/t=100

4 RESULTS AND DISCUSSION

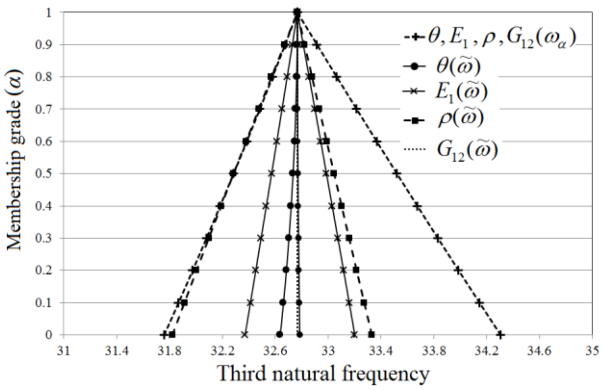
A four layered graphite-epoxy cross-ply (0°/90°/0°/90°) composite cantilever plates is considered in the present computational fuzzy investigation. Table 1 presents the convergence study of non-dimensional fundamental natural frequencies of three layered (θ°/-θ°/θ°) graphite-epoxy untwisted composite plates [14]. The present fuzzy model is employed to determine the non-probabilistic responses by predefined range of variations in input parameters. The fuzzy membership functions are used to determine the first three natural frequencies corresponding to given values of input variables with different degree of fuzziness.



(a)



(b)



(c)

Fig. 3 Fuzzy variation of first three natural frequencies for four layered graphite-epoxy symmetric cross-ply (0°/90°/90°/0°) composite cantilever plate considering E_1=138 GPa, E_2=8.9 GPa, G_{12}=G_{13}=7.1 GPa, G_{23}=2.84 GPa, ρ=3202 Kg/m^3, t=0.006 m, ν=0.3.

The uncertainty propagation of fuzzy variables can be carried out by global optimisation approach. The fuzzy polynomial chaos expansion (PCE) approach is adopted for uncertainty propagation in composite structures wherein the large number of fuzzy input variables are considered to optimize the upper and lower bound. From Figure 3, it is noted that $G_{12}(\tilde{\omega}_\alpha)$ has negligible effect on variation of fundamental natural frequency for cross-ply composite plate. In contrast, the ranges of second and third natural frequencies of cross-ply composite plates are in the order as $\rho(\tilde{\omega}_\alpha) > E_1(\tilde{\omega}_\alpha) > G_{12}(\tilde{\omega}_\alpha) > \theta(\tilde{\omega}_\alpha)$ (in case of only variation of any single input parameter) irrespective of α-cut.

5 CONCLUSIONS

In the present study, uncertainty quantification of first three natural frequencies with fuzzy variables is derived using finite element method. The computational time and cost is reduced by using fuzzy PCE approach. The maximum ranges of first three natural frequencies are consistantly found for combined variation of ply-orientation angle, elastic modulus, mass density and shear modulus compared to individual variation of any input parameter irrespective of fuzzy α-cut. The present study can be extended for future research to deal with more complex system considering a large number of fuzzy variables.

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