UNCECOMP 2015
1[st] ECCOMAS Thematic Conference on
International Conference on Uncertainty Quantification in
Computational Sciences and Engineering
M. Papadrakakis, V. Papadopoulos, G. Stefanou (eds.)
Crete Island, Greece, 25–27 May 2015

NUMERICAL ANALYSIS OF MASONRY STRUCTURES, TAKING INTO ACCOUNT MEASURED GEOMETRIC AND MATERIAL DATA

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Keywords: Masonry, Terrestrial Photogrammetry, Contact, Multi-scale, Homogenization.

Abstract. *Ideas related to the numerical analysis of masonry structures are presented in this article. Terrestrial photogrammetry methods are used for the exact representation of the geometry of the structures. Numerical models are then developed for the study of the mechanical behaviour of masonry. Non-linear finite element analysis models with principles taken from contact mechanics and damage laws are used for the evaluation of the ultimate load and collapse mechanism, in a macroscopic level. Furthermore, computational homogenization is used in a multi-scale framework, for the investigation of the heterogeneous nature of masonry in a mesoscopic level. Classical plasticity laws and/or delamination phenomena are taken into account for the representation of the failure of the material. The effective behaviour is transferred to the macroscopic scale. Applications to real masonry arch bridges and masonry walls are finally presented*

1 INTRODUCTION

Some aspects relates with the numerical analysis of masonry structures are presented in this article. A significant number of masonry structures and special arch bridges in Europe still survive and some of them are still being used, therefore a detailed analysis of these monuments is of great interest. Masonry arches consist of stone blocks and the mortar joints. Blocks have high strength in compression and low strength in tension while mortar has generally low strength. Other mechanical properties (like Young's modulus) are also different between the constitutive materials of these structures. Consequently, a great number of theories have been developed in the past, in order to capture this variation in the mechanical properties of masonry arches. In this framework continuum as well as discontinuous (discrete) models are used and finite element analysis with commercial packages or limit analysis schemes are adopted.

Terrestrial photogrammetry methods are used for the exact representation of the geometry of the structures. Numerical models are then developed for the study of the mechanical behaviour of masonry. Within this paper we report on an application of this technique on a model structure, the Cernadela Bridge in Spain. Non-linear finite element analysis models with principles taken from contact mechanics and damage laws are used for the evaluation of the ultimate load and collapse mechanism, in a macroscopic level.

The resulting combination of techniques allows us evaluate the structural health of an existing complex masonry structure by taking into account geometric data of high precision, mechanical models of adequate complexity and elements of inverse analysis. Further information, like material data from the interior of the structure, is taken into account.

Furthermore, computational homogenization is used in a multi-scale framework, for the investigation of the heterogeneous nature of masonry in a mesoscopic level. Concurrent analysis of the macroscopic and the microscopic structure is performed. According to the classical formulation of the method, two nested boundary value problems are concurrently solved. The initial heterogeneous macroscopic structure is equivalent with a homogeneous one, in each Gauss point of which, an Representative Volume Element (RVE) is corresponding. This RVE includes every heterogeneity and non-linearity of the material. Classical plasticity laws and/or delamination phenomena are taken into account for the representation of the failure of the material. The effective behaviour is transferred to the macroscopic scale. Representative examples related to masonry structures are presented in this paper.

2 TERRESTRIAL PHOTOGRAMMETRY

New photogrammetry techniques allow us measure structures of complex shapes and create accurate models for further structural analysis. Photogrammetry is defined as a method that allows the geometry of objects to be reconstructed from images, where the object has previously impressed. This is possible through the establishment of geometrical relationships between objects coordinates (in 3D space) and image coordinates (in 2D space) into a perspective system. This is usually performed through the collinearity condition that establishes that, at the time of exposure, a point in the object space, the perspective centre and the image coordinates of the point all lie in common straight line. This condition is drawn through the collinearity condition equations that are widely explained in [1]. As a summary of this, we can assume that is possible to obtain the 3D coordinates of the position of any point of object space by knowing the image coordinates of it.

Practical difficulties arising during the mentioned operation were discussed and practical structural analysis and evaluation tasks related to a masonry have already been published in [2].

On the other hand, ground penetrating radar (GPR) is a geophysical method that has been established as one of the most recommended non-destructive methods for routine sub-surface inspections. Regarding the evaluation of masonry structures, the GPR technology has demonstrated its potential to document and measure different inner structural characteristics, such as the dimensioning of wall thicknesses, the detection of internal faults like voids and cracks, as well as pathologies in construction, and also to locate hidden structures and former geometries [3, 4, 5, 6]

2.1 Geometrical modeling of a masonry bridge.

A structural evaluation of a masonry bridge in Spain, the Cernadela Bridge was considered as case study. Close range photogrammetry was used to obtain a set of three dimensional coordinates of points on the surface of the bridge in order to have the geometric basis of the model (fig. 1). Additionally, Ground Penetrating Radar was used to provide complementary data about wall thicknesses and backfill.

The methodology for the 3D modelling of Cernadela Bridge consisted of the following steps: Image acquisition, Topographic survey, GPR survey, Data processing

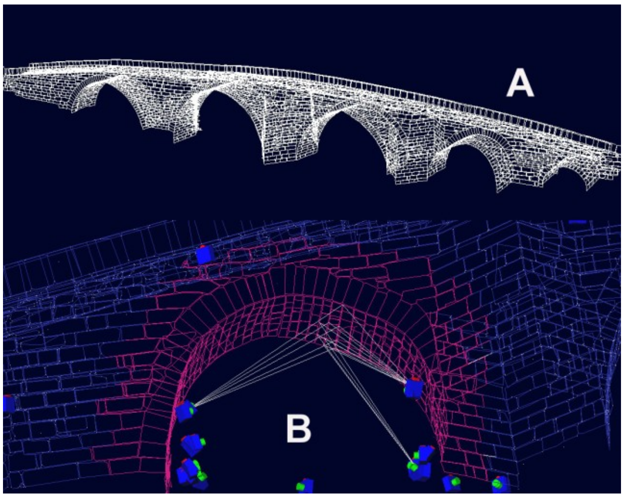

Fig. 1. (a) 3D wireframe model of the whole structure of Cernadela Bridge (b) Detailed model of second vault of the bridge with camera position and intersection rays of some points.

2.2 Creation of cad model

The geometry models initially developed from photogrammetry, are mainly consisted of point clouds and lines. For this reason, they are imported into appropriate computer aided design software with specialised surface processing tools, which are used for the creation of complex surfaces, solids and sets of parts. This improved version of the geometry model is finally imported into classical finite element analysis packages for the structural assessment of the bridge.

3 NON-LINEAR FINITE ELEMENT ANALYSIS

3.1 Interface modeling versus continuous damage mechanics

The finite element model, which has been already used in the past for the study of masonry arches [9, 10], is based on the principles of non-smooth mechanics [7, 8, 9]. Validation of the proposed methodology comes from the comparison between the computational and the real, damaged geometry of the structure. The computational models which have been developed in the past can be roughly divided into two large categories: (a) Discrete models and (b) Continuum models.

In a discrete model formulation the structure is divided into large discrete deformable parts connected with interfaces. The behaviour of the contact surface in each interface is described by a unilateral law, possibly with friction, while the discrete elements are assumed to behave elastically. Detailed discontinuous finite element model incorporating principles taken from non-smooth mechanics like unilateral contact and friction, have been presented among others in [7, 9, 10, 11].

From another point of view, the mechanical behaviour of continuum models is described by a nonlinear constitutive law, where either the masonry is assumed to consist of a single material and its behaviour is described by an inelastic theory (for instance an appropriately modified damage model) [12], or the different mechanical behaviour between stone and mortar and the anisotropy induced by them are taken into account on the basis of a homogenization theory [13].

Experience accumulated from the discontinuous modelling approach, indicated the fact that consideration of potential cracks as macroscopic interfaces, although is a quite realistic method for the representation of the mechanical behaviour, however requires the handling of difficult numerical schemes, in case the method is expanded to large, complex structures with more complicated pattern of interfaces. Moreover, the computational cost towards this effort would be significant for a large scale structure, for instance a three-dimensional model of a multi-ring stone arch bridge. For these reasons, a first attempt for a correlation between the discrete macroscopic approach described above and a continuous damage model, was made by the authors of this study in [14].

3.2 Structural finite element analysis of masonry bridge

Two similar continuum damage models are used for the determination of the ultimate behaviour of the Cernadela stone arch bridge. The size of this structure is quite large and its geometry is complex, as it is consisted of five stone arches. The model is validated by the usage of another continuum model and a two dimensional parametric analysis in a single arch of the bridge. A discrete model formulation is also used for the comparison between the results obtained in the single arch. Finally the smeared crack concrete finite element model is used for the simulation of the whole arch.

Several finite element models have been developed, for the simulation of the Cernadela Bridge. Four of them are used for the simulation of a single arch of the structure and a fifth one for the investigation of the behaviour of the whole bridge. Within the first four models, a parametric investigation of the tensile strength of the masonry and of the width of the arch has been considered. The two damage models as well as the discrete model described above have been used in these analyses. Finally, the proper material parameters have been chosen for implementation on the whole structure.

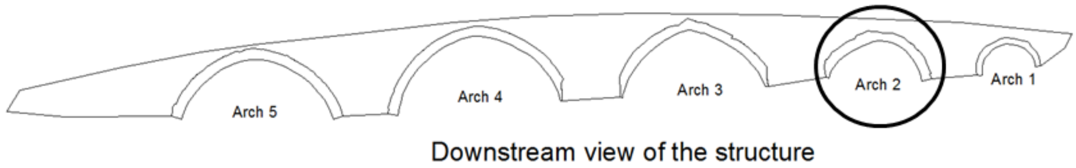

Fig. 2. Geometry of the Cernadela Bridge - The simulated single arch.

In particular, the first smeared crack concrete finite element model is used for the simulation of the second arch (Arch 2) of the Cernadela Bridge in two dimensions, Fig. 2. The main dimensions of the bridge are given below [7]:

- Length of spans (right to left, downstream view): 3.58m, 6.56m, 10.01m, 11.14m, 10.30m.
- Rise of arches (right to left, downstream view): 1.79m, 3.77m, 5.22m, 5.80m, 4.75m.

The finite element analysis model consists of quadrilateral, four-node, plane stress elements with two translational degrees of freedom per node. A typical value for the length of each finite element is 0.03m. A total number of 4725 elements are used. In Fig. 3a a detail of the mesh of the arch is shown.

In parallel, the general-purpose finite element software MSC\Marc is used for the simulation of the second arch of the Cernadela Bridge in two dimensions, Fig. 4. The finite element analysis model consists of quadrilateral, four-node, and plane stress elements with two translational degrees of freedom per node. A total number of 4351 elements and 4661 nodes are used. This particular model has been used in a continuum as well as a discrete formulation, as it will be shown later in this article.

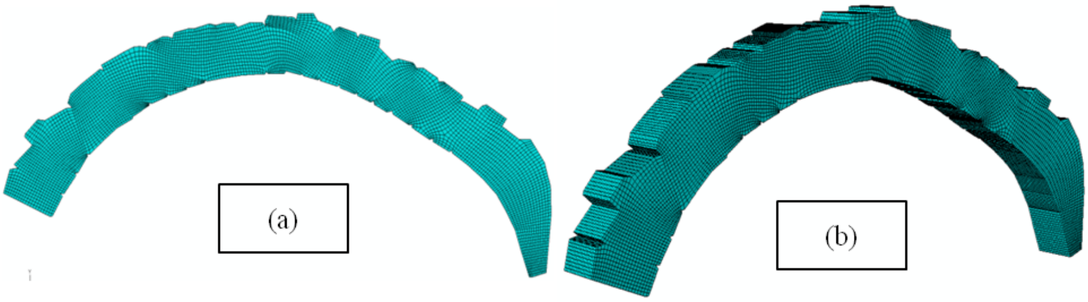

Fig. 3. Mesh of the simulated single arch (a) Two dimensional, (b) Three dimensional model.

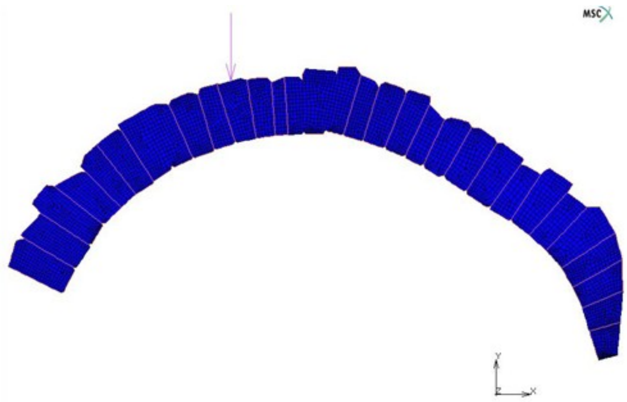

Fig. 4. Mesh of the simulated single arch - Two dimensional model of Marc.

A fourth, three dimensional finite element model has been developed, for the study of the same arch of the bridge, in three dimensions. The width of the arch is considered equal to

0.5m. Three dimensional hexahedral finite elements with three translational degrees of freedom per node have been used. The total number of them is equal to 73520, Fig. 3b. The smeared crack concrete model is used for the investigation of the damage in this three dimensional model. In the described models, loading conditions include self-weight and a concentrated load at the quarter span of the bridge.

In [7] the same arch of the structure was simulated with a discontinuous finite element analysis model, as well as with Ring 2.0 limit analysis software [15, 16]. In both models, a discrete modelling approach was considered, contrary to the present study where continuum damage models are mainly used. Consequently, comparison between the results obtained from the continuum and discrete approach will be considered for this arch of the bridge. Thus, the ultimate (limit) load and the collapse mechanism received from both approaches will be examined. This procedure is used for the validation of the parameters of the used damage models.

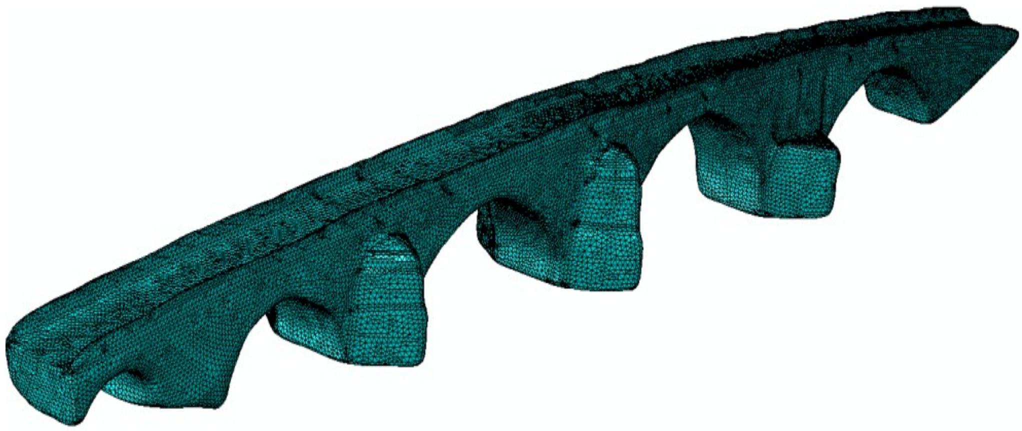

Fig. 5. Mesh of the whole Cernadela Bridge.

Finally, a three dimensional, continuum, finite element model is developed, for the whole geometry of the bridge, Fig. 5. For the investigation of the ultimate behaviour of the structure, the smeared crack concrete model has been used. Three dimensional, tetrahedron finite elements with three translational degrees of freedom per node, are used for the mesh of the model. A great number of finite elements, thus 686087, have been used. The influence of loading in some of the arches or the movement of abutments on the ultimate behaviour of the structure, are investigated. It is noted that the whole structure was also simulated in [17], with a linear finite element analysis model.

3.3 Parameter identification

The mentioned models have been used in the framework of finite element analysis. Within the first smeared crack damage model, Young's modulus has been considered equal to 23GPa, Poisson' s ratio 0.2 and density 2000 kg/m3. The tensile strength of the structure is considered equal to 0.5MPa. Large displacement effects are neglected while the arch is considered to be fixed to the ground. Newton-Raphson incremental iterative procedure has been used for the solution of the non-linear problem.

Using the Marc's alternative finite element continuum model the material parameters are considered as follows: Young's modulus, E=23GPa, Poisson's ration 0.2, density 2000 kg/m3, critical cracking stress, σcr=0.25MPa, tension-softening modulus, Es=2.5GPa, crushing strain, $\varepsilon crush$=0.003 and the shear retention factor equal to 1.0.

For the two dimensional discrete model, the separation procedure along the possible crack interfaces, is based on a critical value of the force normal to the interface. First no tension strength is considered and separation force is zero. Second a low strength of tension equal to 0.25MPa is used.

3.4 Parametric analysis of the single arch of the structure

Results obtained from the study of the second arch of the Cernadela Bridge will be presented in this paragraph. For this reason, continuum damage and discrete models have been developed in two and three dimensions, respectively. The collapse mode and the failure load received from these models, are compared with the corresponding results obtained from a discrete modelling approach presented in [7], in the same arch.

The failure mode which arises from the damage models is the four hinges collapse mechanism, Figs. 6(a) and (b). The same mechanism is received from the discontinuous finite element model presented in [7]. A similar collapse mechanism is obtained from the discrete model formulation which is used in this article. Small differences about the location of the two hinges (at the left side) which are presented are related with the value of separation force, as it is shown in Fig. 7.

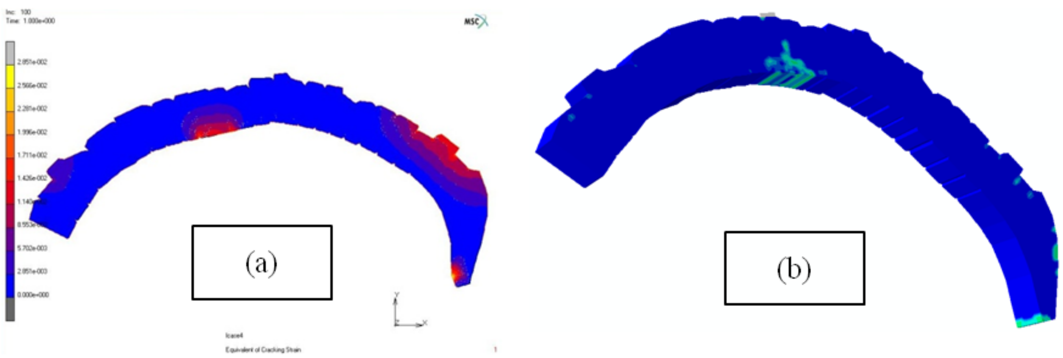

Fig. 6. Collapse mechanism obtained from the continuum finite element models (a) Two dimensional (b) Three dimensional model.

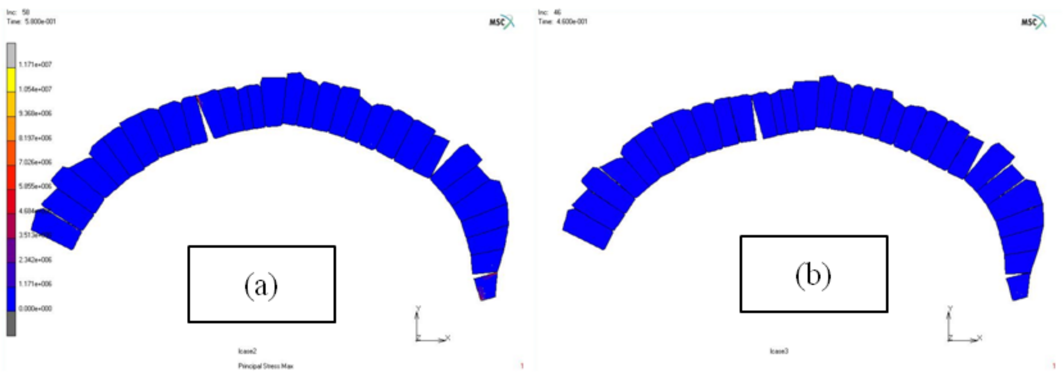

Fig. 7. Collapse mechanism obtained from the discrete, two dimensional finite element model (a) Separation stress = 0.25MPa, (b) Separation stress = 0.00MPa.

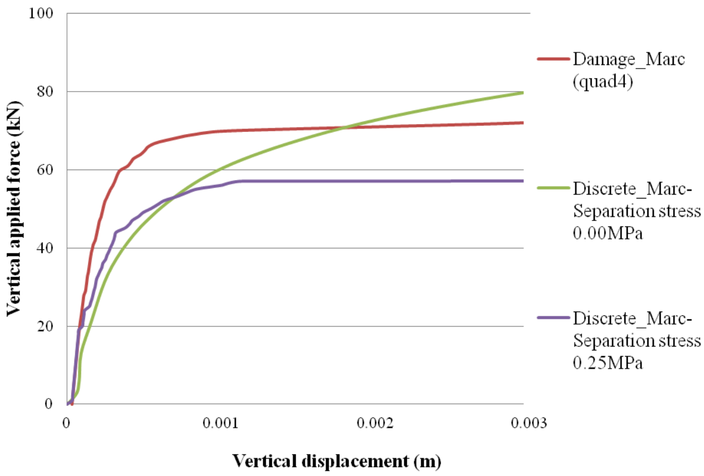

Fig. 8. Force-displacement diagrams for the second arch of the Cernadela Bridge (width of the arch 0.50m).

The influence of the arch width is examined for the two dimensional model considering width equal to 0.5m and 1.0m. The corresponding force-displacement diagram for width equal to .50m is shown in Fig. 8. In Fig. 9 are summarized the force-displacement diagrams, obtained from the damage models used in this article and the discrete models presented in [7]. According to these, the failure loads received from the damage models are found between the corresponding values of the discrete, three dimensional finite element models and the limit analysis model with the Ring software, presented in [7].

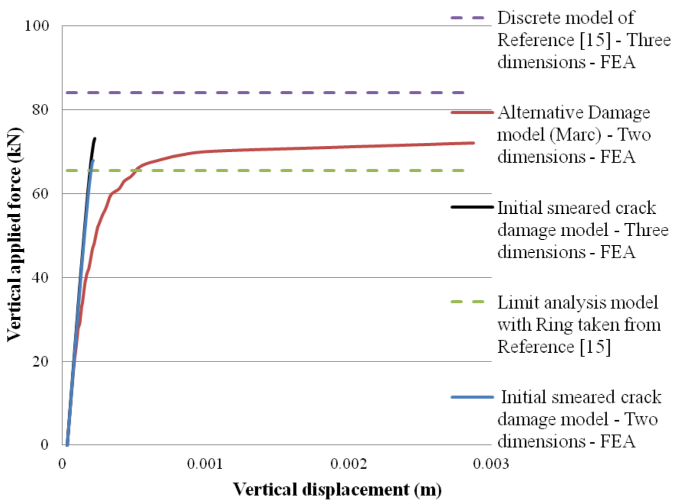

Fig. 9. Summary of the force-displacement diagrams for the second arch of the Cernadela Bridge (width of the arch 0.5m).

3.5 Study of the whole bridge with the damage model.

The smeared crack damage model is used for the investigation of the behaviour of the whole structure. In particular, a non-linear finite element model of the whole bridge has been developed to demonstrate failure on the structure in case movement of abutments or a static, traffic load is applied to the bridge. In [17] a similar investigation was conducted by a linear

finite element model, on the same masonry arch which is used in this study. In [11] the influence of the movement of abutments on the collapse mechanism of two dimensional stone arches was investigated, by developing discrete finite element models.
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Young's modulus has been considered equal to 23GPa, Poisson' s ratio 0.2 and density 2000 kg/m3. The tensile strength of the structure is considered equal to 0.5MPa. Large displacement effects are neglected while the arch is considered to be fixed to the ground. Newton-Raphson incremental iterative procedure has been used for the solution of the non-linear problem. In order to simplify modelling of the structure, the whole bridge has been considered as a unity, thus no distinction between the arch and the fill material has been made. However, it is left for a future work the consideration of the arch and the fill as separate structural units.

Concerning the loading of the structure, three loading steps have been developed. In the first step the dead load of the bridge is considered, while in the second step a uniformly distributed load of 3KN/m3 is applied to the structure. In the third step a concentrate load or a movement of abutments is applied to the bridge.

When a vertical displacement is applied to the fourth abutment of the structure, damage arises in the fourth and the fifth arch according to Fig. 10. Similarly, principle stresses of the linear model presented in [17] become maximum in the same areas of the fourth and fifth arch. In addition, Fig. 11 shows the damage of the fourth arch, in case a traffic static load is applied to it. A close image is obtained by the linear model of [17], for the same loading.

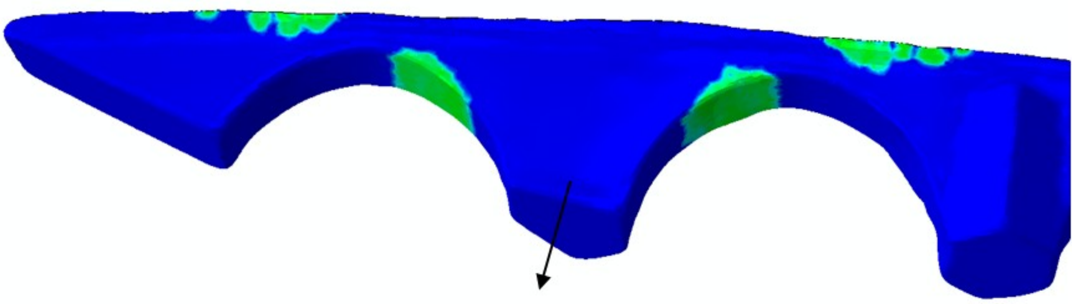


Fig. 10. Damage on the fourth and fifth arch for a vertical displacement of the fourth abutment – Damage model.

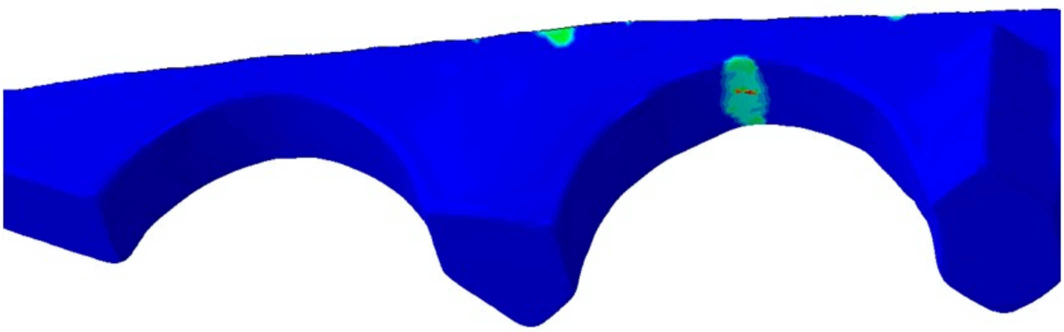


Fig. 11. Damage for a traffic load in the fourth arch – Damage model.

An interesting comment related to the ultimate behaviour of the structure can be made, in case the traffic load of the fourth arch is accompanied with a transverse displacement of the fourth abutment (in a direction vertical to the longitudinal axis of the bridge). According to the upstream view of Fig. 12 the damage in this case is expanded to the spandrel walls of the fourth and fifth arch, contrary to Fig. 11 where damage arises almost exclusively in the middle of the fourth arch. In addition, damage has been expanded to the fourth abutment, according to downstream view of Fig. 13. This demonstrates that the behaviour of the structure is significantly influenced in case a transverse loading is applied to it, for instance after an earthquake.

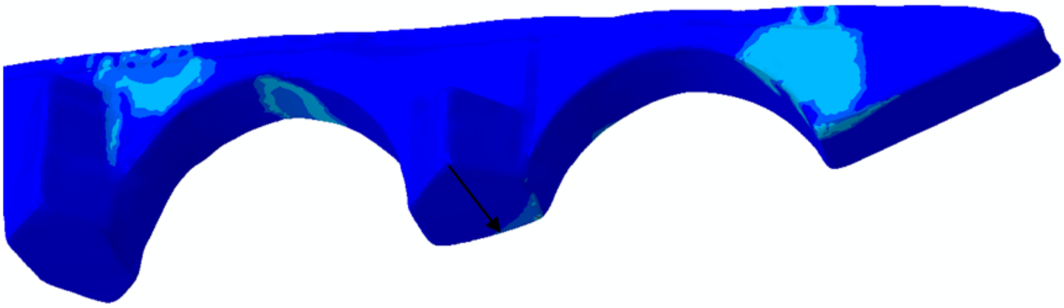


Fig. 12. Damage for a traffic load in the fourth arch and a transverse movement in the fourth abutment – Upstream view.

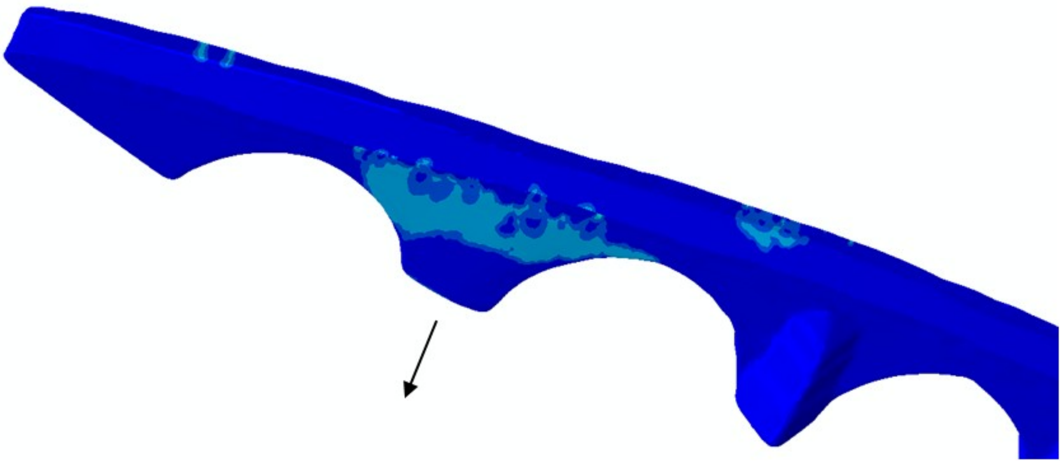


Fig. 13. Damage for a traffic load in the fourth arch and a transverse movement in the fourth abutment – Downstream view.

4 COMPUTATIONAL HOMOGENIZATION

4.1 Introduction

Computational homogenization is used for the investigation of the structural behaviour of complex, heterogeneous structures, by considering a representative microscopic sample of the material, and then projecting the average material characteristics in the macroscopic, structural scale. Several materials, like masonry and composites can be simulated by using computational homogenization.

According to numerical homogenization, a unit cell is explicitly solved and the results are then used for the determination of the parameters of a macroscopic constitutive law [18]. From another point of view, multi-level computational homogenization incorporates a concurrent analysis of both the macro and the microstructure, in a nested multi-scale approach [19-21]. Within this method, the macroscopic constitutive behaviour is determined during simulation, after solving the microscopic problem and transferring the information on the macroscopic scale. This approach, which is generally called FEM^2, offers the flexibility of simulating complex micro-structural patterns, with every kind of non-linearity.

4.2 Description of homogenization method

Parametric finite element analysis, using COMSOL Multiphysics, is used for the simulation of a non-linear masonry Representative Volume Element (RVE), under several loading paths. In each loading path, linear displacement boundary conditions are incrementally applied in the boundaries of the RVE. After solution of the microscopic structure, the average stress is calculated. As a result, a strain-stress database is created. In addition, for each loading path and each loading level, three test incremental loadings are applied to the RVE. Consequently, tangent stiffness information is obtained for each particular loading path and loading level and a second strain-stiffness database is obtained. Based on these databases, and MATLAB-based interpolation for the creation of a metamodel, a computational homogenization model, in a FEM^2 sense, is created for the macroscopic analysis of masonry structures. Comparison with direct heterogeneous macroscopic models shows that the adopted procedure leads to satisfactory results.

The key idea of the present work is to replace the microscopic simulation of the RVE, which is considered within each time step of the computational homogenization method, with two databases containing information related to the stress and the stiffness of the macro model. This information is transferred back to the macroscopic structure.

Thus, instead of solving the RVE in each Gauss point and time step, which is a time consuming procedure, an interpolation of the proper quantity from the databases is considered. This concept has the following steps:

a) Creation of a masonry RVE. It consists of the bricks and the mortar joints, thus the material that connects the bricks (fig. 14). The non-linearity of this model is concentrated on the mortar joints by using a perfect plasticity law. The brick parts are linear.

b) A number of loading paths are developed and applied to the RVE. To do this, plane stress parametric analysis from the structural mechanics module is used. Each loading path consists of a number of increments.

Linear boundary conditions are applied as loading to the boundaries of the RVE. The "Prescribed displacement" option is chosen in the boundary settings.

c) After analysis of each RVE is completed, the average stress is calculated.

d) Steps b) and c) are repeated for each loading path and each loading level, but now three test incremental loadings are applied to the RVE. Then, by incrementally solving the Hooke's law, tangent stiffness information is obtained for the particular loading path and level.

e) Two databases have been created: one that corresponds strains to stresses and another that corresponds strains to stiffness information. These are incorporated in a FEM^2 computational homogenization scheme developed with MATLAB, for the simulation of macroscopic masonry structures.

f) Comparison of the results with direct heterogeneous macroscopic models created in other commercial software packages is used to evaluate the whole procedure.

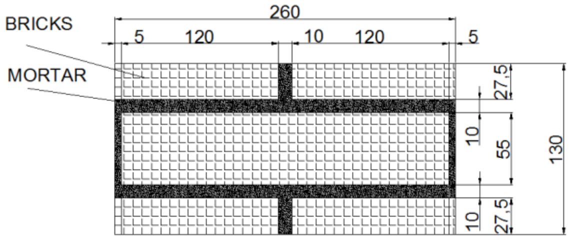


Fig. 14. Geometry of the masonry RVE (dimensions in mm)

4.3 Results of homogenization method to masonry.

Rectangular plane stress elements have been used for the simulation of the model. The out of plane thickness of the structure is taken equal to 70mm. For both the brick and the mortar, isotropic elasticity is considered. Material properties are E_b=4865N/mm^2, n_b=0.09 for the brick and E_m=1180N/mm^2, n_m=0.06 for the mortar joints. A perfect plasticity assumption has been made for the mortar, with a tensile strength of 0.9N/mm^2. The brick is considered linear.

The main idea of the present work, is to replace the simulation of an RVE in each Gauss point and each time step of the macro model, with the usage of the strain-stress and strain-stiffness database, which were created in the previous steps. By adopting this procedure, the method should become faster, as instead of solving a FEM microscopic problem in each Gauss point and time step, the databases and some interpolation method are used in order to obtain the macro stress and the consistent stiffness of the Newton-Raphson method.

For any current value of the macroscopic strain, a stress and a stiffness should be found from the databases previously created. Thus, an interpolation method must be used, to obtain these quantities from the databases. In this work the MATLAB function “TriScatteredInterp” is used, however other possible solutions for the creation of the metamodel (interpolation) can be used.

In particular, parametric analysis results in the development of non-linear stress-strain laws, as it is depicted in Figure 15.

The failure mode of some RVEs is shown in Figure 16.

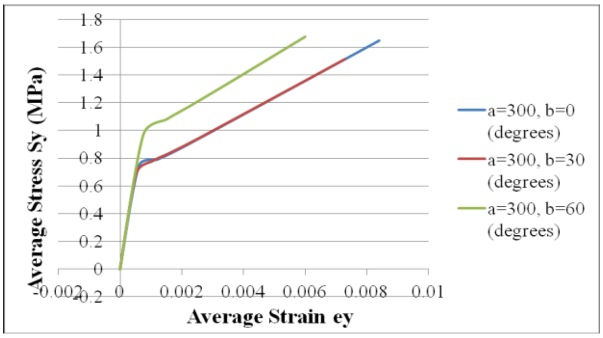


Fig. 15. Average stress-average strain diagrams obtained from parametric COMSOL analysis

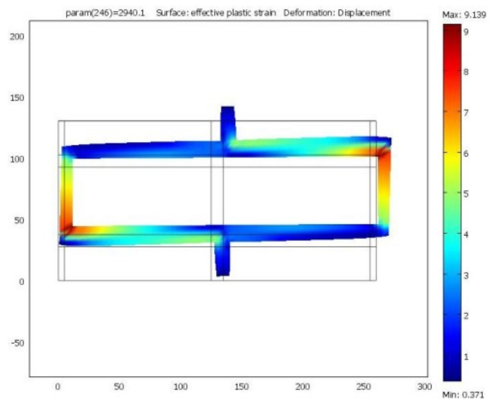


Fig. 16. Effective plastic strain of the RVE, as the parameter of loading is increased

In Figure 17, a similar output is presented, for a bigger masonry wall, with dimensions 1.82x1.69m, fixed vertical left boundary and distributed displacements of 5mm at the right vertical edge, as loading. For the direct macroscopic simulation, MARC software has been used. According to this Figure, the degradation of the strength obtained from the two models, has the same distribution in the domain.

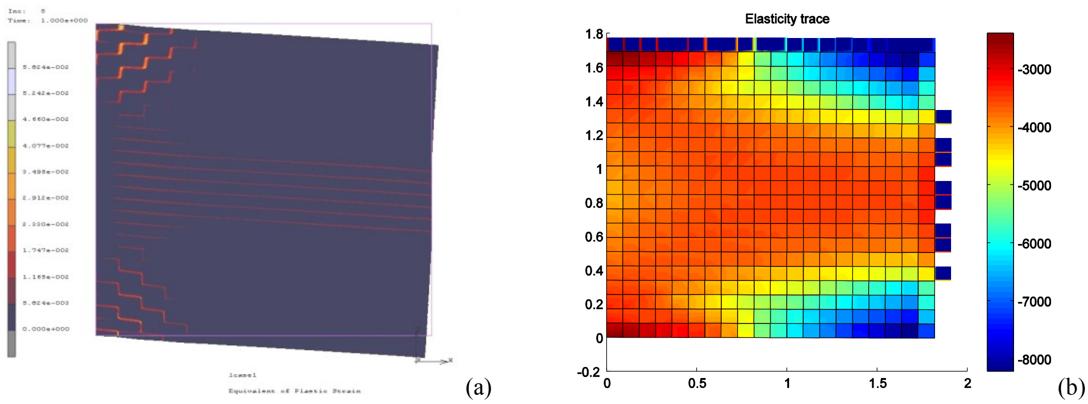
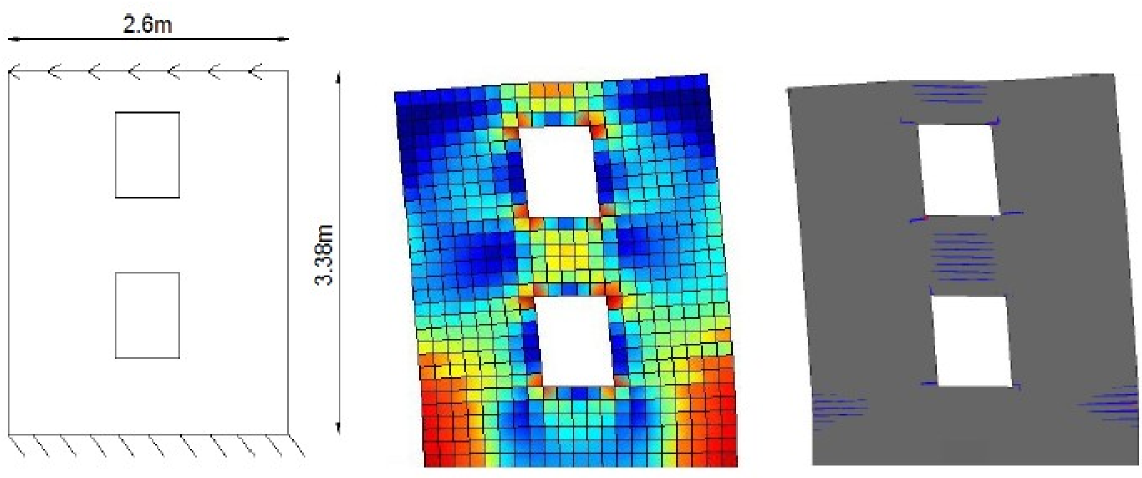


Fig. 17. Degradation of the strength of a bigger macroscopic masonry wall (a) Marc direct macroscopic simulation (b) proposed FEM^2 approach



(a) (b) (c)

Figure 7: (a) The masonry wall with two openings (b) Degradation of the strength proposed multi-scale homogenization (c) Degradation of the strength – direct macroscopic simulation (MARC)

An additional example of a big masonry wall with two openings (windows) has been presented in [22]. A distributed displacement loading of 5mm is applied to the top of the structure, while the bottom of it is fixed, Fig. 18a. In Figs. 18b, 18c is shown that the image of the degradation of the strength is the same for both the multi-scale homogenization and the direct heterogeneous model. In addition, a concentration of plastic strains appears in the corner of the windows, Fig. 18c.

5 CONCLUSIONS

In the present study terrestrial photogrammetry is used for geometry reconstruction of a real masonry arch bridge located in Spain, the metric model is used for subsequent implementation in structural assessment tasks. Terrestrial photogrammetry significantly contributes to the accurate geometric representation of historical structures with milimetric precision. It is important to note some limitations on the use of the photogrammetric method. Although the high level of detail recorded, this may be not enough for detection of movement or subtle displacements of masonry blocks. The main advantage of such surveying method is based on the accuracy reached for the positioning of real masonry elements that will subsequently support the accurate geometric characterization during the finite elements based model of the whole structure. The exact geometry obtained from this method is then used for the investigation of the ultimate behaviour of the structure.

In particular, several two and three dimensional, non-linear models have been developed. To depict the collapse mechanism of the structure, continuum damage models have been used and compared with discrete approaches conducted in the present as well as in older studies.

According to the results, the classical four hinges mechanism arises from the structural analysis of the bridge. Moreover, the influence of parameters such us the width of the structure and the tensile strength of the material in the force-displacement diagrams is shown. Finally, the simulation of the whole structure demonstrates that a possible out of plane movement of abutments will cause significant damage to the bridge.

In parallel, a method for studying heterogeneous structures by taking into account the non-linear behaviour of them was proposed. COMSOL Multiphysics was used to simulate with parametric analysis the non-linear RVE of a masonry structure. Then, the average strain, stress and stiffness were gathered and used in a FEM^2 approach, for the simulation of bigger masonry walls. The results showed a good convergence with direct heterogeneous macroscopic models created with commercial FEM packages (MARC), indicating that the proposed method can be used for the investigation of heterogeneous, non-linear materials like the masonry.

ACKNOWLEDGEMENTS

The work of Dr. Georgios Drosopoulos is being supported by a research project implemented within the framework of the Action «Supporting Postdoctoral Researchers» of the Operational Program "Education and Lifelong Learning" (Action's Beneficiary: General Secretariat for Research and Technology), and co-financed by the European Social Fund (ESF) and the Greek State.

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