

## FINITE ELEMENT MODEL UPDATING OF GEOMETRICALLY COMPLEX STRUCTURE THROUGH MEASUREMENT OF ITS DYNAMIC RESPONSE

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**Keywords:** *Large scale structures, Modal identification, Model updating, Substructuring, Structural Dynamics.*

**Abstract.** *In this work, an integrated reverse engineering strategy is presented that takes into account the complete process, from the developing of CAD model and the experimental modal analysis procedures to computational effective model updating techniques. Modal identification and structural model updating methods are applied, leading to develop high fidelity finite element model of geometrically complex and lightweight bicycle frame, using acceleration measurements. First, exploiting a 3D Laser Scanner, the digital shape of the real bike frame was developed and the final parametric CAD model was created. Next, the finite element model of the frame was created by using quadrilateral shell and hexahedral solid elements. Due to complex geometry of the structure, the developed model consists of about one million degrees of freedom. The identification of modal characteristics of the frame is based on acceleration time histories, which are obtained through an experimental investigation of its dynamic response, in two different states. First, in a support-free state by imposing impulsive loading and second in a fixed-free state by imposing random base excitation with the use of two electrodynamic shakers. A high modal density modal model is obtained. The modal characteristics are then used to update finite element model. Single and multi-objective structural identification methods with appropriate substructuring methods, are used for estimating the parameters (material properties and shell thickness properties) of the finite element model, based on minimizing the deviations between the experimental and analytical modal characteristics (modal frequencies and mode shapes). Direct comparison of the numerical and experimental data verified the reliability and accuracy of the methodology applied.*

## 1 INTRODUCTION

Current industrial design requirements lead frequently to the study, improvement, development and modification of some mechanical parts or even of an entire structure. For many of these parts, there is not available necessary information's, such as drawings or material properties. In order to address this issue, it is necessary to apply a reverse engineering process. In this process, many issues are taken into account, which are related with the developing of FE model, with the experimental modal analysis procedure and with the development of computational effective model updating techniques. The main objective of the present work is to demonstrate the advantages of applying appropriate numerical and experimental methodologies in order to identify the model parameters and develop a high fidelity finite element model of the structure examined.

The equations of motion of mechanical systems with complex geometry are first set up by applying classical finite element techniques. As the order of these models increases, the existing numerical and experimental methodologies for a systematic determination of their dynamic response become inefficient to apply. Therefore, there is a need for the development, improvement and application of new suitable methodologies for investigating dynamics of large scale mechanical models in a systematic and efficient way. Traditionally, in the area of structural dynamics this is done by first employing methodologies that reduce the dimensions of the original system. In this paper examined a time domain reduction method [1-5]. In order to improve the FE model of the structure, structural model updating techniques [16], have been proposed in order to reconcile the numerical (FE) model, with experimental data. Structural model parameter estimation based on measured modal data (e.g. [9-15]) are often formulated as weighted least-squares estimation problems in which metrics, measuring the residuals between measured and model predicted modal characteristics, are build up into a single weighted residuals metric formed as a weighted average of the multiple individual metrics using weighting factors. Standard gradient-based optimization techniques are then used to find the optimal values of the structural parameters that minimize the single weighted residuals metric representing an overall measure of fit between measured and model predicted modal characteristics. Due to model error and measurement noise, the results of the optimization are affected by the values assumed for the weighting factors.

In this work, the applicability and effectiveness of the methods applied, namely the model reduction method and the model updating method, is explored by updating finite element model of a lightweight and geometrically complex bicycle frame, using experimentally identified modal data. Issues related to estimating unidentifiable solutions [17-20] arising in FE model updating formulations are also addressed. A systematic study is carried out to demonstrate the effect of model error, finite element model parameterization, number of measured modes and number of mode shape components on the optimal models and their variability. It is demonstrated that the updated finite element models obtained using measured modal data may vary considerably.

The organization of this paper is as follows. First, overviews the formulation for finite element model updating based on modal data in the following section. In the third section, the experimental device (bicycle frame) is introduced. More specifically, first presented the procedure followed in order to develop the digital shape of the real bike frame and the final parametric CAD model, exploiting a 3D Laser Scanner. Next presented the detailed FE model of the frame and finally a brief review of the experimental set-up and the modal analysis results is given. Finally the parametric studies on updating finite element model of the bicycle frame and the predictions of frequency response functions, based on the optimal models, are presented in the fourth section. Conclusions are summarized in the fifth section.

## 2 FINITE ELEMENT MODEL UPDATING METHOD

Let  $D = \{\hat{\omega}_r, \hat{\phi}_r \in R^{N_o}, r=1, \dots, m\}$  be the measured modal data from a structure, consisting of modal frequencies  $\hat{\omega}_r$  and mode shape components  $\hat{\phi}_r$  at  $N_o$  measured DOFs, where  $m$  is the number of observed modes. Consider a parameterized class of linear structural models used to model the dynamic behavior of the structure and let  $\underline{\theta} \in R^{N_\theta}$  be the set of free structural model parameters to be identified using the measured modal data. The objective in a modal-based structural identification methodology is to estimate the values of the parameter set  $\underline{\theta}$  so that the modal data  $\{\omega_r(\underline{\theta}), \phi_r(\underline{\theta}) \in R^{N_o}, r=1, \dots, m\}$  predicted by the linear class of models at the corresponding  $N_o$  measured DOFs best matches the experimentally obtained modal data in  $D$ . For this, let

$$\varepsilon_{\omega_r}(\underline{\theta}) = \frac{\omega_r^2(\underline{\theta}) - \hat{\omega}_r^2}{\hat{\omega}_r^2} \quad \text{and} \quad \varepsilon_{\phi_r}(\underline{\theta}) = \frac{\|\beta_r(\underline{\theta})\phi_r(\underline{\theta}) - \hat{\phi}_r\|}{\|\hat{\phi}_r\|} \quad (1)$$

be the measures of fit or residuals between the measured modal data and the model predicted modal data for the  $r$ -th modal frequency and mode shape components, respectively, where  $\|\underline{z}\|^2 = \underline{z}^T \underline{z}$  is the usual Euclidean norm, and  $\beta_r(\underline{\theta}) = \hat{\phi}_r^T \phi_r(\underline{\theta}) / \|\phi_r(\underline{\theta})\|^2$  is a normalization constant that guaranties that the measured mode shape  $\hat{\phi}_r$  at the measured DOFs is closest to the model mode shape  $\beta_r(\underline{\theta})\phi_r(\underline{\theta})$  predicted by the particular value of  $\underline{\theta}$ .

To proceed with the model updating formulation, the measured modal properties are grouped into two groups. The first group contains the modal frequencies while the second group includes the mode shape components for all modes. For each group, a norm is introduced to measure the residuals of the difference between the measured values of the modal properties involved in the group and the corresponding modal values predicted from the model class for a particular value of the parameter set  $\underline{\theta}$ . For the first group, the measure of fit  $J_1(\underline{\theta})$  is selected to represent the difference between the measured and the model predicted frequencies for all modes. For the second group, the measure of fit  $J_2(\underline{\theta})$  is selected to represent the difference between the measured and the model predicted mode shape components for all modes. Specifically, the two measures of fit are given by

$$J_1(\underline{\theta}) = \sum_{r=1}^m \varepsilon_{\omega_r}^2(\underline{\theta}) \quad \text{and} \quad J_2(\underline{\theta}) = \sum_{r=1}^m \varepsilon_{\phi_r}^2(\underline{\theta}) \quad (2)$$

The parameter estimation problem is traditionally solved by minimizing the single objective

$$J(\underline{\theta}; \underline{w}) = w_1 J_1(\underline{\theta}) + w_2 J_2(\underline{\theta}) \quad (3)$$

formed by the two objectives  $J_i(\underline{\theta})$ , using the weighting factors  $w_i \geq 0$ ,  $i=1,2$ , with  $w_1 + w_2 = 1$ . The objective function  $J(\underline{\theta}; \underline{w})$  represents an overall measure of fit between the measured and the model predicted characteristics. The relative importance of the residual errors in the selection of the optimal model is reflected in the choice of the weights. The results of the identification depend on the weight values used. The optimal solutions for the parameter set  $\underline{\theta}$  for given  $\underline{w}$  are denoted by  $\hat{\underline{\theta}}(\underline{w})$  [17-20].

### 3 EXPERIMENTAL APPLICATION

The model updating methodologies are applied to update the FE model of real bicycle frame, shown in Figure 1. The frame of the structure is made of Aluminium 6061 with Young's modulus  $E = 69GPa$ , Poisson's ratio  $\nu = 0.3$  and density  $\rho = 2700kg/m^3$ . The frame consists of members like, down tube, top tube, chain stays, seat stays and seat tube. The thicknesses of the cross sections of these parts can be vary along their length.

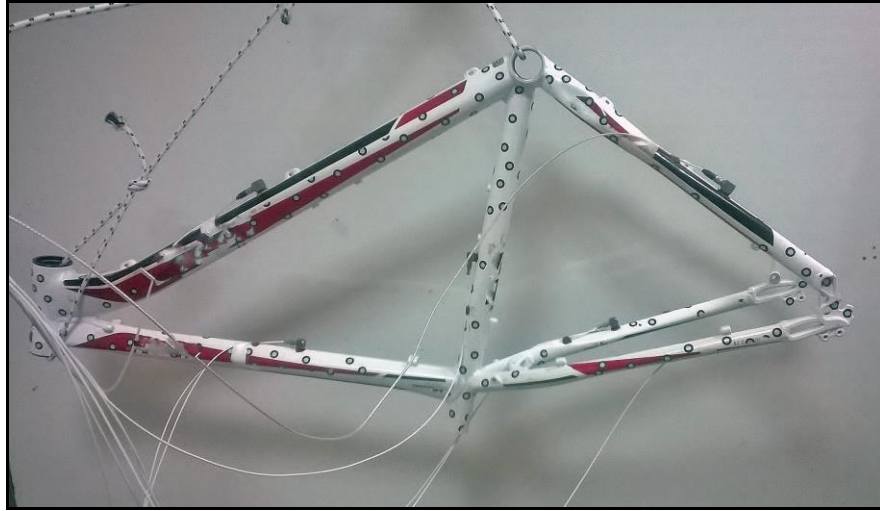


Figure 1: BICYCLE FRAME.

#### 3.1 Digitisation and CAD Model of the Frame

First, exploiting a 3D Laser Scanner, the digital shape of the real bike frame was developed by using the DSR (Digital Shape Reconstruction) method. In this process, four basic steps are being followed in order to collect, process and design the final CAD model, appropriate for the subsequent FE analysis. First, the geometrical data of the bicycle were captured, exploiting the 3D scanner's functionalities, as well as its software tools in order to produce a primary stereo-lithography (STL) file. As a second step, compatible utilities were used to pre-process the initial raw model in order to create the final STL file of the digitized geometry (Figure 2), before designing the CAD surfaces [6, 7, 8].



Figure 2. FINAL STL MODEL OF THE BICYCLE FRAME.

Next, in order to produce the initial CAD model, which is presented in figure 3, a segmentation of the triangulated STL model and NURBS (Non-uniform Rational B-Splines) surface fitting, were applied.

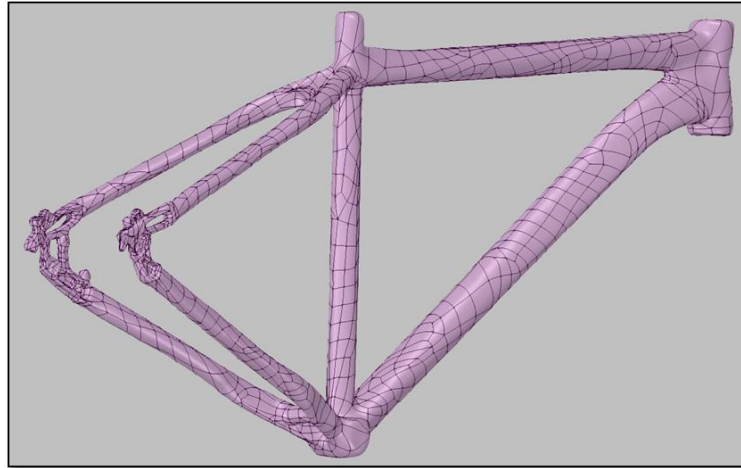


Figure 3. INITIAL CAD MODEL OF THE FRAME AFTER NURBS SURFACE FITTING.

Finally, in order to achieve a full correspondence of the real frame to the CAD model, several details were re-designed and parts of the structure that were recognized as solid or fixed-thickness, were changed accordingly, as depicted in Figure 4. On the other hand, surfaces that would most likely have variable thickness due to mechanical deformation during industrial mass production of the frame were left as surface elements for ease of manipulation in the optimization method.

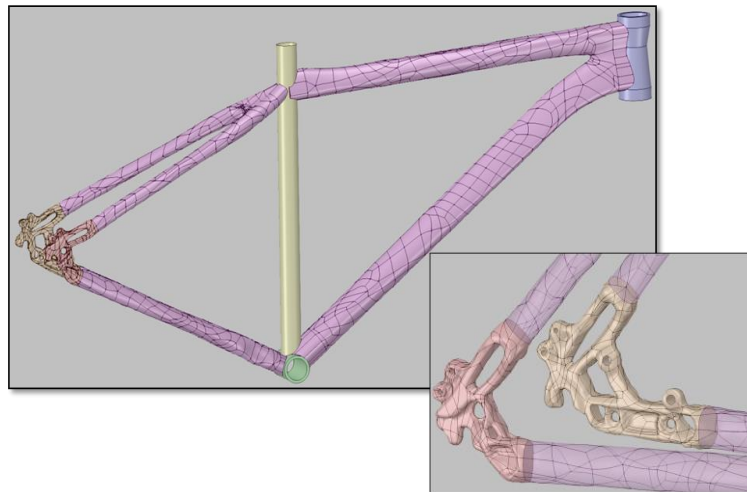


Figure 4. FINAL CAD MODEL.

### 3.2 Finite Element Model

The geometry of the bicycle frame is discretized mainly by shell elements (triangular) and solid elements (tetrahedral). Due to complex geometry of the structure, the total number of degrees of freedom of the resulting model is about one million degrees of freedom (1,000,000). For the development and solution of the finite element model some appropriate software was used [21, 22]. The detailed FE Model of the frame presented in Figure 5.

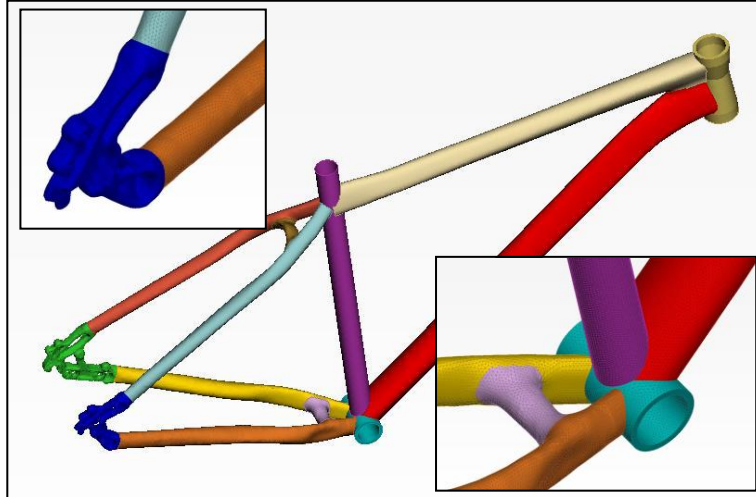


Figure 5. FINITE ELEMENT MODEL OF THE FRAME.

The first eigmode predicted by the nominal finite element model, presented in Figure 6.

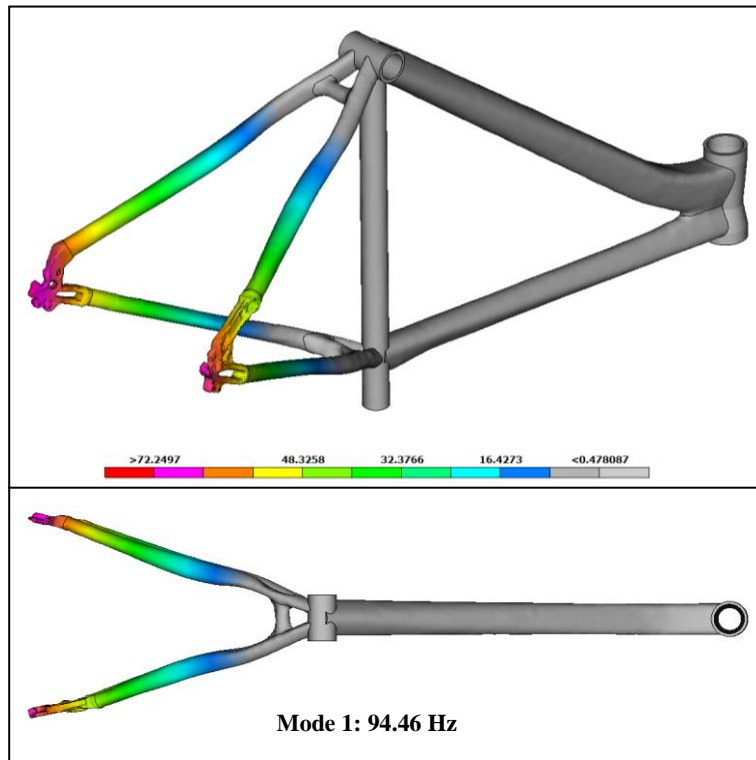


Figure 6. FIRST EIGENMODE PREDICTED BY THE NOMINAL FINITE ELEMENT MODEL.

### 3.3 Experimental Modal Analysis

After development of the nominal finite element model, an experimental modal analysis of the frame was performed to quantify its dynamic characteristics. The frame was hung up with the help of a crane and straps, to approximate free-free boundary conditions for the test.

First, all the necessary elements of the FRF matrix required for determining the response of the frame substructure were determined by imposing impulsive loading [10-14]. The measured frequency range was 0-2048 Hz, which includes the analytical frequency range of interest, 0-600 Hz. An initial investigation indicated that the frame has eleven natural frequencies



in this frequency range. A schematic illustration of the measurement geometry of the test structure is presented in Figure 7a. In this figure, presented the locations and directions of acceleration measurements, when apply an impulsive load in all directions and at several locations. Also in Figure 7b presented the accelerometer locations in the real frame.

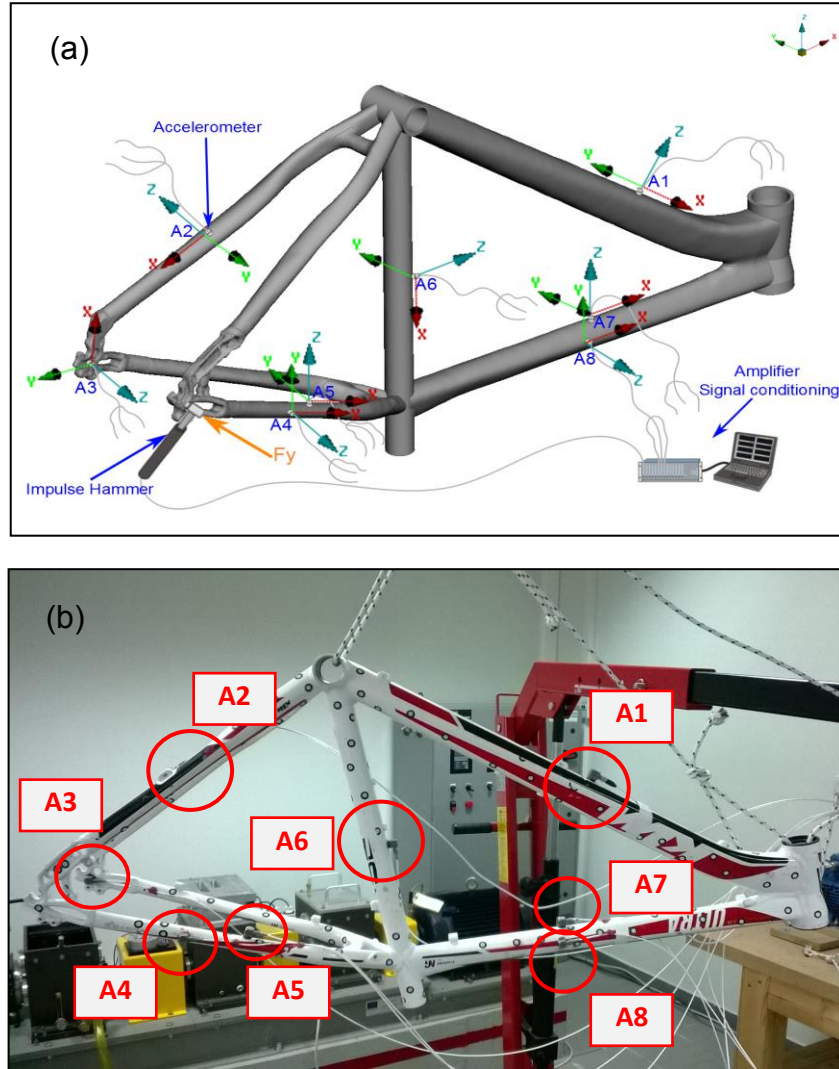


Figure 7. a) SCHEMATIC ILLUSTRATION OF THE MEASUREMENT GEOMETRY b) ACCELEROMETERS LOCATIONS IN REAL FRAME.

For instance, Figure 8 shows the magnitude of one typical element of the FRF matrix before (continuous line) and after (dashed line) application of the Welsh's smoothing method. Based on the measured FR functions, the natural frequencies and the damping ratios of the frame substructure were estimated by applying the "Rational Fraction Polynomial Method" (RFPM). This method has certain attractive merits, especially for systems with high modal density, like the system under consideration [10, 11, 15]. The identified mode shapes have also been recorded so that they can be used for updating the finite element models.

As a second approach, imposing random base excitation with the use of two electrodynamic shakers and determining the response of the frame substructure the FRF matrix was restructured and modal parameters were re-examined. An illustration of the fixed-free arrangement of the bike on the shakers is presented in figure 9.

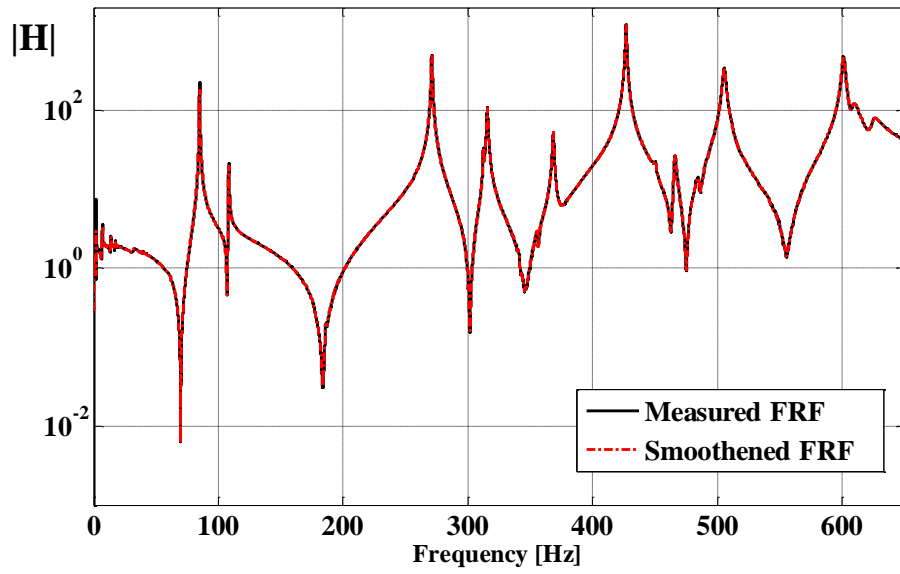


Figure 8. TYPICAL ELEMENT OF THE FRF MATRIX.



Figure 9. FIXED-FREE ARRANGEMENT ON ELECTRODYNAMIC SHAKERS.

As an outcome of the above two procedures, the first column of Table 1 presents the values of the lowest 11 natural frequencies ( $\omega_{FE}$ ) of the frame, while the corresponding damping ratios are included in the fourth column. In the same table, the second column presents the values of the natural frequencies obtained from the analysis of the nominal finite element model ( $\omega_{rNFE}$ ) and the third column compares these frequencies with the corresponding frequencies obtained by the experimental data.



Mode	Identified Modal Frequency	Nominal FE Predicted Modal Frequency	Difference between Identified and FE Predicted Modal Frequencies	Identified Modal Damping Ratio
	$\omega_{rE} [Hz]$	$\omega_{rN_{FE}} [Hz]$	$\frac{\omega_{rN_{FE}} - \omega_{rE}}{\omega_{rN_{FE}}} 100\%$	$\zeta_{rE} (\%)$
1	84.88	96.46	12.01	0.21
2	108.29	118.72	8.78	0.19
3	270.69	319.15	15.19	0.12
4	311.96	357.30	12.69	0.08
5	315.15	358.72	12.15	0.13
6	368.01	414.69	11.26	0.13
7	425.73	491.11	13.31	0.12
8	450.25	494.05	8.87	0.16
9	465.19	530.79	12.36	0.16
10	484.45	545.44	11.18	0.35
11	504.86	575.88	12.33	0.23

Table 1. MODAL FREQUENCIES AND MODAL DAMPING RATIOS OF THE FRAME.

The errors determined between the nominal FE model and the experimental measurements are not insignificant, indicating that the FE model updating process is necessary.

## 4 FINITE ELEMENT MODEL UPDATING

### 4.1 FE model parameterization

The parameterization of the finite element model of the experimental vehicle are introduced in order to demonstrate the applicability of the proposed finite element model updating method. The parameterized model consisting of thirteen parts which are shown in Figure 10. The first four parts (P1, P2, P3 and P4) consist of solid elements, while the remaining nine parts (P5-P13) consist of shell elements. At each of these parts are used as design variables the thickness of the shell elements, the Young's modulus and the density. Thus, the final number of the design parameters are thirty five (35) variables.

In Table 2 presented the initial values that have been set in each parameter, which are identical to the nominal FE model, with the upper and lower limits, which were selected to be used for the optimization process. In particular, for most values of densities was chosen as lower limit, the value  $2,500 Kg/m^3$  and as upper limit, values between  $3,500 Kg/m^3$  and  $3,700 Kg/m^3$ . Also, for most values of Young's modulus of the shell elements, was selected as lower limit, the value  $60GPa$  and as upper limit, the value of  $70GPa$ . For the solid elements, these limits ranged between the values  $68-72GPa$ . In the second column presented the nominal values, as well as the upper and lower limits for the thicknesses of the shell elements. Finally the last column of the table shows the step of design, which is set at 1% of the respective previous value for all cases.

The finite element model is updated using the lowest eleven identified modal frequencies and mode shapes shown in Table 1. The identified mode shapes include components at all 8 sensor locations. Additionally, we define as design response and total weight of the model, in order to be taken into consideration during the optimization process.

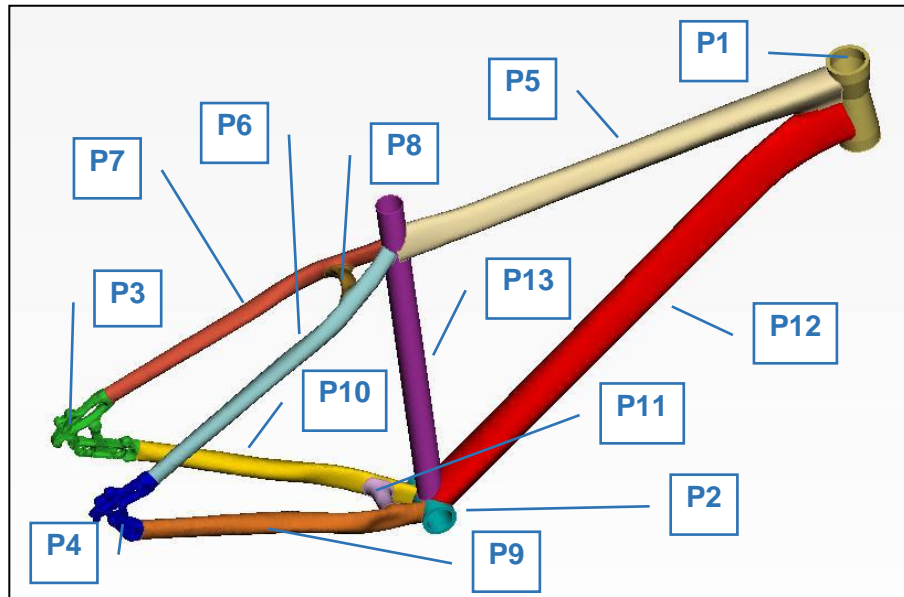


Figure 10. PARTS OF THE PARAMETERIZED FE MODEL OF THE FRAME.

Part	Initial Thickness [mm]	Initial Density [kg/m <sup>3</sup> ]	Initial Young's Modulus [Gpa]	Move Limit
	LB - UB	LB - UB	LB - UB	
P1	-	2700 <b>2500 – 3700</b>	69 <b>68 – 72</b>	1%
P2	-	2700 <b>2500 – 3500</b>	69 <b>68 – 72</b>	1%
P3	-	2700 <b>2500 – 3700</b>	69 <b>68 – 72</b>	1%
P4	-	2700 <b>2500 – 3700</b>	69 <b>68 – 72</b>	1%
P5	2 <b>1.60 – 2.30</b>	2700 <b>2500 – 3500</b>	69 <b>68 – 70</b>	1%
P6	1.7 <b>1.60 – 2.30</b>	2700 <b>2500 – 3500</b>	69 <b>68 – 70</b>	1%
P7	1.7 <b>1.60 – 2.30</b>	2700 <b>2500 – 3500</b>	69 <b>68 – 70</b>	1%
P8	2.5 <b>2.50 – 5.50</b>	2700 <b>2500 – 3500</b>	69 <b>68 – 72</b>	1%
P9	1.7 <b>1.60 – 2.30</b>	2700 <b>2500 – 3500</b>	69 <b>68 – 70</b>	1%
P10	1.7 <b>1.60 – 2.30</b>	2700 <b>2500 – 3500</b>	69 <b>68 – 70</b>	1%
P11	2.5 <b>2.50 – 5.50</b>	2700 <b>2500 – 3500</b>	69 <b>68 – 72</b>	1%
P12	2 <b>1.60 – 2.30</b>	2700 <b>2500 – 3500</b>	69 <b>68 – 70</b>	1%
P13	2 <b>1.60 – 2.50</b>	2700 <b>2500 – 3500</b>	69 <b>68 – 70</b>	1%

Table 2. DESIGN VARIABLES AND OPTIMIZATION DESIGN LIMITS.

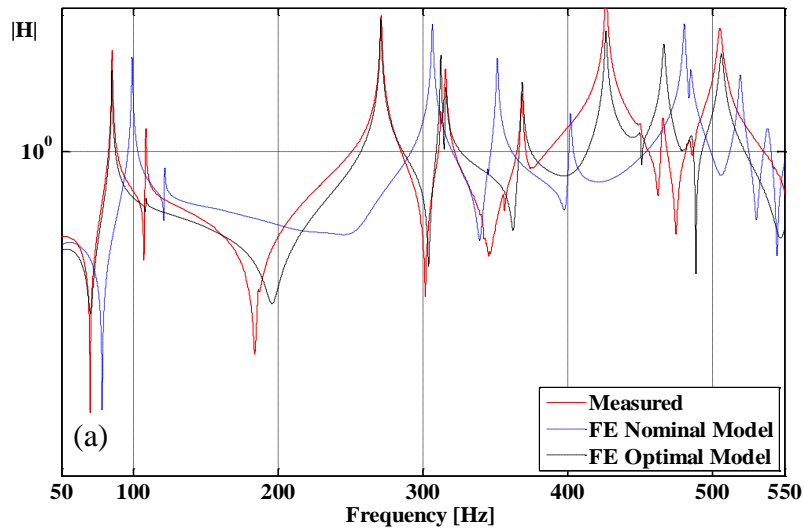
## 4.2 FE Model Updating Results

The results from the FE model updating method are shown in Table 3. In this table presented a comparison between identified ( $\omega_{rE}$ ) and optimal FE predicted modal frequencies ( $\omega_{rO_{FE}}$ ).

Mode	Identified Modal Frequency	Optimal FE Predicted Modal Frequency	Difference between Identified and FE Predicted Modal Frequencies
	$\omega_{rE} [Hz]$	$\omega_{rO_{FE}} [Hz]$	$\frac{\omega_{rO_{FE}} - \omega_{rE}}{\omega_{rO_{FE}}} 100\%$
1	84.88	84.89	0.01
2	108.29	108.29	0.00
3	270.69	270.61	0.03
4	311.96	312.16	0.06
5	315.15	315.47	0.10
6	368.01	368.01	0.00
7	425.73	426.01	0.07
8	450.25	449.51	0.17
9	465.19	465.59	0.09
10	484.45	484.38	0.02
11	504.86	505.23	0.07

Table 3. COMPARISON BETWEEN IDENTIFIED AND OPTIMAL FE PREDICTED MODAL FREQUENCIES

The FRF predicted by the optimal FE model (black dashed line) are compared in Figures 11a-b with the FRF computed directly from the measured data (red continuous line) at two indicative measurement locations in the frequency range [50Hz, 550Hz]. The FRF of the initial nominal model (blue dashed dot line) is also shown in these figures to be inadequate to represent the measured FRF. Compared to the FRF of the initial nominal model, it is observed that the updated optimal model tend to considerably improve the fit between the model predicted and the experimentally obtained FRF close to the resonance peaks.



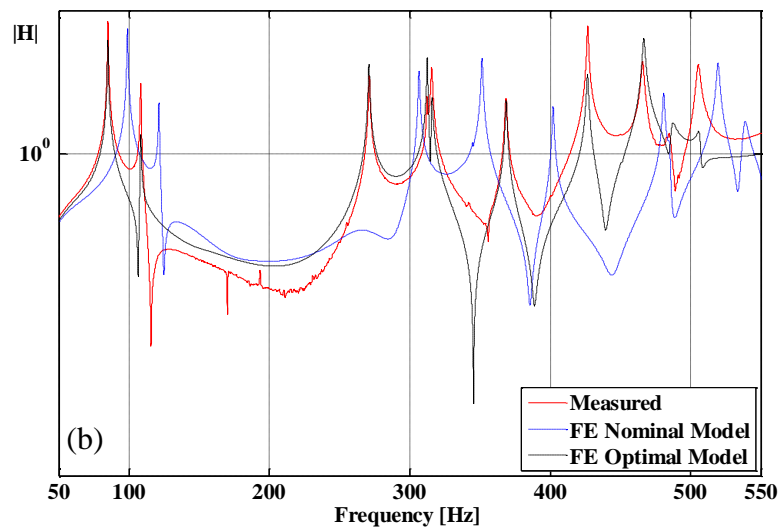


Figure 11. COMPARISON BETWEEN MEASURED, NOMINAL AND OPTIMAL FRF IN TWO TYPICAL ELEMENTS OF THE FRF MATRIX.

## 5 SUMMARY

An integrated reverse engineering strategy was presented that takes into account the complete process, from the developing of CAD model and the experimental modal analysis procedures to computational effective model updating techniques. Numerical and experimental methodologies were applied in order to identify the model parameters and develop a high fidelity finite element model of the structure examined. The applicability and effectiveness of the methods applied, namely the model reduction method and the model updating method, is explored by updating finite element model of a lightweight and geometrically complex bicycle frame, using experimentally identified modal data. Direct comparison of the numerical and experimental data verified the reliability and accuracy of the methodology applied.

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