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OUTPUT–ONLY SCHEMES FOR JOINT INPUT–STATE–PARAMETER ESTIMATION OF LINEAR SYSTEMS

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Abstract. *The subject of predicting structural response, for control or fatigue assessment purposes, via output only vibration measurements is an emerging topic of Structural Health Monitoring. The subject of estimation of the states of a partially observed dynamic system within a stochastic framework has been studied by many scientists and there are well developed algorithms to manage both linear and nonlinear state-space models. Dealing with structural systems, the system states comprise the response displacements and velocities at the degrees of freedom of the structure. On one hand, in practical cases it is difficult or sometimes impossible to measure structural displacements and velocities for continuous monitoring purposes. On the other hand, recent developments in highly accurate low consumption wireless MEMS accelerometers permit continuous and accurate acceleration measurements when dealing with structural systems. Dealing with operational conditions the uncertainties stemming from the absence of information on the input force, model inaccuracy and measurement errors render the state estimation a challenging task, with research to achieve a robust solution still in progress. Eftekhar Azam et al. [1] have proposed a novel dual Kalman filter to accomplish the task of joint input-state estimation for linear time invariant systems. In this study, the extension of such a scheme is considered for the joint input-state and parameter estimation of linear systems.*

1 INTRODUCTION

This paper focuses on development of algorithms that facilitates the problem of full response predictions for structural systems with uncertain properties via sparse output-only vibration measurements. The latter is particularly useful within the context of fatigue estimation. The concept of using the estimated structural response for fatigue damage prediction was first proposed by Papadimitriou *et al.* [2], where a method was presented that relies on the Kalman filter for estimating power spectral densities of the strain in the body of the structure, in this manner predicting the remaining fatigue life. To identify the fatigue damage, a time history of the stresses in the hotspot points of the structure is required. To estimate the stress in a point of interest, the displacement field at that point is needed; therefore, a reliable state estimate is crucial for an accurate fatigue damage prediction.

The topic of estimation of the states of a partially observed dynamic system in a stochastic framework has been studied by many scientists and there are well developed algorithms to manage both linear (e.g. the Kalman filter [3]) and nonlinear (e.g. the particle filter [4], the unscented Kalman filter [5, 20]) state-space models. In order to yield accurate estimates of state of the system, in addition to sparse output measurements, the latter algorithms require accurate knowledge of the input to the system. However, in most operational conditions the input is unknown and is practically impossible to measure. Recently, the problem of estimating the states of the system in presence of unknown input has gained interest among the researchers. Gillijns and De Moor proposed a new filter for joint input and state estimation for linear systems with direct transmission, that is globally optimal in the minimum-variance unbiased sense [6]. Lourens *et al.* [7] have proposed an extension of the method developed in Gillijns and De Moors work to cope with the numerical instabilities that arise when the number of sensors surpasses the order of the model. Lourens *et al.* [8] have proposed an augmented Kalman filter for unknown force identification in structural systems, and concluded that the augmented Kalman filter is prone to numerical instabilities due to un-observability issues of the augmented system matrix. Naets *et al.* have proposed application of dummy displacement measurements in combination with AKF for stable force estimation [9]. Eftekhar Azam *et al.* have proposed a dual Kalman filter (DKF) for simultaneous estimate of the input and state of the linear time invariant dynamic systems via sparse acceleration measurements [1].

Hernandez [10] proposed an observer that possesses similar characteristics to the Kalman filter in the sense that it minimizes the trace of the state error covariance matrix. The main notion behind the algorithm is that the proposed observer can be implemented as a modified linear finite element model of the system, subject to collocated corrective forces proportional to the measured response. The latter filter requires the spectral density of the input for optimizing the applied gain. Bernal and Ussia have proposed a sequential deconvolution for input estimation in linear time invariant systems [11]. Kazemi Amiri and Bucher have developed a new parametric impulse response matrix utilized for nodal wind load identification by response measurement [12]. The methods and techniques mentioned herein require a model of the system for estimation of response at non-collocated degrees-of-freedom (DOF). In many cases, the parameters of the mathematical models need to be synchronously updated due to causes such as degradation, or environmental influence, resulting into a joint state and parameter identification problem. In doing so, it is a common practice to augment the state vector with the unknown parameters of the system and estimate the dynamics of the resulting augmented state vector. Within such a context, Eftekhar Azam and Mariani have studied the use of sigma-point Kalman filter and particle filters for identification of nonlinear softening constitutive models [13]. Chatzi and Smyth have applied evolutionary particle filters for identification of states and parameters of a dynamic structure represented by a Bouc-Wen model [14]. In subsequent work, Chatzis *et al.* [21] have explored the coupling of Stochatsic Subspace methods with the UKF for joint state and parameter identification. Eftekhar Azam and Mariani have applied a hybrid extended particle filter for identification of nonlinear parameters of a four story shear-type building [15].

In works reviewed in latter paragraphs the focus has been set either on state-input or state-parameter estimation. This work, overviews existing schemes and proposes a novel approach for simultaneous prediction of input, states and parameters of a structural system. In related work, Naets *et al.* have proposed an estimation technique which employs physical models to perform coupled state/input/parameter estimation [16]. In order to obtain a modeling technique which permits the identification of a wide range of parameters in a generic fashion at a low computational burden, the use of a parametric model reduction technique is proposed. The reduced model is coupled to an extended Kalman filter (EKF) with augmented states for the unknown inputs and parameters. In this study, an extension to the dual Kalman filter scheme proposed by Eftekhar Azam et al. is pursued [1]. The proposed algorithm takes advantage of a dual Kalman filter for estimating the input in a first stage, and an Unscented Kalman Filter (UKF) for jointly estimating the states and parameters of the system in a second stage.

As mentioned in the preceding paragraph, in this study the UKF is adopted for the state-parameter estimation stage. In the realm of automatic control, the Extended Kalman Filter (EKF) has been deemed as the de facto standard for online state and parameter estimation. The EKF is based on successive linearizations of the nonlinear state-space equations at each time instant and application of the standard Kalman filter to the linearized equations. The EKF has been successfully applied to many problems; however, in presence of severe nonlinearities the performance of the EKF can be drastically affected. Moreover, the procedure of the linearization demands an estimate of the jacobian of the nonlinear function, which may not be always practical [17]. To address the shortcomings of the EKF, Julier and Uhlmann have proposed the unscented Kalman filter, where the statistics of the state and observation process are propagated directly through a minimal set of quadrature points [5]. The UKF requires several direct analyses of the numerical model of the system and is may become computationally cumbersome. In alleviating the latter issue, Eftekhar Azam *et al.* have proposed a parallelization scheme for the UKF [5], and have studied the scalability and efficiency issues when the parallelization is pursued within a shared-memory (OpenMP) architecture [18].

In what follows, in Sec. 2 the mathematical formulation of the problem and the relevant notations are introduced. In Sec.3 the novel state, input and parameter estimation method is outlined. In Subsection 3.1 the dual Kalman filter approach for input estimation based on acceleration observation is explained. Subsequently, Subsection 3.2 overviews the unscented Kalman filter implementation for state-parameter estimation and Subsection 3.3 summarizes the initialization, measurement update and time update phases of the proposed scheme. Section 4 is devoted to the demonstration of numerical results obtained by applying the proposed method to a simulated example, while in Section 5 the results are summarized and some guidelines for further research are provided.

2 PROBLEM FORMULATION

2.1 Preliminaries

A linear structural system with n DOFs can be represented by a second–order vector differential equation as

$$\mathbf{M}\ddot{\mathbf{u}}(t)+\mathbf{C}\dot{\mathbf{u}}(t)+\mathbf{K}\mathbf{u}(t)=\mathbf{f}(t)=\mathbf{S}_p\mathbf{p}(t) \tag{1}$$

in which $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}\in\mathbb{R}^{n\times n}$ are the mass, viscous damping and stiffness matrices, respectively, $\mathbf{u}$(t) $\in\mathbb{R}^n$ is the vibration displacement vector, and $\mathbf{f}$(t) $\in\mathbb{R}^n$ is the vector of excitations expressed as a superposition of time histories $\mathbf{p}(t)\in\mathbb{R}^m$ $(m\leq n)$ that act on specific DOFs of the structure, as indicated by the influence matrix $\mathbf{S}_p\in\mathbb{R}^{n\times m}$. In Eq. 1 it is assumed that the stiffness matrix is amenable to unknown/unmodelled changes, due to material degradation or environmental influences, which result in a shift of the system properties.

By defining the $[2n\times 1]$ state vector as $\mathbf{x}(t)=[\mathbf{u}^T(t)\ \ \dot{\mathbf{u}}^T(t)]^T$, a state–space representation of Eq. 1 is given by

$$\dot{\mathbf{x}}(t)=\mathbf{A}_c\mathbf{x}(t)+\mathbf{B}_c\mathbf{p}(t) \tag{2a}$$
$$\mathbf{d}(t)=\mathbf{G}_c\mathbf{x}(t)+\mathbf{J}_c\mathbf{p}(t) \tag{2b}$$

where the state and the transmission matrices are defined as

$$\mathbf{A}_c=\begin{bmatrix}\mathbf{O}_n & \mathbf{I}_n\\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C}\end{bmatrix},\quad \mathbf{B}_c=\begin{bmatrix}\mathbf{0}\\ \mathbf{M}^{-1}\mathbf{S}_p\end{bmatrix} \tag{3}$$

and the output and feedforward matrices assume the form

$$\mathbf{G}_c=\begin{bmatrix}\mathbf{S}_d & \mathbf{O}\\ \mathbf{O} & \mathbf{S}_v\\ -\mathbf{S}_a\mathbf{M}^{-1}\mathbf{K} & -\mathbf{S}_a\mathbf{M}^{-1}\mathbf{C}\end{bmatrix},\quad \mathbf{J}_c=\begin{bmatrix}\mathbf{0}\\ \mathbf{0}\\ \mathbf{S}_a\mathbf{M}^{-1}\mathbf{S}_p\end{bmatrix} \tag{4}$$

The matrices of Eq. 4 have been formulated by considering that the output vector $\mathbf{d}(t)$ may contains combined vibration displacement, velocity and acceleration measurements from specific DOFs, in a way that is described by the selection matrices $\mathbf{S}_d$, $\mathbf{S}_v$ and $\mathbf{S}_a$, of appropriate dimensions.

Assuming constant inter–sample behaviour of the input signal $\mathbf{p}(t)$ (e.g., zero–order hold principle), the discrete–time equivalent of Eq. 2 is expressed as

$$\mathbf{x}_{t+1}=\mathbf{A}\mathbf{x}_t+\mathbf{B}\mathbf{p}_t \tag{5a}$$
$$\mathbf{d}_t=\mathbf{G}\mathbf{x}_t+\mathbf{J}\mathbf{p}_t \tag{5b}$$

with t now denoting the discrete–time index, $t=0,T_s,2T_s,\ldots$ (T_s (s) is the sampling period), $\mathbf{A}=e^{\mathbf{A}_cT_s}$, $\mathbf{B}=\left[\mathbf{A}-\mathbf{I}\right]\mathbf{A}_c^{-1}\mathbf{B}_c$, $\mathbf{G}=\mathbf{G}_c$ and $\mathbf{J}=\mathbf{J}_c$.

2.2 The estimation problem

If the stiffness matrix $\mathbf{K}$ depends on a set of parameters stored in vector $\boldsymbol{\theta}=[k_1\ \ k_2\ \ldots\ k_n]^T$, the problem considered herein pertains to the online estimation of $\boldsymbol{\theta}$, $\mathbf{x}_t$ and $\mathbf{p}_t$, under the availability of (i) noise–corrupted observations $\mathbf{d}_t$ and (ii) the mass and damping matrices of the structure. Mathematically, this problem is represented by the following augmented state–space model

$$\mathbf{z}_{t+1} = \mathbf{A}^a\mathbf{z}_t + \mathbf{B}^a\mathbf{p}_t + \mathbf{v}_t \tag{6a}$$
$$\mathbf{d}_t = \mathbf{G}^a\mathbf{z}_t + \mathbf{J}^a\mathbf{p}_t + \mathbf{w}_t \tag{6b}$$

with $\mathbf{z}_t = [\mathbf{x}_t^T \;\; \boldsymbol{\theta}_t^T]^T \in \mathbb{R}^{3n}$ denoting the augmented state vector, $\mathbf{v}_t$ and $\mathbf{w}_t$ the zero mean process and measurement noises with covariance matrices $\mathbf{Q}$ and $\mathbf{R}$, respectively, and $\mathbf{A}^a$, $\mathbf{B}^a$, $\mathbf{G}^a$ and $\mathbf{J}^a$ the augmented state–space matrices defined as

$$\mathbf{A}^\alpha = \begin{bmatrix} \mathbf{A} & \mathbf{O} \\ \mathbf{O} & \mathbf{I} \end{bmatrix}, \; \mathbf{B}^\alpha = \begin{bmatrix} \mathbf{B} \\ \mathbf{0} \end{bmatrix}, \; \mathbf{G}^\alpha = \begin{bmatrix} \mathbf{G} & \mathbf{0} \end{bmatrix}, \; \mathbf{J}^\alpha = \begin{bmatrix} \mathbf{J} \\ \mathbf{0} \end{bmatrix} \tag{7}$$

3 THE STATE, INPUT AND PARAMETER ESTIMATION PROCEDURE

The proposed algorithm takes advantage of a DKF for estimating the input and of an UKF for estimating both the states and the unknown parameters of the structure.

3.1 Input estimation: the DKF

The method introduces a fictitious process equation that describes the input force as

$$\mathbf{p}_{t+1} = \mathbf{p}_t + \mathbf{v}_t^p \tag{8}$$

where $\mathbf{v}_t^p$ is a zero mean white Gaussian process with an associated covariance matrix $\mathbf{Q}^p$. In this way a new state–space model can be obtained using Eqs. 6b. 7, in which the observed quantity is $\mathbf{d}_t$, the unknown state is $\mathbf{p}_t$ and where the actual sought–for state $\mathbf{z}_t$ plays the role of a known input to the system, when an estimate becomes available through the UKF:

$$\mathbf{p}_{t+1} = \mathbf{p}_t + \mathbf{v}_t^p \tag{9a}$$
$$\mathbf{d}_t = \mathbf{J}^\alpha\mathbf{p}_t + \mathbf{G}^\alpha\mathbf{z}_t + \mathbf{w}_t \tag{9b}$$

Thus, through the implementation of the standard Kalman filter an online estimation of $\mathbf{p}_t$ can be obtained. To this, the measurement update step calculates the input gain, mean and covariance as,

$$\left(\mathbf{J}^\alpha\mathbf{P}_t^{p-}\mathbf{J}^{\alpha\,T} + \mathbf{R}\right)\mathbf{G}_t^p = \mathbf{P}_t^{p-}\mathbf{J}^{\alpha\,T} \tag{10a}$$
$$\mathbf{p}_t = \mathbf{p}_t^- + \mathbf{G}_t^p\left(\mathbf{d}_t - \mathbf{G}^\alpha\mathbf{z}_t^- - \mathbf{J}^\alpha\mathbf{p}_t^-\right) \tag{10b}$$
$$\mathbf{P}_t^p = \mathbf{P}_t^{p-} + \mathbf{G}_t^p\mathbf{J}^\alpha\mathbf{P}_t^{p-} \tag{10c}$$

where $\mathbf{p}_t^-$, $\mathbf{P}_t^{p-}$ and $\mathbf{z}_t^-$ denote predictions of the input mean, input covariance and state mean at time $t-1$, respectively. Accordingly, during the time update step, the input mean and covariance predictions for time $t+1$ are provided by

$$\mathbf{p}_{t+1}^- = \mathbf{p}_t \tag{11a}$$
$$\mathbf{P}_{t+1}^- = \mathbf{P}_t^p + \mathbf{Q}^p \tag{11b}$$

3.2 State and parameter estimation: the UKF

The UKF is employed herein for obtaining a solution to the joint state and parameter estimation problem [5, 14, 17]. The nonlinearity herein stems from the binomial products of system states $\mathbf{x}$ and parameters $\boldsymbol{\theta}$ in the system matrices of Eq. 6. Starting from this latter state–space formulation and under the availability of measurement data at time t, that is, the input estimate $\mathbf{p}_t$ and the output $\mathbf{d}_t$, a set of sigma points is calculated (refer to the next section for all undefined quantities)

$$\mathbf{Z}_t^- = \left[\mathbf{z}_t^- \ \ \dots \ \ \mathbf{z}_t^-\right] + \sqrt{c}\left[\mathbf{0} \ \ \sqrt{\mathbf{P}_t^-} \ \ -\sqrt{\mathbf{P}_t^-}\right] \tag{12}$$

and directed to Eq. 6b to calculate a set of output vectors

$$\widehat{\mathbf{D}}_t^- = g\left(\mathbf{Z}_t^-, t\right) \tag{13}$$

Accordingly, the output mean and covariance, as well as the cross covariance between the state and the output are calculated by,

$$\hat{\mathbf{d}}_t = \widehat{\mathbf{D}}_t^- \boldsymbol{\mu}_z \tag{14a}$$

$$\mathbf{P}_t^{dd} = \widehat{\mathbf{D}}_t^- \mathbf{M} \widehat{\mathbf{D}}_t^{-T} + \mathbf{R} \tag{14b}$$

$$\mathbf{P}_t^{pd} = \mathbf{Z}_t^- \mathbf{M} \widehat{\mathbf{D}}_t^T \tag{14c}$$

respectively, while the filter gain $\mathbf{K}_t$ is calculated by

$$\mathbf{P}_t^{dd}\mathbf{K}_t = \mathbf{P}_t^{pd} \tag{15}$$

The updated state mean and covariance are estimated by

$$\mathbf{z}_t = \mathbf{z}_t^- + \mathbf{K}_t\left[\mathbf{d}_t - \hat{\mathbf{d}}_t\right] \tag{16a}$$

$$\mathbf{P}_t = \mathbf{P}_t^- + \mathbf{K}_t \mathbf{P}_t^{dd} \mathbf{K}_t^T \tag{16b}$$

In the time update step a new set of sigma points is calculated

$$\mathbf{Z}_t = \left[\mathbf{z}_t \ \ \dots \ \ \mathbf{z}_t\right] + \sqrt{c}\left[\mathbf{0} \ \ \sqrt{\mathbf{P}_t} \ \ -\sqrt{\mathbf{P}_1}\right] \tag{17}$$

and directed to Eq. 6a to calculate a set of state vectors

$$\widehat{\mathbf{Z}}_t = f\left(\mathbf{Z}_t, 1\right) \tag{18}$$

Then, a prediction of the state mean and covariance for time $t+1$ is obtained by

$$\mathbf{z}_{t+1}^- = \widehat{\mathbf{Z}}_t \boldsymbol{\mu}_z \tag{19a}$$

$$\mathbf{P}_{t+1}^- = \widehat{\mathbf{Z}}_t \mathbf{M} \widehat{\mathbf{Z}}_t^T + \mathbf{Q} \tag{19b}$$

3.3 The proposed DKF-UKF estimation algorithm

The steps of the proposed method are as follows:

INITIALIZE

1. Set initial values for the input vector: $\mathbf{p}_0$ and $\mathbf{P}_0^p$
2. Set initial values for the augmented state vector: $\mathbf{z}_0$ and $\mathbf{P}_0$
3. Set the parameters of the UKF (n_z the size of the augmented state vector) [19]:
   - $\alpha = 1$, $\beta = 2$, $\kappa = 0$
   - $\lambda = \alpha^2(n_z+\kappa) - n_z$, $c = \alpha^2(n_z+\kappa)$
   - $W_m^0 = \lambda/(n_z+\lambda)$, $W_m^i = 1/2(n_z+\lambda)$, $i = 1,2,\ldots,2n_z$
   - $W_c^0 = \lambda/(n_z+\lambda) + (1-\alpha^2+\beta)$, $W_c^i = W_m^i$, $i = 1,2,\ldots,2n_z$
   - $\boldsymbol{\mu}_z = [W_m^0 \ \ldots \ W_m^{2n_z}]^T$
   - $\mathbf{M} = \left(\mathbf{I} - [\boldsymbol{\mu}_z \ \ldots \ \boldsymbol{\mu}_z]\right) \times diag(W_c^0 \ \ldots \ W_c^{2n_z}) \times \left(\mathbf{I} - [\boldsymbol{\mu}_z \ \ldots \ \boldsymbol{\mu}_z]\right)^T$

UPDATE at time t (when measured output $\mathbf{d}_t$ is available)

1. Calculate input gain: $\left(\mathbf{J}^\alpha \mathbf{P}_t^{p-} \mathbf{J}^{\alpha\, T} + \mathbf{R}\right)\mathbf{G}_t^p = \mathbf{P}_t^{p-}\mathbf{J}^{\alpha\, T}$
2. Update input mean and covariance:

$$\mathbf{p}_t = \mathbf{p}_t^- + \mathbf{G}_t^p\left(\mathbf{d}_t - \mathbf{G}^a\mathbf{z}_t^- - \mathbf{J}^\alpha\mathbf{p}_t^-\right)$$
$$\mathbf{P}_t^p = \mathbf{P}_t^{p-} + \mathbf{G}_t^p\mathbf{J}^\alpha\mathbf{P}_t^{p-}$$

3. Calculate sigma points: $\mathbf{Z}_t^- = \left[\mathbf{z}_t^- \ \ldots \ \mathbf{z}_t^-\right] + \sqrt{c}\left[\mathbf{0} \ \sqrt{\mathbf{P}_t^-} \ -\sqrt{\mathbf{P}_t^-}\right]$
4. Propagate sigma points through the output equation: $\widehat{\mathbf{D}}_t^- = g\left(\mathbf{Z}_t^-, t\right)$
5. Calculate output mean and covariance:

$$\hat{\mathbf{d}}_t = \widehat{\mathbf{D}}_t^-\boldsymbol{\mu}_z$$
$$\mathbf{P}_t^{dd} = \widehat{\mathbf{D}}_t^-\mathbf{M}\widehat{\mathbf{D}}_t^{-T} + \mathbf{R}$$

6. Calculate cross covariance between state and output: $\mathbf{P}_t^{pd} = \mathbf{Z}_t^-\mathbf{M}\widehat{\mathbf{D}}_t^{-T}$
7. Calculate state gain: $\mathbf{P}_t^{dd}\mathbf{K}_t = \mathbf{P}_t^{pd}$
8. Update state mean and covariance:

$$\mathbf{z}_t = \mathbf{z}_t^- + \mathbf{K}_t\left[\mathbf{d}_t - \hat{\mathbf{d}}_t\right]$$
$$\mathbf{P}_t = \mathbf{P}_t^- + \mathbf{K}_t\mathbf{P}_t^{dd}\mathbf{K}_t^T$$

PREDICT at time t

1. Predict input mean and covariance for $t+1$:

$$\mathbf{p}_{t+1}^{-} = \mathbf{p}_t$$
$$\mathbf{P}_{t+1}^{-} = \mathbf{P}_t^{p} + \mathbf{Q}^{p}$$

2. Calculate sigma points: $\mathbf{Z}_t = \left[\mathbf{z}_t \ \ \ldots \ \ \mathbf{z}_t\right] + \sqrt{c}\left[\mathbf{0} \ \ \sqrt{\mathbf{P}_t} \ \ -\sqrt{\mathbf{P}_t}\right]$

3. Propagate sigma points through the state equation: $\widehat{\mathbf{Z}}_t = f\left(\mathbf{Z}_t, t\right)$

4. Predict state mean and covariance for $t+1$:

$$\mathbf{z}_{t+1}^{-} = \widehat{\mathbf{Z}}_t \boldsymbol{\mu}_z$$
$$\mathbf{P}_{t+1}^{-} = \widehat{\mathbf{Z}}_t \mathbf{M} \widehat{\mathbf{Z}}_t^{T} + \mathbf{Q}$$

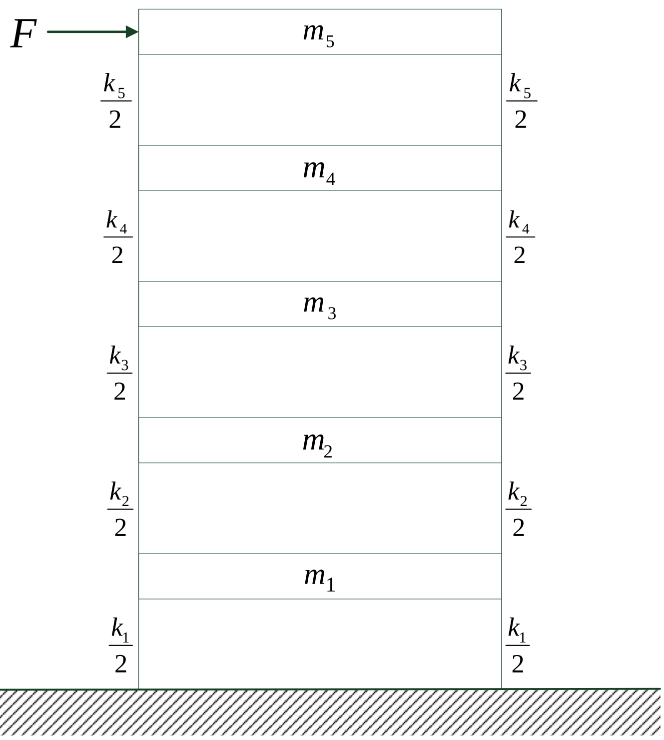

Figure 1: The simulated five–story shear frame structure.

4 APPLICATION

The proposed DKF–UKF framework is now applied to the input, state and parameter estimation problem of a simulated five–story shear frame (Fig. 1) subject to wind excitation, using limited output observation. Each story is modeled as a lumped mass, while the vertical columns are modeled as massless springs with equivalent stiffness. Table 1 illustrates the selected physical parameters and the associated vibration modes of the shear building. Regarding the latter,

Table 1: Physical and modal parameters of the building

	Physical Space			Modal Space	
Story	m_j (Mgr)	k_j (kN/m)	**Mode**	f_n (Hz)	ζ_n $(\%)$
1	10	10000	1	1.566	1.000
2	8	10000	2	4.400	1.362
3	8	9000	3	6.829	1.553
4	8	9000	4	8.922	1.814
5	8	9000	5	10.259	1.000

energy dissipation in the form of Rayleigh damping has been adopted, with the first and the last mode being characterized by 1% modal damping. It is assumed that the shear velocity field is such that the corresponding wind pressures are non–negligible only in the highest story, producing thus the force excitation that is depicted at Fig. 1.

The simulation data are obtained through the discretization of the structural equation (in fact its state–space representation) into the state–space model of Eq. (5), at a sampling period $T_s = 0.0025s$, using the zero–order hold. A zero mean Gaussian white noise process with variance 10kN is used as excitation. Throughout the simulation both the state and the output equations are noise–corrupted by zero mean Gaussian white noise processes of covariance matrices

$$\mathbf{Q}^s = 10^{-12}\mathbf{I}_{10} \text{ and } \mathbf{R} = diag(4\cdot 10^{-3}, 10^{-4}, 10^{-4})$$

respectively. The size of $\mathbf{R}$ stems from the fact that the vibration acceleration response from only the first, the fourth and the last story is considered available. The augmented state equation's noise covariance matrix $\mathbf{Q}$ is set as

$$\mathbf{Q} = diag(\mathbf{Q}^s, 10^{-8}\mathbf{I}_5)$$

and the one of the fictitious Eq. 7 as $\mathbf{Q}^p = 10^{-8}$.

The initial parameter vector and its covariance matrix are set as

$$\theta_0 = [6000\ \ 6000\ \ 6000\ \ 6000\ \ 6000]^T$$
$$\mathbf{Q}_0^\theta = diag(0.2\cdot 10^{-5}, 1.0\cdot 10^{-5}, 0.8\cdot 10^{-5}, 0.8\cdot 10^{-5}, 2.0\cdot 10^{-5})$$

respectively, while the initial state vector $\mathbf{x}_0$ is set to zero and $\mathbf{Q}_0^x = 10^{-15}\mathbf{I}_{10}$. Thus,

$$\mathbf{z}_0 = [\mathbf{x}_0\ \ \boldsymbol{\theta}_0]^T$$
$$\mathbf{P}_0 = diag(\mathbf{Q}_0^x, \mathbf{Q}_0^\theta)$$

Figures 2–5 display the outcome of a 300s simulation. In general, the results are very encouraging, as the estimated quantities follow the true ones with a high degree of accuracy. In particular, the input force estimate (Fig. 2) follows the real force quite accurately already from the start of the simulation time and before the convergence of the unknown parameters. Correspondingly, similar performance is observed in the displacement and velocity states, aside form minor discrepancies (Figs. 3–4). Finally, the unknown stiffness parameters have also been successfully identified (Fig. 5 and Tab. 2) with an exception of k_5, which seems to converge in a value lower (approximately 15%) than its nominal value. However, this does not affect the estimation of the directly affected states, indicating a lesser influence of this parameter on the model performance under the particular excitation, additionally verifying the proposed method's robustness.

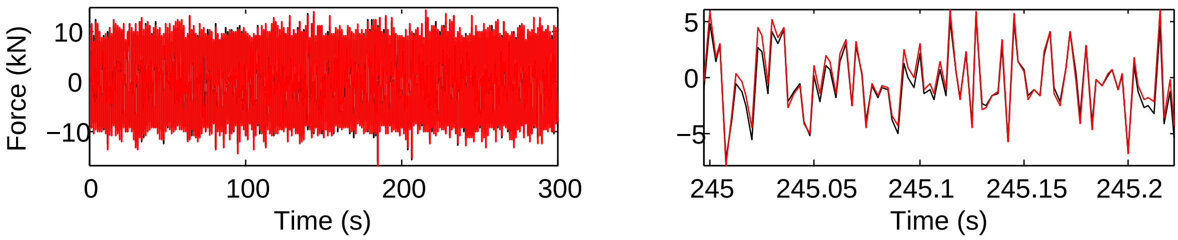

Figure 2: Actual (black) versus estimated (red) input force. Left figure: total simulation time. Right figure: approximately 0.25s detail.

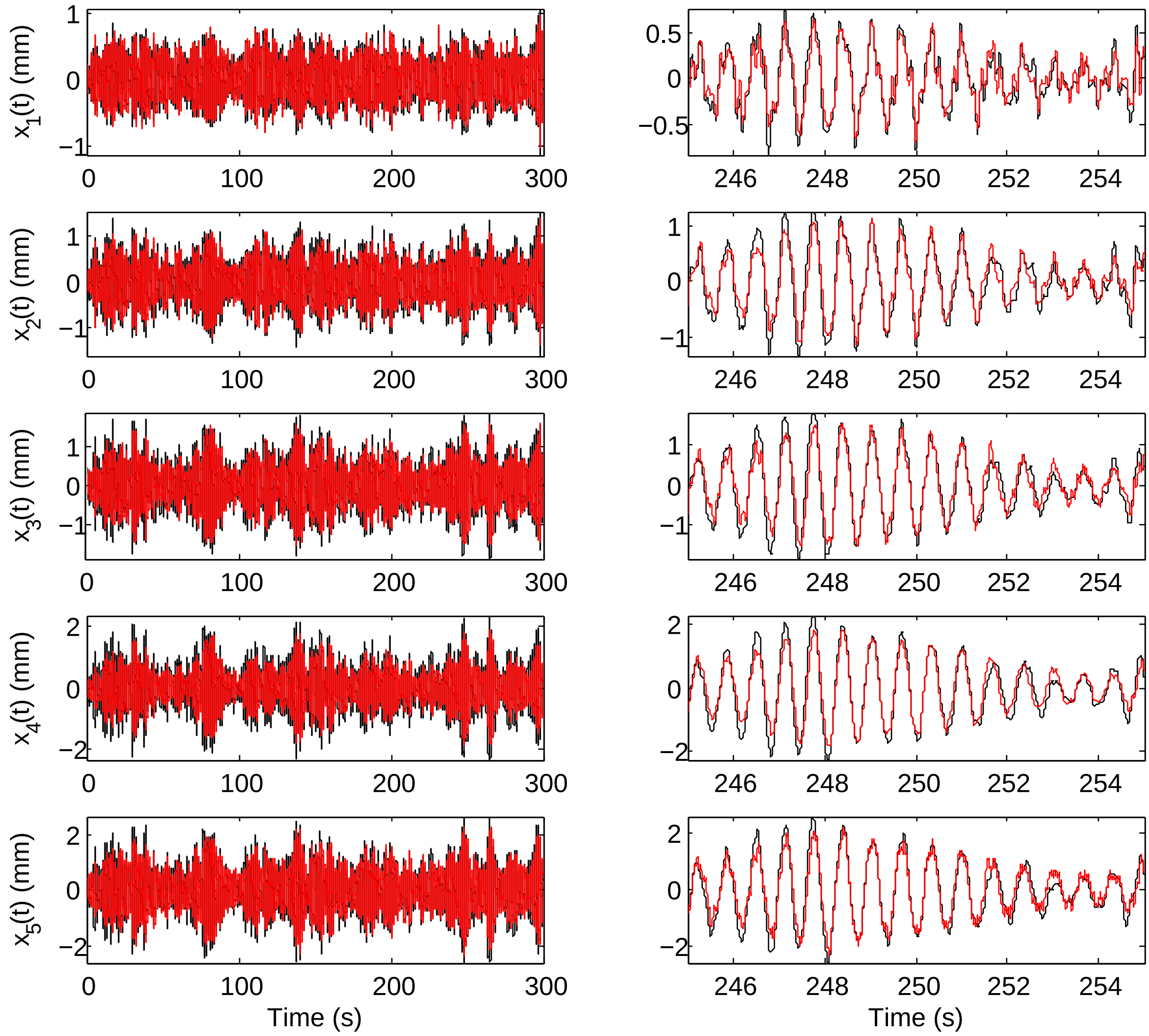

Figure 3: Actual (black) versus estimated (red) displacements. Left column: total simulation time. Right column: approximately 10s detail.

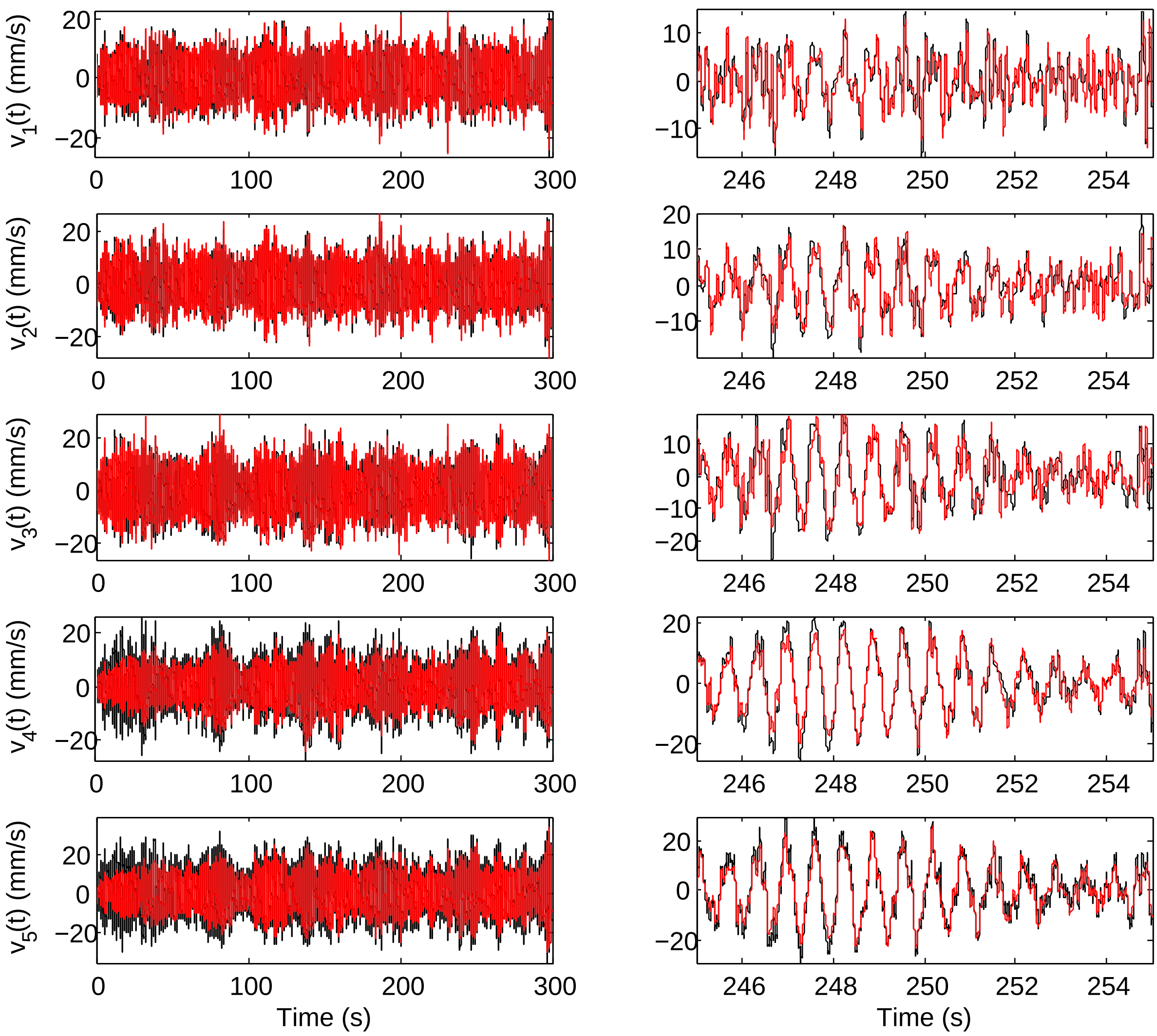

Figure 4: Actual (black) versus estimated (red) velocities. Left column: total simulation time. Right column: approximately 10s detail.

5 CONCLUSIONS

The joint input–state–parameter estimation problem of a structural system of uncertain properties using limited, noise–corrupted observations was the main pursuit of the current study. To this end, a DKF–UKF framework was developed and applied to a simulated frame subject to wind loading. The proposed scheme introduces a fictitious process equation that aims at resembling the unknown dynamics of the structural excitation and designs a DKF for the measurement and time update of the unknown input. This input is then forwarded to an augmented state–space model, the state vector of which contains both the original states of the structure and a vector of unknown structural parameters. Since the latter two quantities are nonlinearly related, the corresponding state–parameter estimation problem is handled by the UKF.

The illustrated results indicate a robust performance under purely random excitation and suggest further exploration, towards a number of issues that deserve further investigation. Among others, the applicability and the robustness of the fictitious process equation for the description of the unknown input needs validation and adjustment over a range of different input classes

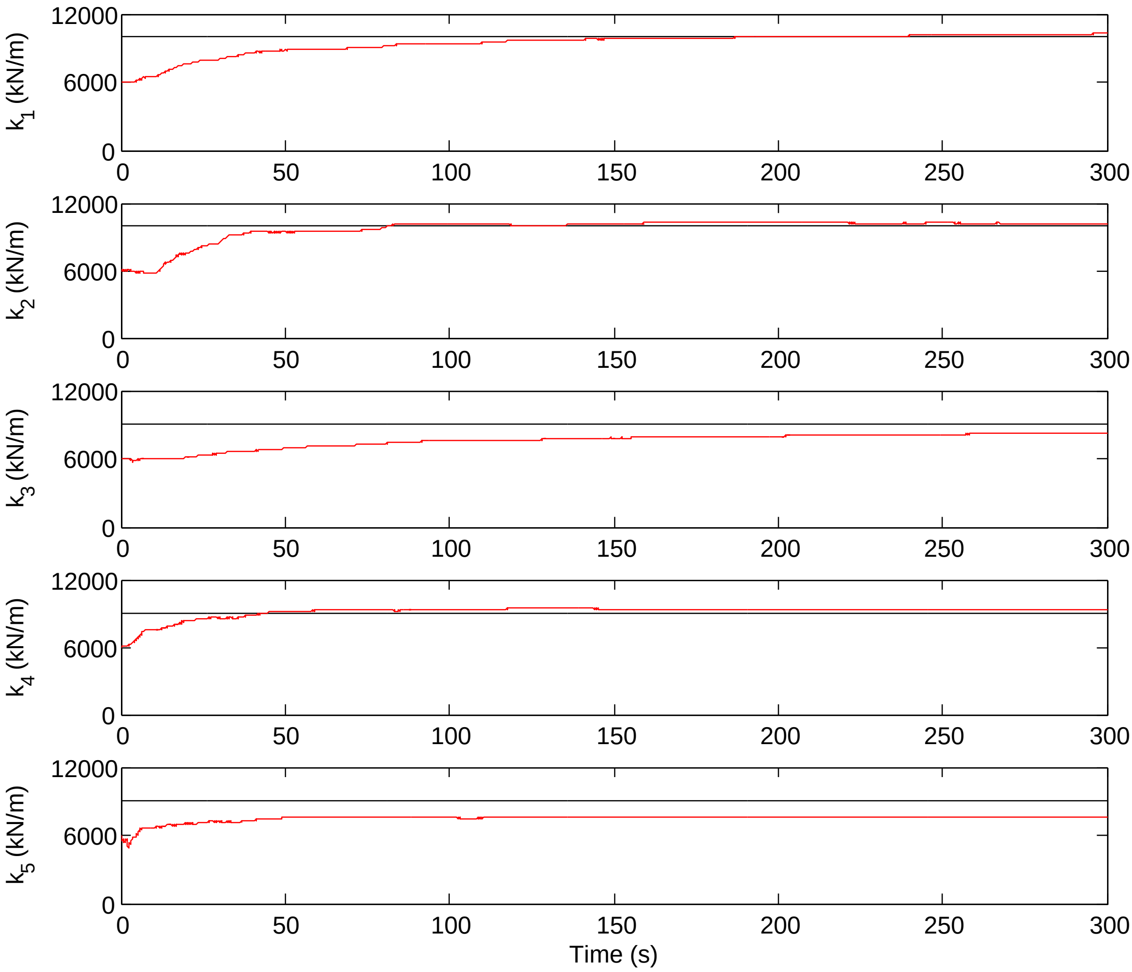

Figure 5: Actual (black) versus estimated (red) story stiffness.

that include purely sinusoidal and/or nonstationary characteristics, as well as for different level of system uncertainties, and limited observation sets. Extensions to time–varying structures is also a topic of current research undertaken by the authors, in an effort to formulate a rigorous and robust framework for fatigue prediction of structures under realistic conditions.

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Table 2: Percentage errors between actual and estimated story stiffness. Units in kN/m.

	k_1	k_2	k_3	k_4	k_5
True	10000	10000	9000	9000	9000
Estimated	10257	10152	8206	9314	7609
Error (%)	2.567	1.515	8.818	3.485	15.450

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