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Two Dimensional Random Field of Concrete Compressive Strength for RC Frames

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Abstract: *The basic concept of random field theory is applied to three reinforced concrete frames. The three frames have the same concrete grade, distributed reinforcement and curing condition. By using the rebound experimental testing, the concrete compressive strength of each test zone involved in the beam and column of frames is obtained for the development of two-dimensional random field. The homogeneous assumption is verified by estimating the*

autocorrelation function of the concrete compressive strength. The two-dimensional power spectral density function is then modeled for the concrete compressive strength by introducing the assumption of the quadrant symmetric correlation structure and separable correlation structure. The stochastic harmonic function method is finally introduced to decompose the target power spectral density of the concrete compressive strength to generate a set of stochastic samples for the stochastic dynamic analysis of reinforced concrete structures.

1 INTRODUCTION

The importance of the stochastic approach for the large-scale engineering structure is receiving lately considerable attention with the development of computational methods and analytical procedures. Influence of the inherent uncertainties on the inelastic behavior of reinforced concrete structures is continuously gaining its significance. The traditional deterministic approach, which is widely used in the engineering practice, cannot address a rational treatment of these uncertainty quantifications. Based on the mean or extreme values of these system uncertainties, results obtained from the deterministic analysis could not represent all possible scenarios. Therefore, the deterministic approach cannot assure the design of reinforced concrete structures is an optimum result. However, the stochastic methods with the help of the powerful computing resources and enhanced solution algorithms provide a possibility in providing a confidence interval at the given confidence level for the nonlinear responses of reinforced concrete structures.

Stochastic finite element method (SFEM) is the main technique in the computational stochastic method to consider the inherent uncertainties, which is an extension of the classical deterministic finite element method (Kleiber et al., 1992). The SFEM uses the finite element theory to treat the structural properties as a set of random variables. According to a wide range of literature review, there are normally three most important alternative formulations among SFEM: the perturbation approach (Liu et al., 1986); the spectral stochastic finite element method and the Monte Carol simulation (Papadrakakis et al., 1996). However, these approaches aim at the second-order statistical quantities of the structural response, which is inadequate to understand the probabilistic information of the structural responses comprehensively. Recently, the probability density evolution method (PDEM) developed by Li and Chen (Li et al., 2004; 2006) with the ability to obtain the instantaneous probability density function (PDF) of the structural response and its evolution, is available for the general stochastic dynamical systems. It has been recognized that the proposed method has considerably advantageous for the existent SFEM (Li et al., 2008; 2009).

However, there is another fundamental issue that needs to be addressed in details, which is denoted by the modeling of uncertainty quantifications. Characteristic examples of those uncertainty quantifications are Young modulus and concrete compressive strength. These uncertainties are usually spatially distributed and correlated over the region of the structure. Their probability distributions and the correlation structure actually should be determined by the experimental measurement. However, due to the lack of the relevant experimental data, assumptions are usually taken in the inelastic analysis of SFEM. The assumption selected by the researchers makes the type of the random field falling into two categories: the homogeneous random field and the non-homogeneous random field. The homogeneous random field occurs naturally due to its simplicity and feasibility. The spectral representation method (Shinozuka et al., 1991) and the Karhunen-Loeve expansion method (Huang et al., 2001) are two main alternative methods to decompose the homogeneous random field in practical application. For the non-homogeneous random field, the SFEM becomes more difficult in the decomposition, although it more conforms to the practical engineering. As a result, the correlation structure is usually subjective to be determined and the physical background of the random field is normally not taken into consideration. Therefore, it is necessary to conduct more experimental measurements on the determination of two-dimensional random field for the reinforced concrete structures.

As such, a rebound experiment testing is firstly presented for three reinforced concrete frames. The three frames have the same concrete grade, distributed reinforcement and curing condition.

Interested material parameter is purely related to the concrete compressive strength. Other concrete parameters can be then estimated from the magnitude of the concrete compressive strength. Furthermore, the autocorrelation function (ACF) of the concrete compressive strength is developed to examine the spatial variability of this significant parameter. Then the stochastic harmonic functions (SHFs) method is introduced for generating a set of stochastic samples with the obtained ACF. The SHFs method avoids the disadvantages of the existing mathematical expansions, such as the Karhunen-Loeve decomposition method and the spectral representation method, it has been verified that only several components of the SHFs are needed to describe the probabilistic information of the quantity of interest adequately. So the computational efficiency is greatly improved because other methods generally involve a large number of random variables to deal with in practical application.

2 REBOUND EXPERIMENT

The correlation structure of the concrete parameter is the foundation of the SFEM for the stochastic dynamical analysis of reinforced concrete structures. Beyond to be direct measured; it is often determined by the assumption. Presently, there is no standard experimental equipment and few conclusions have been made from experimental testing to provide a correlation structure for the concrete compressive strength parameter. So the ACF of the concrete compressive strength is often determined by researchers optionally. Therefore, a rebound experimental testing is introduced to measure the correlation structure of the concrete compressive strength directly. The rebound experimental testing is selected because it provides a large number of closely-spaced and accurate data but has little damage on the experimental specimens. Besides, the rebound testing is relatively cheap, quick and ideal for the comparative and uniformity assessment of the concrete parameters. It should be noted that the rebound experimental testing is based on the principle that the rebound height of an elastic mass depends on the hardness of the surface upon which it impinges, so the measured data give a direct measure about the relative hardness within the limited zone, which will be empirically related to the concrete compressive strength.

Rebound Test Equipment and Procedure

The Schmidt Rebound Hammer has been used worldwide for many years to measure the concrete compressive strength in engineering practice. In testing, the spring-controlled hammer mass slides on the plunger within a tubular housing. The plunger then retracts against the spring when pressed against the concrete surface till the hammer impacts, where the spring-controlled mass rebounds and holds in position on a scale by depressing the locking button. Part of Energy is absorbed during the process due to the internal friction within the concrete body. Finally the scale reading will be recorded as the rebound number.

Based on the "Technical Specification for Inspection of Concrete Compressive Strength by Rebound Method" (JGJ/T 23-2001), the beam-column member of each frame is divided into several test zone. The surface of each zone should be smooth, clean and dry. Each zone takes over an area not exceeding 0.04 *m* square. The centre-to-centre spacing between the adjacent zones should be 300 *mm*. At each test zone, it is necessary to take approximately 16 readings. Then the average rebound number at each test zone is computed by removing three maxima readings and three minima readings. For reducing the operator bias, a grid mesh is necessary for locating these impact points at each test zone as shown in Figure 1. It can be seen from the figure that each beam has five average rebound numbers while each column has ten average rebound numbers. According to the appendix A. in JGJ/T 23-2001, these rebound numbers should be converted to the concrete compressive strength by using the empirical relationship.

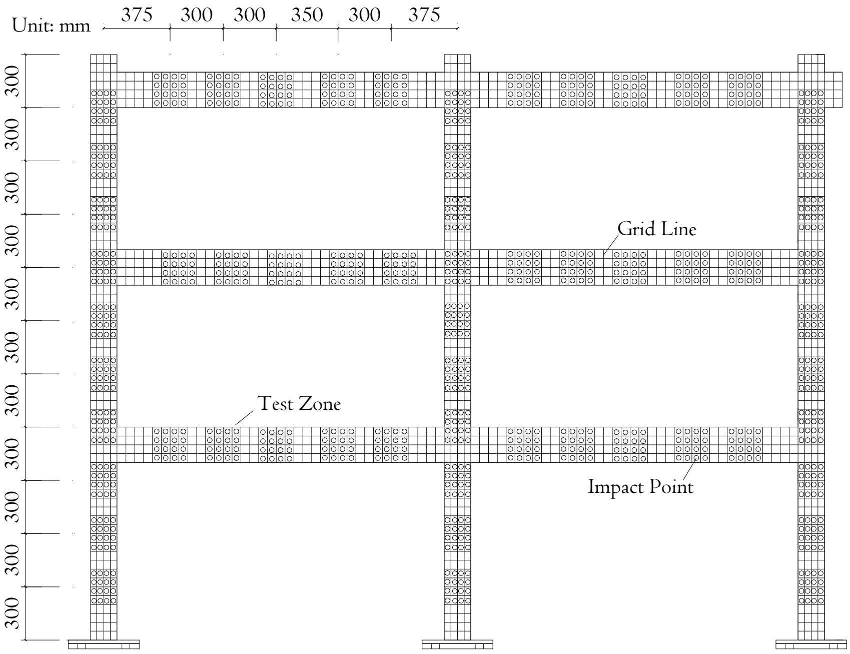


Figure 1 Rebound locations on three reinforced concrete frame structures

Concrete Compressive Strength Located at RC Frame

There are six beams and three columns for each reinforced concrete frame. It is naturally assumed that each beam-column member is a sample of one-dimensional random field. Therefore, the beam has six samples and the column has three samples. The mean value (MV), standard deviation (SD) and coefficient of variation (COV) of the concrete compressive strength located at different position on these members is obtained by using the rebound experimental testing as shown in Table 1 – Table 2. It is observed that the MV of the concrete compressive strength located on beams is much lower than the MV of the strength located on columns. But the COV is on the contrary. The reason may be attributed to fact that there are different compaction degrees in construction process between the reinforced concrete beams and columns. For calibrating the accuracy of the obtained result from the rebound experimental testing, an un-axial compressive testing is further conducted with a batch of concrete prisms. Results from the concrete prisms testing reveal that the mean strength of the concrete compressive strength is 37.17 MPa and the corresponding COV is 9.24%. Apparently, the concrete compressive strength using the rebound experimental testing is much lower than the concrete compressive strength from the un-axial compressive testing, but the COV is vice versa. It can be explained by the fact that results of the bound experimental testing are controlled by more factors, especially the relationship between the hardness and concrete compressive strength is empirical, so that more uncertainties would be included and presented in the rebound experimental testing. Actually, the practical engineering investigations have also verified that the concrete rebound strength is on the low side about 30%.

Position (m)	MV (MPa)	SD (MPa)	COV (%)
1.25	18.71	1.71	9.12
0.95	19.44	1.95	10.02
0.60	19.70	2.24	11.39
0.30	20.00	2.44	12.18
0.00	18.76	1.90	10.12

Table 1 Concrete compressive strength located at different position on beams

Position (m)	MV (MPa)	SD (MPa)	COV (%)
2.70	22.84	1.67	7.30
2.40	23.20	1.65	7.09
2.10	22.82	1.12	5.90
1.80	22.06	1.64	7.41
1.50	22.47	2.02	8.98

1.20	22.82	1.60	7.03
0.90	23.31	1.50	6.43
0.60	25.09	2.03	8.42
0.30	22.19	1.31	5.91
0.00	21.09	3.01	15.3

Table 2 Concrete compressive strength located at different position on columns

3 MODELING OF RANDOM FIELD

The spatial variability of the concrete compressive strength should be quantified by the ACF. Some assumptions are made in the development of the ACF for the simplicity and feasibility. The basic one is the homogeneous random field assumption, which means all the probabilities depend on the relative distance but not the absolute location of each point. The homogeneous random field assumption is generally written as

$$m(u) = Const. \tag{1}$$

$$R(u,u') = R(u-u') = R(\tau) \tag{2}$$

where *m(u)* is the mean value of the random field at location u ; $R(u,u')$ is the ACF of the random field; $\tau = u - u'$ is the distance between the location u and u'. In engineering practice, the distance τ is usually limited within a certain region because of the upper and lower bounds on the distance intervals where the structural property applies. Therefore, Eqn. (1) and (2) can be rewritten as

$$m(u_i) = \frac{1}{N}\sum_{k=1}^{N} B_k(u_i) = E[B(u_i)] \quad i = 1,2,\cdots,m \tag{3}$$

$$R(u_i, u_i + \tau) = \frac{1}{N}\sum_{k=1}^{N} B_k(u_i) B_k(u_i + \tau) = E[B(u_i)B(u_i + \tau)] \quad i = 1,2,\cdots,m \tag{4}$$

where N is the total number of samples; m is the total number of data points; $B_k(u_i)$ is the concrete compressive strength at location u_i; $E[]$ is the expected value. Following these, the procedure for obtaining the ACF of the concrete compressive strength is described as shown in Figure 2.

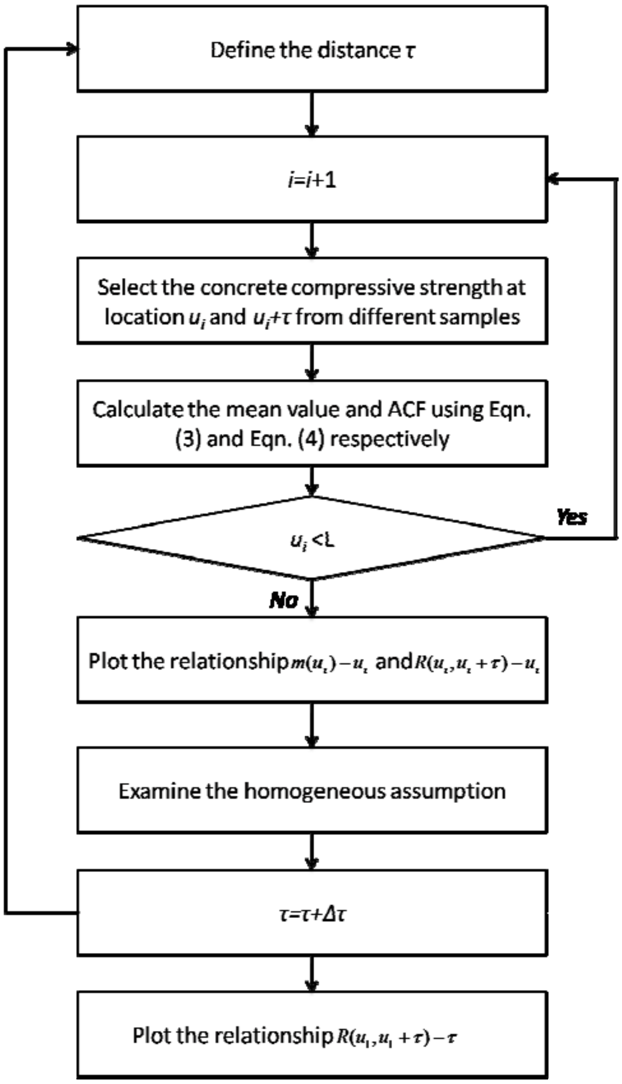


Figure 2 Procedure of obtaining the ACF of the concrete compressive strength

Taken an example by defining the distance of τ at 0.3 *m*, the relationship of $m(u_i)-u_i$ and $R(u_i,u_i+\tau)-u_i$ is shown in Figure 3 - Figure 4 for reinforced concrete beams and columns respectively.

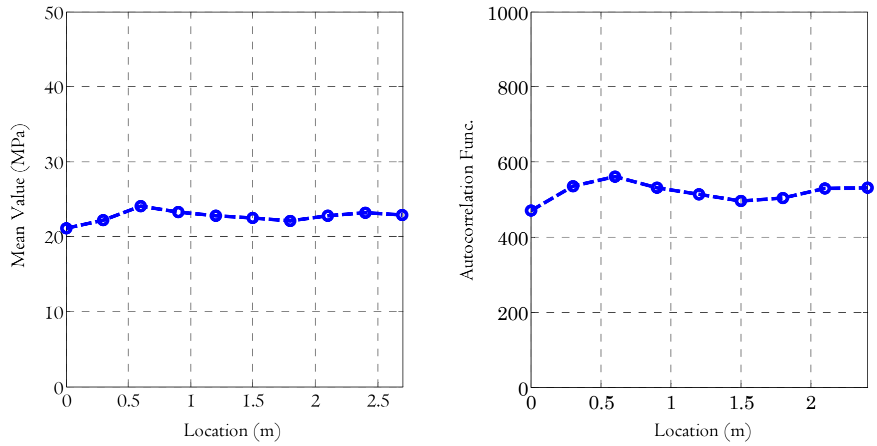


Figure 3 Verification of homogeneous assumption for beams

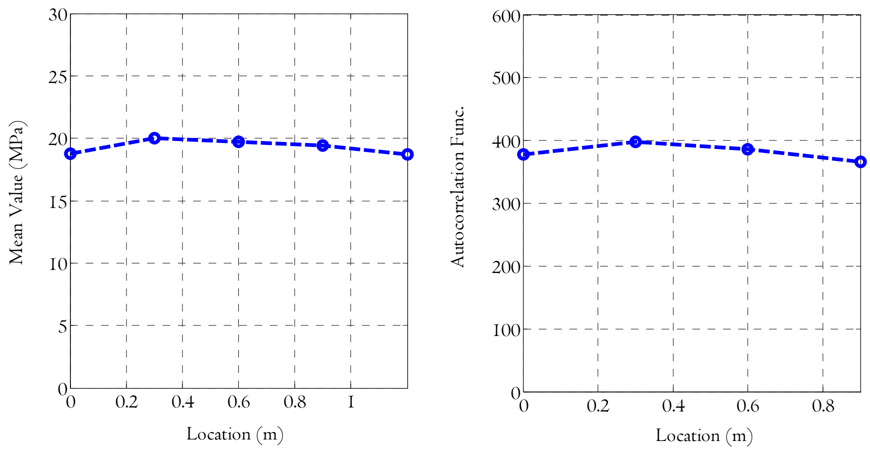


Figure 4 Verification of homogeneous assumption for columns

It is observed that the fluctuation along with the location is relatively slight; indicating the homogeneous assumption is satisfied for the concrete compressive strength. As a result, the ACF of the concrete compressive strength can be estimated after calibrating the experimental data with zero-mean-value as

$$R(\tau) = \frac{1}{N}\sum_{k=1}^{N} B_k(u_0)B_k(u_0+\tau) \tag{5}$$

where u_0 is the original location of beams and columns. Due to the covariance function takes the same expression as the ACF because of the zero-mean-value calibration, the autocorrelation coefficient (ACE) of the concrete compressive strength is then defined by

$$\rho(\tau) = \frac{R(\tau)}{\sigma^2} = \frac{R(\tau)}{R(0)} \tag{6}$$

where σ is the variance of the samples. Figure 5 and Figure 6 shows the ACE for beams and columns respectively.

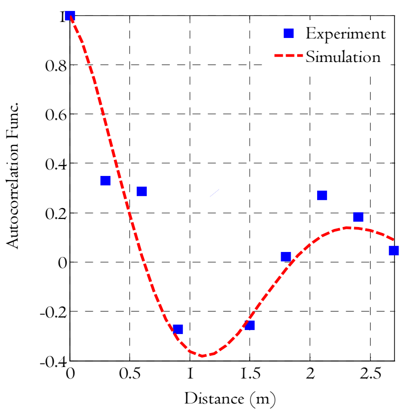


Figure 5 ACE of columns

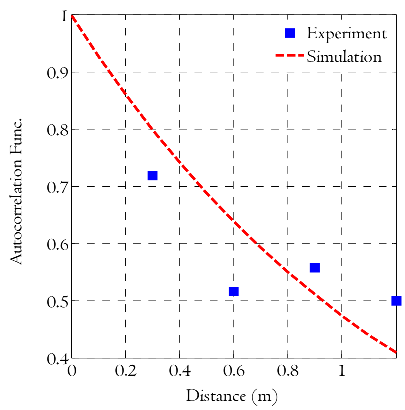


Figure 6 ACE of beams

After using the least square method to fit the experimental data, the ACE of beams and columns can be expressed as:

$$\rho(\tau) = \exp\left(-\frac{\tau}{1.217}\right)\cos(2.547\tau) \quad \tau \geq 0 \qquad \text{For columns} \tag{7}$$

$$\rho(\tau) = \exp\left(-\frac{\tau}{1.34}\right) \quad \tau \geq 0 \qquad \text{For beams} \qquad (8)$$

If introducing the assumption of the quadrant symmetric correlation structure and incorporating the separable correlation structure, a two-dimensional random field can be proposed for the concrete compressive strength of reinforced concrete frames as

$$R(\tau_1, \tau_2) = 11.765 \exp\left[-\left(\frac{|\tau_1|}{1.34} + \frac{|\tau_2|}{1.217}\right)\right] \cos\left(2.547|\tau_2|\right) \qquad (9)$$

where τ_1 and τ_2 are the distance at the horizontal and vertical direction respectively; 11.765 is the square of the variance that determined from the prism testing. Figure 7 illustrates the two-dimensional ACF of the concrete compressive strength for reinforced concrete frames.

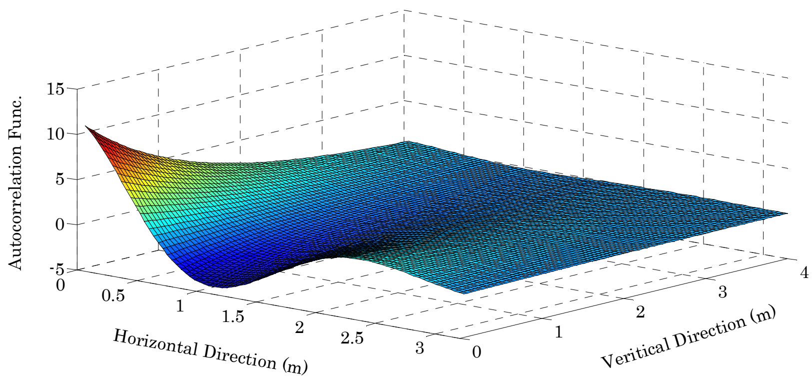


Figure 7 ACF of concrete compressive strength for reinforced concrete frames

The power spectral density (PSD) function as shown in Figure 8 is further obtained by introducing the two-dimensional Wiener-Khinchine transform pair, giving the expression as

$$S(\omega_1,\omega_2)=\frac{1}{(2\pi)^2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}R(\tau_1,\tau_2)e^{-i(\omega_1\tau_1+\omega_2\tau_2)}d\tau_1 d\tau_2 \tag{10}$$

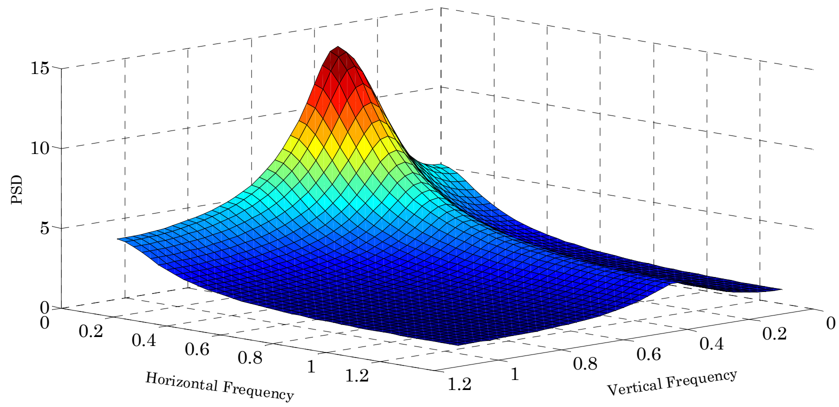


Figure 8 PSD function of concrete compressive strength

4 STOCHASTIC HARMONIC FUNCTIONS

Several methods such as the Karhunen-Loeve decomposition and spectral representation method are available in literatures to generate the stochastic samples of the two-dimensional random field for concrete compressive strength. However, those mathematical expressions have certain disadvantages in practical application. One of the most important aspects is the truncation issue from an infinite series. Usually, in order to avoid the deviation and make the PSD of the generated random field approximating at the target PSD in accuracy, hundreds of components are needed in decomposition, which lead to a large number of basic random variables involved. It is therefore cumbersome to handle so many random variables in numerical simulation. As a result, the SHFs method developed by Chen et al. is presented in this paper to generate the two-dimensional stochastic samples for the concrete compressive strength with the prescribed target PSD function (Chen et al., 2011; 2013). The method decomposes the two-dimensional random field by using a set of pre-defined basic random variables. Several components of the SHFs are adequate for generating the two-dimensional stochastic samples. It therefore improves the computational efficiency greatly. The SHFs method considers the two-dimensional random field as a linear superposition of a series of stochastic harmonic functions as

$$f(x_1,x_2)=\sqrt{2}\sum_{n_1=1}^{N_1}\sum_{n_2=1}^{N_2}\Big[A_{n_1n_2}\cos(K_{1n_1}x_1+K_{2n_2}x_2+\Phi^1_{n_1n_2}) \\ +\overline{A}_{n_1n_2}\cos(K_{1n_1}x_1-K_{2n_2}x_2+\Phi^2_{n_1n_2})\Big] \tag{11}$$

where the PDF of random frequencies K_{1n_1} and K_{2n_2} can be written as

$$p_{K_{in_i}}(K_i)=\begin{cases}\dfrac{1}{K^p_{in_i}-K^p_{in_{i-1}}}=\dfrac{1}{\Delta K_{in_i}} & K_i\in\left(K^p_{in_{i-1}},K^p_{in_i}\right],i=1,2\\ 0 & others\end{cases} \tag{12}$$

The amplitudes $A_{n_1n_2}$ and $\overline{A}_{n_1n_2}$ are the function of random frequencies respectively.

$$A_{n_1n_2}=\sqrt{2S(K_{1n_1},K_{2n_2})\Delta K_{1n_1}\Delta K_{2n_2}} \tag{13}$$

$$\overline{A}_{n_1n_2}=\sqrt{2S(K_{1n_1},-K_{2n_2})\Delta K_{1n_1}\Delta K_{2n_2}} \tag{14}$$

where $S(K_{1n_1},K_{2n_2})$ and $S(K_{1n_1},-K_{2n_2})$ are the target PSD of the two-dimensional random field. For a case study, the generated samples of the concrete compressive strength are illustrated in Figure 9 by selecting the stochastic harmonic components with $N_1=N_2=6$。

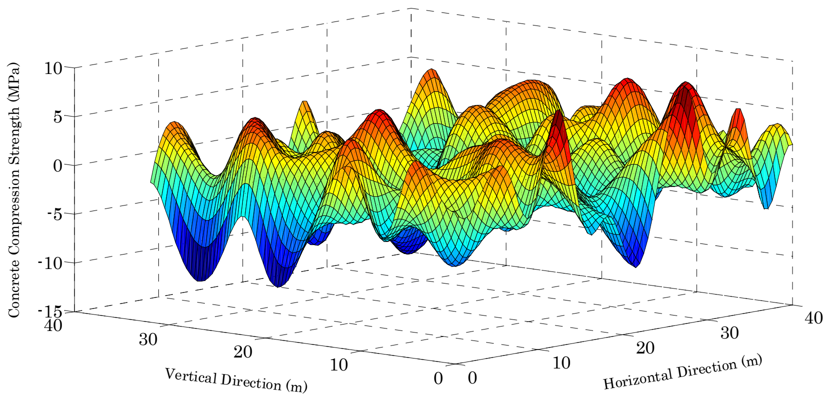
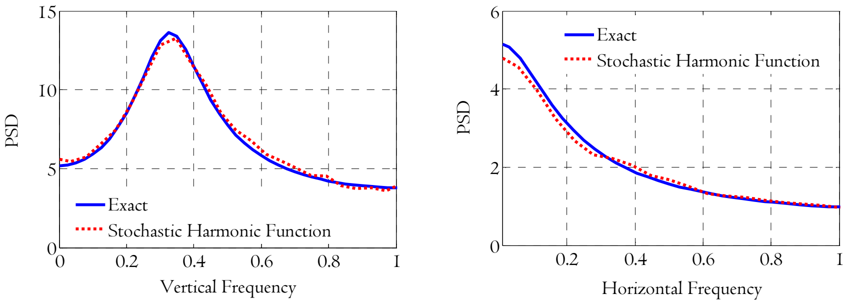


Figure 9 The No. 100 sample of the concrete compressive strength

The cross-section of the PSD together with the target PSD when K_1=0 and K_2=0 are calculated as shown in Figure 10. It is observed that the PSD of the SHFs method accords well with the target PSD.



$K_1 = 0$ $K_2 = 0$

Figure 10 Cross-section of the SHFs PSD and the target PSD

5 CONCLUSION

The concrete compressive strength of three reinforced concrete frames which have the same concrete grade, distributed reinforcement and curing condition is investigated by using the rebound experimental testing. A two-dimensional autocorrelation function of the concrete compressive strength is developed after introducing the basic concept of the random field theory. The stochastic harmonic function method is then presented to decompose the autocorrelation function of the concrete compressive strength for generating a set of stochastic samples for the stochastic dynamical analysis of reinforced concrete frames. The work can be further integrated with the probability density evolution method (PDEM) to investigate the stochastic nonlinear mechanical behaviors of reinforced concrete structures.

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