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MULTIDIMENSIONAL STOCHASTIC MATERIAL MODELING AT LARGE DEFORMATIONS CONSIDERING PARAMETER CORRELATIONS

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Abstract. *In many engineering applications the material behavior, e.g. hyper-elasticity and plasticity, is described by appropriate mathematical models. However, these become uncertain due to different types of uncertainty such as variation in the manufacturing process, measurement errors and missing or incomplete information on material properties. This contribution presents a framework for nonlinear elastic stochastic material model at large deformations. As a key idea uncertainty of the material is described by parameters, which are modeled as stochastic variables.*

To this end, 150 specimens for three different rubber materials are experimentally investigated in tensile tests. Based on experimental results 1000 artificial data are generated by aid of an ARMA process [1]. The artificial data are used for parameter identification of an Ogden material model for rubber materials . Furthermore, statistical analysis of material parameters including their correlations is studied. The number of material parameters define the dimension of the stochastic space. Usually, the stochastic material parameters are considered as stochastically independent. However, in our work we consider the dependency including the correlation obtained from experimental data.

The hyperelastic stochastic material parameters are expanded with the multivariate PCE. In this context, the stresses depend on stochastic variables. To determine the corresponding PC coefficients for non-independent stochastic material parameters we use a Cholesky decomposition. As a numerical example we consider the static problem for uniaxial tension of the rectangular plate. This structure is investigated in order to represent the experimental setup conditions.

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1 INTRODUCTION

Most materials in the engineering science are heterogeneous, where polycrystallines and composites are typical examples. The heterogeneity leads to uncertainty in the material properties due to the manufacturing process. When assessing the reliability of components, different uncertainties are distinguished. In our contribution we concentrate on the aleatoric uncertainty caused by material parameters, which is characterized by random variables. Therefore, the mechanical system must be described by stochastic partial differential equations (SPDEs). The solution of the mathematical problem can be determined by the stochastic numerical material model, where the system response of the model renders a distribution with statistical mean and variance. A ubiquitous strategy for its solution is the Monte Carlo method [2, 3]. An alternative to reduce the computational effort, is the spectral method by Ghanem and Spanos [4], which is considered in this paper. In this context the Polynomial Chaos Expansion (PCE) is often apply to represent random variables with a series of random Hermite polynomials. A necessary condition for these polynomials is the orthogonality of them, which implies that random variables do not correlate. In order to consider arbitrary polynomials, the Gram Schmidt algorithm is implemented [5].

Research areas for the stochastic modeling are: linear elasticity of solids and mechanics [4], plasticity of solids and mechanics [6, 7], large deformations [8, 9], fluid flow [10, 11, 12], flow-structure interactions [13, 14] and linear convection problems [15]. An open task is the application of the modeling of rubber-like materials, such as natural rubber, which is constitutively represented by an Ogden model and is therefore the focus of this paper. Another challenge is to calculate eigenvalues of the right Cauchy-Green tensor using PC arithmetic.

The uncertainty is considered by random material parameters, which are modeled as stochastic variables. From experimental data the distribution of the random material parameters are generated by parameter identification of each experimental result. It is well known that material parameters of the Ogden model correlate with each other, which leads to correlated random variables. However, the PCE and the determination of PC coefficients require uncorrelated random variables. Therefore, in this work, correlations are determined from statistics of material parameters and are employed for the transformation into uncorrelated random variables.

The organization of this paper is as follows: Section 2 describes the basics of the multivariate PCE. In Section 3 constitutive equations of hyperelasticity are summarized. Finally, Section 4 presents a numerical example, where stochastic system results are compared with results from experimental investigations.

2 MULTIVARIATE POLYNOMIAL CHAOS EXPANSION

In this work uncertainty is modeled by stochastic random variables. In this context the probability space is denoted by $(\Omega, \Sigma, \mathbb{P})$, where Ω is the set of elementary events, Σ is the σ-algebra and $\mathbb{P}$ is the probability measure. Let ω be an element of Ω. One possibility is to expand random variables with the multivariate PCE, see [16]. This involves a basis of known random functions with deterministic coefficients, which is of course an advantage compared e.g. with the Karhunen-Loeve Expansion. Then, the PCE of an arbitrary random variable $X(\omega)$ can be formulated as

$$X(\omega) = \sum_{\underline{\alpha}} \hat{X}_{\underline{\alpha}} \Psi_{\underline{\alpha}}(\underline{\theta}(\omega)), \tag{1}$$

where $\hat{X}_{\underline{\alpha}}$ are multivariate deterministic PC coefficients and $\underline{\alpha}$ is a multi-index. The PC basis functions $\Psi_{\underline{\alpha}}(\underline{\theta}(\omega))$ are described by multivariate polynomials with uncorrelated standard distributed random variables $\underline{\theta}$, which can be expressed as a tensor product

$$\Psi_{\underline{\alpha}}(\underline{\theta}) = \prod_{i=1}^{m} \psi_{\alpha_i}(\theta_i). \tag{2}$$

In Eq. (2) ψ_{α_i} are univariate polynomials, where $\alpha_i \in \mathbb{N}_0$ is the degree of the polynomial and $m \in \mathbb{N}_0$ specifies the stochastic dimension and / or the number of uncorrelated variables θ_i. As an example Table 1 summarizes multi-index polynomials for a stochastic dimension with $m = 4$.

$\Psi_{[0,0,0,0]} = {\psi_0}^4$	$\Psi_{[2,0,0,0]} = \psi_2(\theta_1)$	$\Psi_{[0,1,1,0]} = \psi_1(\theta_2)\,\psi_1(\theta_3)$	
$\Psi_{[1,0,0,0]} = \psi_1(\theta_1)$	$\Psi_{[1,1,0,0]} = \psi_1(\theta_1)\,\psi_1(\theta_2)$	$\Psi_{[0,1,0,1]} = \psi_1(\theta_2)\,\psi_1(\theta_4)$	
$\Psi_{[0,1,0,0]} = \psi_1(\theta_2)$	$\Psi_{[1,0,1,0]} = \psi_1(\theta_1)\,\psi_1(\theta_3)$	$\Psi_{[0,0,2,0]} = \psi_2(\theta_3)$	$\dots$
$\Psi_{[0,0,1,0]} = \psi_1(\theta_3)$	$\Psi_{[1,0,0,1]} = \psi_1(\theta_1)\,\psi_1(\theta_4)$	$\Psi_{[0,0,1,1]} = \psi_1(\theta_3)\,\psi_1(\theta_4)$	
$\Psi_{[0,0,0,1]} = \psi_1(\theta_4)$	$\Psi_{[0,2,0,0]} = \psi_2(\theta_2)$	$\Psi_{[0,0,0,2]} = \psi_2(\theta_4)$	

Table 1: Multi-index polynomials for the stochastic dimension $m = 4$

A more practical choice instead of the multivariate PCE in Eq. (1) is the single-index representation

$$X \approx \sum_{k=0}^{P} \hat{X}_k \Psi_k(\theta_1, \theta_2, \dots, \theta_m). \tag{3}$$

The multi-index $\underline{\alpha}$ is now reformulated as a single-index k such that the maximum number of PC terms is calculated as

$$P + 1 = \frac{(m+p)!}{m!p!}, \tag{4}$$

where $p = \sum_{i=1}^{m} \alpha_i$ describes the polynomial order. Using Eq. (4) with $m = 4$ and $p = 2$, a single-index representation of Eq. (2) is given in Table 2, where the PC basis Ψ_k, $k = 0, ..., P$, are described by univariate polynomials.

2.1 Correlated input represented by uncorrelated variables

As mentioned above, standard distributed random variables θ_i of Eq. (2) must be uncorrelated. Therefore, the aim of this section is the transformation of correlated into uncorrelated random variables, using the Cholesky decomposition for an arbitrary matrix $\underline{A} = \underline{L}\,\underline{L}^T$. The following vectors are defined

$$\underline{X}(\underline{\theta}) = [X_1(\underline{\theta}), X_2(\underline{\theta}), \dots, X_m(\underline{\theta})]^T, \quad \bar{\underline{X}} = [\mathbb{E}(X_1), \mathbb{E}(X_2), \dots, \mathbb{E}(X_m)]^T, \tag{5}$$

where $X_i, i = 1, \dots, m$ are correlated random variables and $\mathbb{E}(X_i)$ are corresponding expected values. The covariance matrix of $\underline{X}$ can be calculated with

$$\underline{\Sigma}_{\underline{X}\,\underline{X}} = \mathrm{cov}(\underline{X}) = \mathbb{E}\left((\underline{X} - \bar{\underline{X}})(\underline{X} - \bar{\underline{X}})^T\right) = \underline{L}\,\underline{L}^T, \tag{6}$$

Order p	Single-index k	Ψ_k
0	0	$\Psi_0 = \Psi_{[0,0,0,0]}$
1	1	$\Psi_1 = \Psi_{[1,0,0,0]}$
	2	$\Psi_2 = \Psi_{[0,1,0,0]}$
	3	$\Psi_3 = \Psi_{[0,0,1,0]}$
	4	$\Psi_4 = \Psi_{[0,0,0,1]}$
2	5	$\Psi_5 = \Psi_{[2,0,0,0]}$
	6	$\Psi_6 = \Psi_{[1,1,0,0]}$
	7	$\Psi_7 = \Psi_{[1,0,1,0]}$
	8	$\Psi_8 = \Psi_{[1,0,0,1]}$
	9	$\Psi_9 = \Psi_{[0,2,0,0]}$
	10	$\Psi_{10} = \Psi_{[0,1,1,0]}$
	11	$\Psi_{11} = \Psi_{[0,1,0,1]}$
	12	$\Psi_{12} = \Psi_{[0,0,2,0]}$
	13	$\Psi_{13} = \Psi_{[0,0,1,1]}$
	14	$\Psi_{14} = \Psi_{[0,0,0,2]}$

Table 2: Single-index representation of the multivariate polynomials for a stochastic dimension $m = 4$ and polynomial order $p = 2$

where $\underline{L}$ is the lower triangular matrix of the Cholesky decomposition. For standardized uncorrelated random variables, the covariance matrix is trivial and equal to the identity matrix $\underline{I}$. Therefore, Eq. (6) is reformulated as

$$\underline{L}^{-1}\,\underline{\Sigma}_{\underline{X}\,\underline{X}}\,\underline{L}^{-T} = \mathbb{E}\Big(\underbrace{\underline{L}^{-1}\,(\underline{X}-\bar{\underline{X}})}_{\underline{Y}}\;\underbrace{(\underline{X}-\bar{\underline{X}})^T\,\underline{L}^{-T}}_{\underline{Y}^T}\Big) = \underline{\Sigma}_{\underline{Y}\,\underline{Y}}, \tag{7}$$

where $\underline{Y}(\underline{\theta}) = [Y_1(\theta_1), Y_2(\theta_2), \ldots, Y_m(\theta_m)]^T$ in Eq. (7) in contrast to $\underline{X}$ in Eq. (6) contains standardized uncorrelated zero-mean random variables $Y_i(\theta_i)$. From Eq. (7) we conclude the relation

$$\underline{X} - \bar{\underline{X}} = \underline{L}\,\underline{Y}, \tag{8}$$

where random variables $\underline{X}$ and $\underline{Y}$ are expanded with the PCE according to Eq. (3)

$$\sum_{k=0}^{P} \hat{\underline{X}}_k\,\Psi_k(\underline{\theta}) - \hat{\underline{X}}_0 = \sum_{k=1}^{P} \hat{\underline{X}}_k\,\Psi_k(\underline{\theta}) = \sum_{k=1}^{P} \underline{L}\,\hat{\underline{Y}}_k\,\Psi_k(\underline{\theta}). \tag{9}$$

In Eq. (9) we exploit $\hat{\underline{X}}_0 = \bar{\underline{X}}$ and $\hat{\underline{Y}}_0 = \underline{0}$, due to the fact that $\underline{Y}$ contains zero-mean random variables. Comparing coefficients of Eq. (9) renders

$$\hat{\underline{X}}_k = \underline{L}\,\hat{\underline{Y}}_k, \qquad \forall\, k \in \{1, \ldots, P\}, \tag{10}$$

where the PC coefficients $\hat{\underline{X}}_k$ are formulated as PC coefficients $\hat{\underline{Y}}_k$ of uncorrelated random variables. Finally, the polynomials Ψ_k in Eq. (3) have to be specified. In the following sections two different types of polynomials Ψ_k will be discussed.

2.2 Multivariate Hermite polynomials

In general, a normal distribution of experimental data is assumed. Therefore, we use Hermite polynomials, which are based on standardized uncorrelated normally distributed random variables θ_i. The first five univariate Hermite polynomials have the form

$$\psi_0 = 1; \quad \psi_1 = \theta; \quad \psi_2 = \theta^2 - 1; \quad \psi_3 = \theta^3 - 3\theta; \quad \psi_4 = \theta^4 - 6\theta + 3. \tag{11}$$

Due to the orthogonality property of Hermite polynomials it follows, see [17]

$$\mathbb{E}(\Psi_{\underline{0}}) = 1, \qquad \mathbb{E}(\Psi_{\underline{\alpha}}\Psi_{\underline{\beta}}) = \underline{\alpha}!\,\delta_{\underline{\alpha}\underline{\beta}}, \qquad \mathbb{E}(\Psi^2_{\underline{\alpha}}) = \underline{\alpha}!, \qquad \mathbb{E}(\Psi_{\underline{\alpha}}) = 0 \quad \forall\, \underline{\alpha} \neq \underline{0}, \tag{12}$$

where $\mathbb{E}$ and $\underline{\alpha}!$ denote the expected value and the factorial with

$$\underline{\alpha}! = \prod_{i=1}^{m} \alpha_i!. \tag{13}$$

These Hermite polynomials are used to calculate PC coefficients $\hat{\underline{Y}}_k$ of Eq. (10).

2.3 Input parameter based polynomials

An alternative to Hermite polynomials are input parameter based (IPB) polynomials. In contrast to Hermite polynomials, these are based on standardized uncorrelated distributed random variables and we set $\theta_i = Y_i$. According to Eq. (2) this yields

$$\Psi_{\underline{\alpha}}(\underline{Y}) = \prod_{i=1}^{m} \psi_{\alpha_i}(Y_i), \tag{14}$$

where ψ_{α_i} depends on Y_i. An orthogonal set of univariate polynomials can be computed with the Gram-Schmidt algorithm [18]

$$\psi_0 = 1, \qquad \psi_l(Y_i) \;= e_l(Y_i) - \sum_{k=0}^{l-1} \frac{\mathbb{E}\left(e_l(Y_i) \cdot \psi_k(Y_i)\right)}{\mathbb{E}\left(\psi_k(Y_i) \cdot \psi_k(Y_i)\right)}, \qquad l = 1, 3, \ldots, p. \tag{15}$$

In Eq. (15) polynomials $e_l(Y_i) = (Y_i)^l$ are of degree l. The advantage of these polynomials is that the PC coefficients $\hat{Y}_{ik}$ of the i-th entry of $\hat{\underline{Y}}_k$ are known as

$$\hat{Y}_{ik} = \begin{cases} \delta_{ik} & \text{if} \quad k = 1, \ldots, m \\ 0 & \text{else} \end{cases} \tag{16}$$

and must not be calculated by aid of Hermite polynomials. Then, Eq. (3) leads to

$$X_i \approx \sum_{k=0}^{P} \hat{X}_{ik} \Psi_k(Y_1, Y_2, \ldots, Y_m), \tag{17}$$

where X_i depends on IPB polynomials $\Psi_k(Y_1, Y_2, \ldots, Y_m)$. The corresponding PC coefficients $\hat{X}_{ik}$ can be be formulated using Eq. (10) and Eq. (16)

$$\hat{X}_{i0} = \bar{X}_i, \qquad \hat{X}_{ik} = L_{ik}, \qquad \hat{X}_{ik} = 0 \quad \forall\, k \in \{(m+1), \ldots, P\}, \tag{18}$$

where L_{ik} describes the (i, k) entry of the lower triangular matrix, which is calculated from the Cholesky decomposition of $\underline{\Sigma}_{\underline{X}\,\underline{X}}$ in Eq. (6).

3 A stochastic variation of Ogden's material model

The derivation of the constitutive relations of the stochastic Ogden model is analogous to the deterministic problem and is therefore not presented in detail. For this reason, we will refer to relevant literature [19, 20, 21, 22, 23]. Thus, for the stochastic strain energy function we obtain

$$U(\tilde{\lambda}_i, J, \omega) = \underbrace{\sum_{p=1}^{2}\sum_{i=1}^{3}\frac{\mu_p(\omega)}{\alpha_p(\omega)}\left(\tilde{\lambda}_i(\omega)^{\alpha_p(\omega)} - 1\right)}_{U^{iso}(\tilde{\lambda}_1,\omega)} + \underbrace{\sum_{p=1}^{N}\frac{K}{2}(J(\omega)-1)^{2p}}_{U^{vol}(J,\omega)}, \tag{19}$$

where $\tilde{\lambda}_i$ are the eigenvalues of the deviatoric right Cauchy-Green tensor $\mathbf{C}$, J is the Jacobi determinant of the deformation gradient $\mathbf{F}$, K is the bulk modulus and α_p, μ_p, $p = 1, 2$ are additional material parameters. The only differences between Eq.(19) and the well known deterministic strain energy function is that $\alpha_1, \mu_1, \alpha_2, \mu_2, \tilde{\lambda}_i$ and J in Eq.(19) are stochastic variables and depend on ω. Then, exploiting the second law of thermodynamics, where no dissipation is considered for hyper elastic materials, the first and second Piola-Kirchhoff stress tensor $\mathbf{P}$ and $\mathbf{S}$ are obtained as

$$\mathbf{F}^{-1}\mathbf{P} = \mathbf{S}(\tilde{\lambda}_i, J, \omega) = \mathbf{S}^{\text{iso}}(\tilde{\lambda}_i, \omega) + \mathbf{S}^{\text{vol}}(J, \omega) = 2\left(\frac{\partial U^{iso}(\tilde{\lambda}_i, \omega) + \partial U^{vol}(J, \omega)}{\partial \mathbf{C}(\omega)}\right). \tag{20}$$

According to [24] expressions of the second Piola-Kirchhoff stress tensor $\mathbf{S}$ is considered. As mentioned in the introduction, one challenge of the stochastic modeling of Ogden's material model is the calculation of eigenvalues λ_i of the right Cauchy-Green tensor

$$\mathbf{C}(\omega) = \sum_{i=1}^{3}\lambda_i^2(\omega)\mathbf{N}_i(\omega)\otimes\mathbf{N}_i(\omega) = \sum_{i=1}^{3}\lambda_i^2(\omega)\mathbf{M}_i(\omega), \tag{21}$$

where $\mathbf{N}_1(\omega)$ and $\mathbf{M}_i(\omega) = \mathbf{N}_i(\omega)\otimes\mathbf{N}_i(\omega)$ are eigenvectors and eigenvector basis, respectively. The eigenvalues and eigenvectors of deterministic problems can be calculated easily by standard numerical methods. For stochastic problems the following eigenvector problem must be solved

$$\mathbf{C}(\omega)\mathbf{N}_i(\omega) - \lambda_i^2(\omega)\mathbf{N}_i(\omega) = \mathbf{0}, \quad i = 1, 2, 3, \tag{22}$$

$$\mathbf{N}_i^T(\omega)\mathbf{N}_i(\omega) = 1, \quad i = 1, 2, 3. \tag{23}$$

For plane stress problems Eq. (23) is simplified as described in [25]. The system of equations can be solved by using PC-arithmetics, such as product and addition in combination with an optimization algorithms. Basic operations of the PC arithmetic can be found in [26].

4 STATISTICAL ANALYSIS OF OGDEN'S PARAMETERS AND THE RESPONSE

Tensile tests are used to identify material parameters of natural rubber. Based on 150 experimental force-displacement curves, 1000 artificial data are generated as described in [1].

4.1 Distribution and Cholesky decomposition of material parameters

For all artificial data sets parameter identifications are performed such that a distribution of each parameter is obtained. In Figure 1 the cumulative density function (CDF) of Ogden's material parameters $\mu_1(\omega), \alpha_1(\omega), \mu_2(\omega)$ and $\alpha_2(\omega)$ are illustrated. The solid line represents

the CDF of the random parameter, where as the dashed line shows the Gaussian distribution. Parameters α_1 and α_2 are dimensionless quantities and μ_1 and μ_1 are given in MPa. The bulk modulus is fixed to $K = 1000\,\text{MPa}$ for all parameter identifications. All random parameters are stored in the vector

$$\underline{\kappa}(\omega) = [\alpha_1(\omega), \mu_1(\omega), \alpha_2(\omega), \mu_2(\omega)]^T. \tag{24}$$

With $\underline{\kappa}(\omega)$ from Eq. (24), the expected values and the lower triangular matrix for the covariance matrix in Eq. (6) can be calculated as

$$\underline{\bar{\kappa}} = \begin{bmatrix} \bar{\alpha}_1 \\ \bar{\mu}_1 \\ \bar{\alpha}_2 \\ \bar{\mu}_2 \end{bmatrix} = \begin{bmatrix} -6,197 \\ -0,077 \\ -0,058 \\ -5,891 \end{bmatrix}, \; \underline{L}_{\underline{\kappa}\,\underline{\kappa}} = \begin{bmatrix} 2.0813 & 0 & 0 & 0 \\ -0.0046 & 0.0375 & 0 & 0 \\ 0.0091 & 0.0079 & 0.0219 & 0 \\ 0.3635 & 0.6167 & -0.0119 & 0.1695 \end{bmatrix} \cdot 10^{-1}. \tag{25}$$

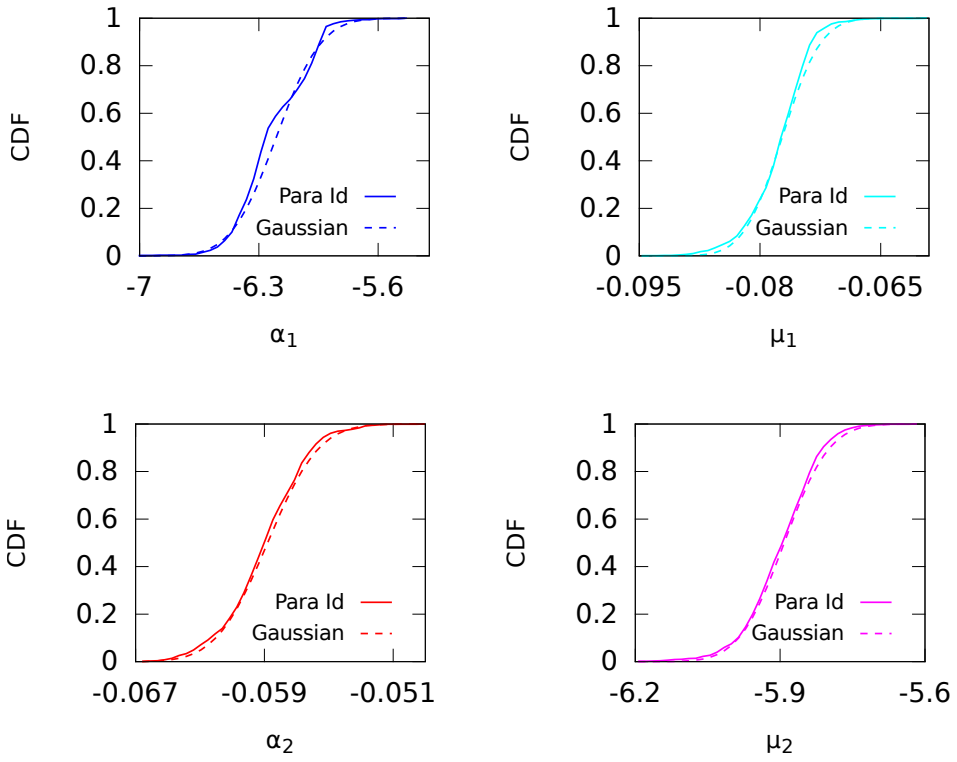

Figure 1: Cumulative density functions for material parameters α_1, μ_1, α_2 and μ_1 from 1000 deterministic parameter identifications

4.2 PC coefficients of material parameters

In order to determine the PC coefficients, the correlated inputs $\underline{\kappa}$ are first transformed into uncorrelated random variables $\underline{Y}$, as described in Eq. (7). When Hermite polynomials are used, PC coefficients of Y are determined using the collocation method [27]. Eq. (10) can be used to calculate the PC coefficients of $\underline{\kappa}$. Alternatively, the IPB polynomials can be used as described in Section 2.3. In this case, PC coefficients correspond to the expectation values $\underline{\bar{\kappa}}$ and the lower

triangular matrix $\underline{L}_{\underline{\kappa}\,\underline{\kappa}}$ with

$$\underline{\hat{\kappa}} = \begin{bmatrix} \hat{\alpha}_{10} & \hat{\alpha}_{11} & 0 & 0 & 0 \\ \hat{\mu}_{10} & \hat{\mu}_{11} & \hat{\mu}_{12} & 0 & 0 \\ \hat{\alpha}_{20} & \hat{\alpha}_{21} & \hat{\alpha}_{22} & \hat{\alpha}_{23} & 0 \\ \hat{\mu}_{20} & \hat{\mu}_{21} & \hat{\mu}_{22} & \hat{\mu}_{23} & \hat{\mu}_{24} \end{bmatrix} = \begin{bmatrix} \bar{\underline{\kappa}} & \underline{L}_{\underline{\kappa}\,\underline{\kappa}} \end{bmatrix}, \text{ only for IPB polynomials,} \tag{26}$$

see also relations in Eq. (18). PC coefficients for both types of polynomials in Section 2.2 and Section 2.3 are summarized in Table 3.

	(p, P)	k	Ψ_k	$\hat{\alpha}_{1k}$	$\hat{\mu}_{2k}$	$\hat{\alpha}_{2k}$	$\hat{\mu}_{2k}$
HP	$(1,5)$	1	1	-6.19857	-0.07724	-0.05881	-5.89198
		2	θ_1	0.20225	-0.00031	0.00096	0.03813
		3	θ_2	-0.00844	0.00368	0.00087	0.05811
		4	θ_3	-0.00028	-0.00019	0.00211	-0.00478
		5	θ_4	0.01105	0.00023	0.00004	0.01894
IPB	$(1,5)$	1	1	-6.19752	-0.07723	-0.05881	-5.89157
		2	Y_1	0.20814	-0.00047	-0.00091	0.03635
		3	Y_2	0	0.00375	-0.00079	0.06167
		4	Y_3	0	0	0.00219	$-0,00119$
		5	Y_4	0	0	0	0.01696

Table 3: PC coefficients of material parameters for Hermite and IPB polynomials

4.3 Comparison of numerical simulation with experiments

As a numerical example we consider the static problem for uniaxial tension of a rectangular plate, with the same condition as in the experimental investigations. We apply the stochastic material model for both types of polynomials, Hermite and IPB polynomials. The stochastic model can then be performed using the specific PC coefficients. For comparing the numerical and experimental results, a 95% confidence interval of the 1.PK-stretch curves is considered as can be seen in Figure 3. In addition, Figure 4 shows the CDFs of the first Piola-Kirchhoff at displacements of $25\,\text{mm}$ and $85\,\text{mm}$.

5 CONCLUSION

In this paper the stochastic modeling at large deformations for the Ogden material model is presented. A major focus is the consideration of correlated material parameters in the PCE. In this context, the basic idea are to represent correlated by uncorrelated random variables. This ensures that each variable can be developed independently and the corresponding PC coefficients can be calculated. Two types of polynomials, Hermite and IPB polynomials, are presented. In contrast to Hermite polynomials, where PC coefficients have to be determined numerically, IPB polynomials are calculated based on input variables, which render exact PC coefficients. In the numerical example, results for both methods are compared with experimental results. We conclude that a good agreement is observed using Hermite and IPB polynomials based on correlated random material parameters.

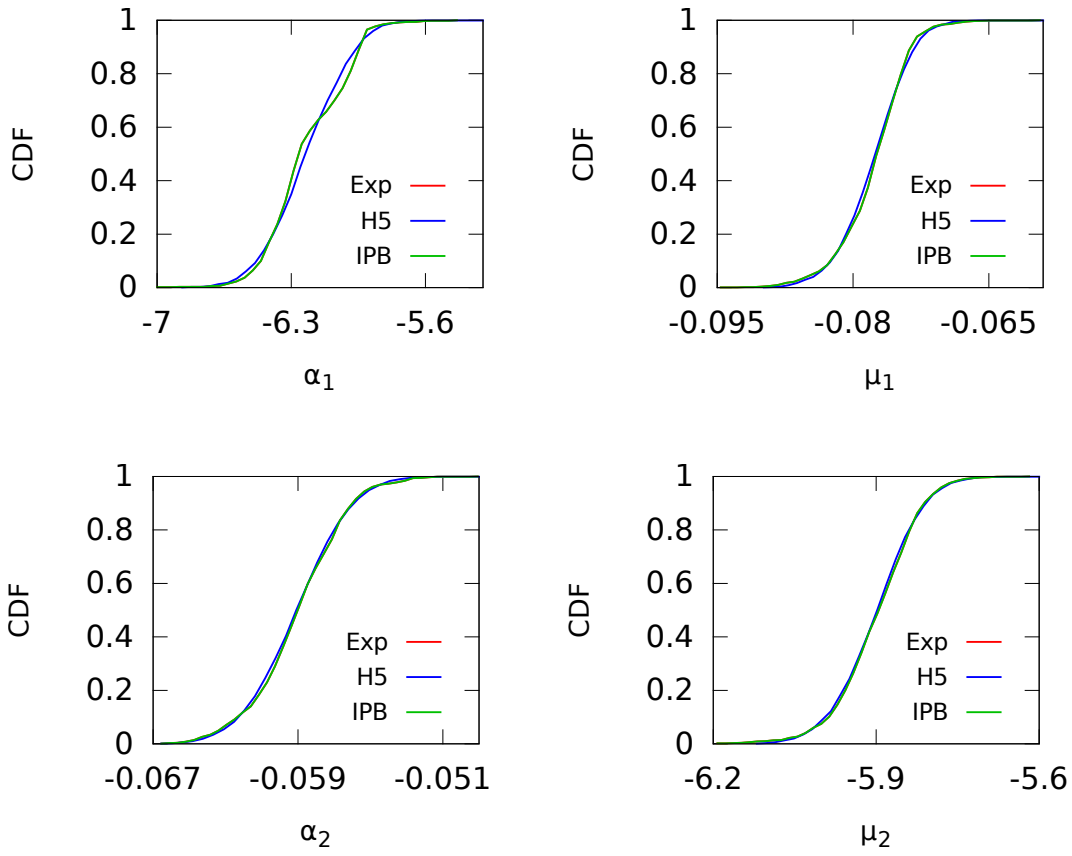

Figure 2: Density functions for material parameters α_1, μ_1, α_2 and μ_1 from experiments and modeling with both Hermite and IPB polynomials

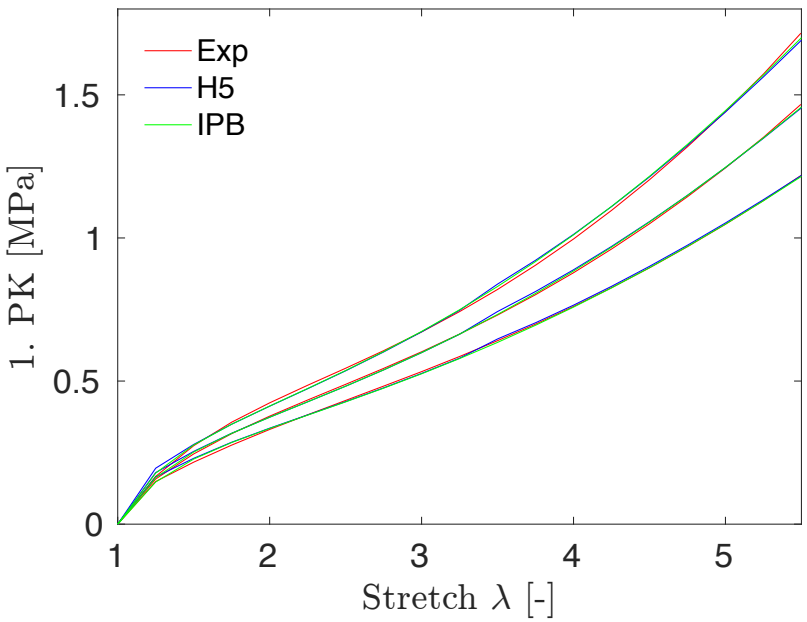

Figure 3: 95% confidence interval of the 1.PK-stretch curves

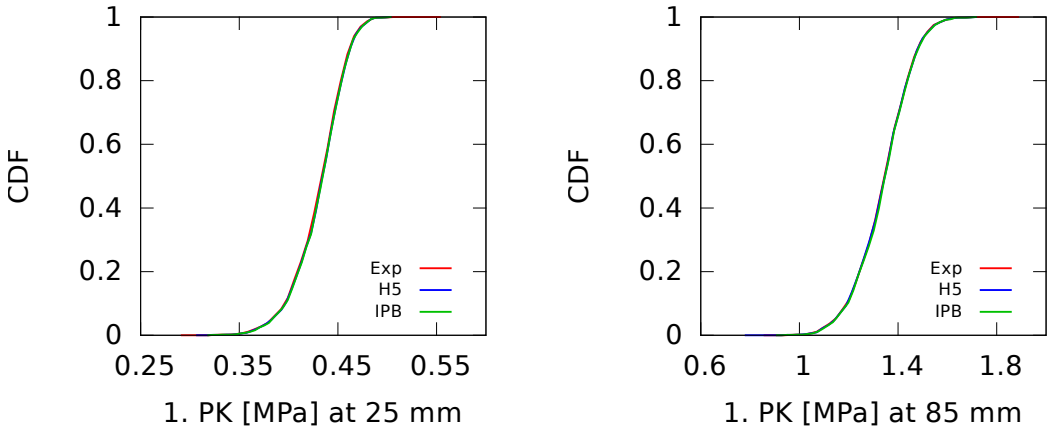

Figure 4: 1.PK at the displacement 25 mm and 85 mm

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