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AN APPROACH TO ROBUST DESIGN OF A CRUMPLE-ZONE STRUCTURE USING FUZZY ARITHMETIC

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Abstract. *Mathematical modeling and numerical simulation of crashworthy structures are state-of-the-art tools in automotive design. During the design phase, system parameters are only partially known or even unknown, leading only to a limited predictive significance of the simulation results. In this paper, an approach to robust design of a crumple-zone structure using fuzzy arithmetic is presented. The crumple zone is extracted from a full scale finite-element model of a Ford Taurus. In order to reduce the computation time, a surrogate model based on sparse-grid interpolation is derived. Using a sampling approach for fuzzy arithmetic, the simulation and analysis of the fuzzy-parametrized system is realized. The multivariate fuzzy output includes dependencies of the parameters and is therefore suitable for defining a robustness criterion. This leads to a multi-objective optimization problem including fuzzy-valued uncertainties to achieve a robust design of the crumple-zone structure.*

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1 INTRODUCTION

Traffic accident is still one of the most prevalent causes of death. According to data of the Global Health Obsevatory (GHO), provided by the World Health Organization (WHO), about 1.25 million deaths were caused by traffic accidents in the year 2013 [1]. In Germany alone, about 3500 people died by traffic accidents in 2015, and the recent statistics show an equal trend for the year 2016 [2]. Against this background, passenger safety plays a vital role in automotive research and development. Besides active safety systems, e.g. traction control systems and autonomous emergency braking, which are designed to avoid accidents, the crashworthiness of vehicles, and therefore, passive safety systems are crucial factors in protecting driver and passengers. In case of a frontal crash, the crumple-zone structure of the car plays an important role for the passenger safety and, thus, to prevent fatal and lethal injuries. Its main function is the absorbtion of the kinetic energy of a driving car by plastic deformation. In this respect, the resulting acceleration, which affects the passengers and acts as a loading quantity, needs to be kept below a critical value. Furthermore, the deformation of the crumple zone must not be to large, for example to prevent the penetration of the driver cabin by the engine mount.

Nowadays, mathematical modeling and numerical simulation of crashworthy structures are state-of-the-art tools in automotive design. However, the steadily increasing refinement and complexity of the models lead to a significant increase in computing time. To manage this drawback, reduction techniques, simplification procedures and surrogate models are used to obtain acceptable results within a reasonable time. In return, however, these simplification and approximation procedures give rise to uncertainty in the models, potentially supplemented by uncertainty due to insufficient or vague knowledge about parameters or crash conditions. Additionally, during the initial design phase, some parameters may be kept intentionally vague because of unknown requirements and specifications to achieve a later optimal design.

Common approaches to deal with uncertainties in a numerical way are based on probability theory. It is able to treat stochastic processes and models with parametric uncertainties triggered by randomness. The outcome usually provides a probabilty distribution based on a large sample size. However, if there is no variability and randomness of quantities, probability theory cannot handle these uncertainties appropriately. To cope with imprecision or a lack of information, interval analysis and possibility theory are best practice [3]. One way to handle these uncertainties in a possibilistic way is the use of fuzzy-valued model parameters instead of crisp ones and the application of fuzzy arithmetic to perform the uncertainty analysis.

In this paper, the crumple-zone structure, as an extracted substructure of a full-scale finite element model of a car, is modeled by uncertain, fuzzy-valued design parameters. With this fuzzy arithmetical approach, the influence of the design parameters on the acceleration and the absorbed kinetic energy during a frontal crash is inverstigated. With the help of the introduced methods, a fuzzy two-dimensional fuzzy set, i.e. a binary fuzzy relation, of the acceleration and the absorbed energy can be determined, showing that the resulting values are not independent of each other. On this basis, a robustness measure can be defined which can be used for further investigations.

To accomplish robust design, a trade-off problem between nominal optimality and robustness against uncertainty, caused by fuzzy-valued quantities for objective functions and constraints, must be solved. On this basis, a reliable and robust design for the crumple-zone structure can be achieved, depending on some predefined level of confidence.

2 CRUMPLE-ZONE MODEL

The crumple-zone structure is the most relevant part for passenger safety of a vehicle in case of a frontal crash. Due to high loads in the event of an impact, highly nonlinear processes have to be taken into account, including contacts, large deformations and nonlinear material behavior. To model the dynamic behavior of the structure, the finite-element method proves to be a suitable tool. In this work, the commercial finite-element code LS-DYNA is used [4]. In the following sections, the model structure and the model uncertainties during the design process are introduced and discussed.

2.1 Model Structure

In this work, the finite-element model of a Ford Taurus is used. The full-scale model is provided by the National Crash Analysis Center (NCAC) of the George Washington University under a contract with the FHWA and NHTSA of the US DOT. It consists of approximately 2.7 million degrees of freedom [5] and is, even with modern high performance cluster, too big for performing a large number of evaluations within a reasonable time. Therefore, only the crumple-zone structure, as the most relevant part of the full model, will be considered. The identified, relevant parts for this work are shown in Figure 1.

To obtain reasonable results, the extracted parts have to be preprocessed by adding additional constraints, boundary conditions and lumped masses. To model the inertia properties of the full vehicle, a lumped mass $m_{\mathrm{L}} = 500$kg is added to the structure. The nodes on the backside of the structure are constrained, restricting the translation in vertical and lateral direction. To take account of the lateral stiffness of the structure resulting from the engine mount, additional rigid links are placed in lateral direction. The properties of the extracted and modified crumple-zone structure and the full-scale model are summarized in Table 1.

Model	Elements	Nodes	Parts	Total mass
Full model	973416	922007	804	1739 kg
Crumple-zone structure	33717	32067	17	546 kg

Table 1: Properties of the finite-element model.

2.2 Model Uncertainties

During product development, some design parameters are still vague or only partially known, which implies some uncertainty to the model parameters. As an example for uncertain design parameters, the sheet thickness of three different parts of the crumple-zone structure are selected. The chosen parts are colored in blue in Figure 1. Hence, the system equations, which need to be solved, reformulates to

$$\begin{aligned} \mathbf{M}\left(\tilde{\boldsymbol{p}}\right)\ddot{\boldsymbol{u}}\left(t\right) + \mathbf{K}\left(\boldsymbol{u}(t),\tilde{\boldsymbol{p}}\right) &= \boldsymbol{f}\left(t\right) \\ \boldsymbol{z}\left(t\right) &= \mathbf{C}\boldsymbol{u}(t), \end{aligned} \qquad (1)$$

with neglected damping, $\boldsymbol{u}(t)$ being the vector of state variables and $\tilde{\boldsymbol{p}}$ denoting the vector of uncertain parameters. In this example, there exist no precisely studied random observations for the sheet thicknesses, but a lack of knowledge and therefore, missing information. To incorporate

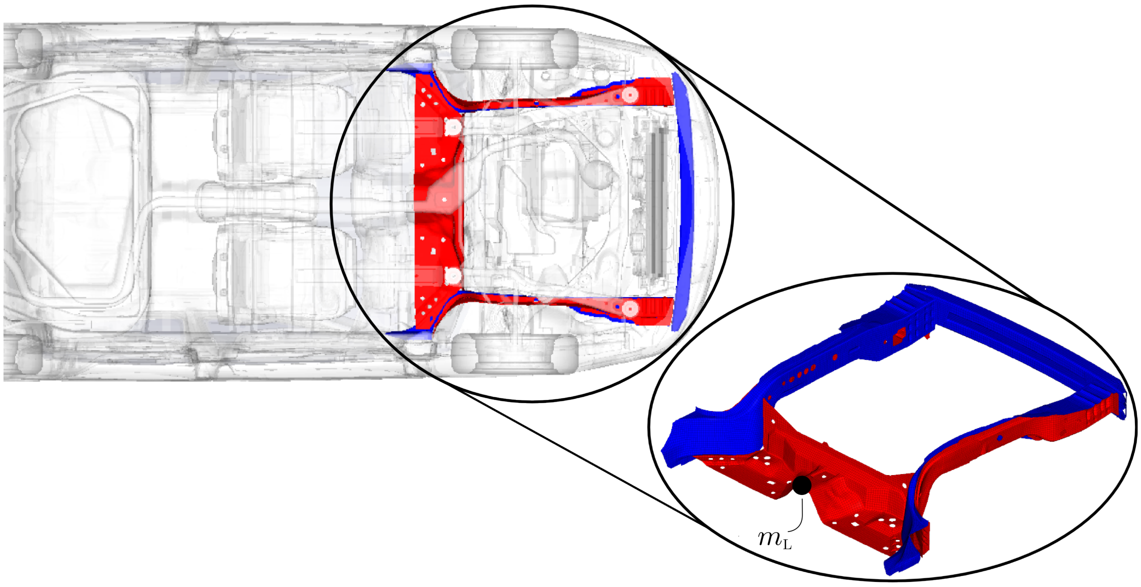


Figure 1: Finite-element model of the Ford Taurus and extracted crumple-zone structure (colored).

those uncertainties of epistemic kind, appropriate methods are needed. Common probabilistic methods are not suitable to deal with uncertainties of the mentioned kind, because they rely on precise frequentist data and are not capable of modeling the occurring imprecision. As a possibilistic approach to incorporate those epistemic uncertainties, fuzzy arithmetic can be used. In fuzzy arithmetic, uncertain parameters are modeled in terms of fuzzy numbers $\tilde{\boldsymbol{p}}$ with the entries $\tilde{p}_i,\ i = 1, 2, \ldots, n$, of the form $\tilde{p}_i = \{(x, \mu_{\tilde{p}_i}(x_i))|x_i \in \mathbb{R}, \mu_{\tilde{p}_i}(x_i) \in [0, 1]\}$ and with $\mu_{\tilde{p}_i}$ being the membership functions. As an example, a triangular fuzzy number is shown in Figure 2 with $\bar{x}_i$ denoting the nominal value, a_i the left-side deviation and b_i the right-side deviation, respectively. The fuzzy-valued sheet thicknesses are modeled by symmetric triangular fuzzy numbers with worst-case deviations a_i and b_i of $\pm 10\%$ of their nominal values $\bar{x}_i$.

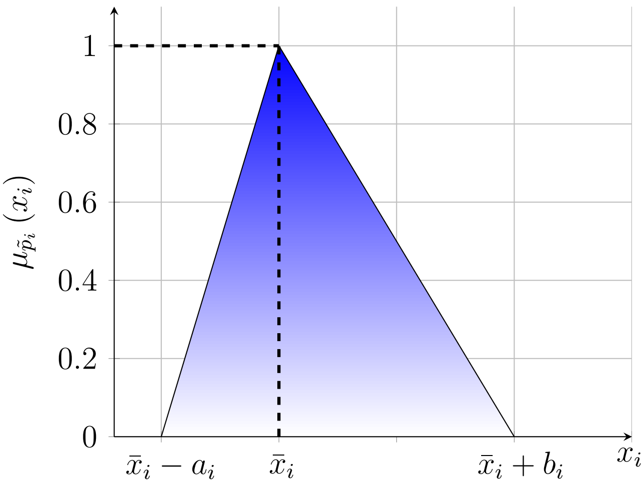


Figure 2: Trianglular fuzzy number $\tilde{p}_i$.

3 FUZZY ARITHMETIC

The analysis of dynamical systems can be extended to the inclusion of uncertain parameters by the use of fuzzy numbers. Without loss of generality, the fuzzy-parametrized system of Equation 1 can be described as

$$\tilde{\boldsymbol{q}}(t) = F\left(\tilde{\boldsymbol{p}}, \boldsymbol{u}, t\right) \tag{2}$$

with the fuzzy-valued parameters $\tilde{\boldsymbol{p}}$. The membership value of the result $\mu_{\tilde{\boldsymbol{q}}}(\boldsymbol{z})$ can theoretically be determined based on the extension principle introduced by ZADEH [6]. The membership function of the, in general, multivariate fuzzy output reads as

$$\mu_{\tilde{\boldsymbol{q}}}\left(\boldsymbol{z}\right) = \begin{cases} \sup\limits_{\boldsymbol{z}=F(\boldsymbol{x},\boldsymbol{u},t)} \mu_{\tilde{\boldsymbol{p}}}\left(\boldsymbol{x}\right) & \text{if } \exists\ \boldsymbol{z} = F(\boldsymbol{x}, \boldsymbol{u}, t), \\ 0 & \text{else.} \end{cases} \tag{3}$$

It provides a possibilitic distribution of the outcomes with respect to their membership values, including worst-case bounds and nominal values. Hereinafter, two different implementations will be introduced to practically perform an uncertainty analysis by the use of fuzzy arithmetic.

3.1 The Transformation Method

One problem-independent implementation of fuzzy arithmetic is the Transformation Method introduced in [7]. Based on an α-cut approach, the membership functions of the input fuzzy parameters $\tilde{p}_i$ are decomposed into sets of nested intervals $X_i^{(j)} = [a_i^{(j)}, b_i^{(j)}]$ related to $m+1$ equally spaced levels of membership $\mu_j = j/m$ with $j = 0, 1, \ldots, m$. The obtained intervals are then transformed into arrays $\hat{X}_i^{(j)}$ of the input samples containing the lower and upper interval bounds in a predefined scheme. After the evaluation of the model with the parameter combinations based on the arrays, the results are comprised by the arrays $\hat{Z}^{(j)}$ and are retransformed into output intervals $Z^{(j)} = \left[a^{(j)}, b^{(j)}\right]$. In the last step, the output intervals are recomposed into the output fuzzy number $\tilde{q}$. The Transformation Method resorts to a cartesian grid for the evaluation steps. The union of the single-input combinations results in a non-regular grid for both, the reduced and the general Transformation Method.

3.2 The Sampling Approach of Fuzzy Arithmetic

In order to handle multi-dimensional fuzzy outputs and to solve implicit problems, the so-called sampling method, as introduced in [8], is presented and illustrated. For this sampling approach, a discretization of the membership functions is not needed. In order to receive the proper fuzzy arithmetical solution, the generation of appropriate sample points is moved to the very beginning of the evaluation. This is contrary to the common α-cut approaches. In case of an explicit and non-adaptive sampling, this can be achieved in a straightforward manner. The multivariate membership function of n fuzzy numbers is obtained by

$$\mu_{\tilde{\boldsymbol{p}}} = t(\cdots t(\mu_{\tilde{p}_1}, \mu_{\tilde{p}_2}) \cdots, \mu_{\tilde{p}_n}) \tag{4}$$

with the operator $t(\cdot)$ representing a t-norm. In case of completely independent parameters, the minimum operator can be used. It yields a conservative estimation of the multivariate membership function and is therefore suitable if no information about the interdependency is present. In this work, the fuzzy parameters are considered as completely independent. In the following,

the general procedure, given a specific sampling sequence, will be described. How to obtain this specific sequence will be outlined later.
In the explicit case, the system is described by the mapping $f : \boldsymbol{X} \rightarrow \boldsymbol{Z}$ with the parameter domain $\boldsymbol{X} \in \mathbb{R}^n$ and the output domain $\boldsymbol{Z} \in \mathbb{R}^l$. The sampling sequence of N points is

$$[\boldsymbol{x}] = \left[\boldsymbol{x}^{[i]}\right]_{i=1}^{N} = \left[\left(x_1^{[i]}, \ldots, x_n^{[i]}\right)\right]_{i=1}^{N}, \boldsymbol{x}^{[i]} \in \mathbb{R}^n , \tag{5}$$

and has to be constructed from the parameter space. With a pointwise evaluation of the function $f\left(\boldsymbol{x}^{[i]}\right)$, the corresponding pairs $\boldsymbol{z}^{[i]}$ in the fuzzy output domain are computed. Additionally, the given joint membership function is to be evaluated, yielding potential membership values of $\boldsymbol{z}$, and thus, potential boundary points of the membership function

$$\mu = \left[\mu^{[i]}\right]_{i=1}^{N} = \left[\mu_{\tilde{\boldsymbol{p}}}\left(\boldsymbol{x}^{[i]}\right)\right]_{i=1}^{N} \tag{6}$$

of the result. This is in contrast to the above-mentioned Transformation Method, which already provides points that lie on the membership curve. Because not all obtained points are valid candidates for the memberhsip curve, an additional step, namely the reconstruction of the membership function, has to be performed, where the valid pairs $[\hat{\boldsymbol{z}}, \hat{\mu}]$ are extracted from the results. For $\boldsymbol{z} \in \mathbb{R}$, the simplest way to obtain the valid points of the membership curve is to construct a convex hull with piecewise linear functions. This is already a sufficient approximation in most cases. However, there are many additional possibilities to obtain an appropriate boundary curve, e.g. by approximating of the α-cut bounds or the membership function [8].
In order to receive appropriate samples, the sampling of the parameter domain can be done in multiple ways. However, there exist some specific points which should definitely be included in the sampling sequence, independent of the chosen method: the nominal value of the input fuzzy relation with the joint membership $\mu_{\tilde{\boldsymbol{p}}}$ to obtain the nominal value of the output domain, and the vertex points of the nested multi-dimensional hyper cuboids $\left(\bar{\boldsymbol{X}} \subset \boldsymbol{X}\right)$ for constant α-cuts, resulting in the same pattern as in the above-mentioned reduced Transformation Method. This leads to the proper fuzzy-valued results in case of monotonic behavior and includes the nominal value of the input fuzzy relation $(\mu_{\tilde{\boldsymbol{p}}} = 1)$, as a degenerated hypercuboid. Additionally, uniformly distributed random points can be included in the sampling sequence to handle even non-monotonic functions. Besides the random pattern, also regular patterns schemes or irregular pattern, like the general Transformation Method, can be used.

Reconstruction of a multi-dimensional fuzzy output

For a multi-dimensional fuzzy output, as e.g. the two-dimensional eigenvalues in the complex plane, the reconstruction step of the sampling method has to be adapted. One way is to project the obtained tuples $[z_1, z_2, \ldots, z_l, \hat{\mu}]$ on a predefined, cartesian grid. The output domain can be decomposed using hierarchical, recursive decomposition techniques. In the two-dimensional case, this can be achieved by using the quadtree decomposition [9]. The decomposition is completed, if a predefined minimum size of the quadtree elements is reached, or no output value is found. Using the extension principle by ZADEH, the supremum of the membership values of all fuzzy outputs lying in a quadtree element is determined. With this formulation, a piecewise constant membership function for the multi-dimensional fuzzy sets can be contructed, which is already a good approximation and contains valuable information. Observe that in the multi-dimensional case the output is in general not a fuzzy number because the convexity criterion does not necessarily hold.

4 RESPONSE SURFACE MODELING

Despite the increasing performance capacities of modern computer systems and high performance clusters the above-mentioned methods still need an enormous number of system evaluations to provide reasonable results. Even moderately complex systems with only a few fuzzy parameters turn out to be challenging in terms of computation time. The two major influences on the overall computation time are on the one hand the complexity of the actual model and, thus, the computation time of a single evaluation, and on the other hand the number of system evaluations based on the chosen fuzzy arithmetical approach. The latter is strongly dependent on the number of fuzzy-valued parameters. Hence, the number of system evalutions, for a given resolution, grows exponentionally with the dimension of the fuzzyparameter space. The so-called course of dimensionality impedes the straightforward evaluation of high-dimensional models. In order to reduce the computional cost, one idea is to set up a surrogate model which is able to represent the response of the system for the range of the fuzzy-valued parameters in a decent manner. There exist several approaches to obtain a suitable surrogate model, such as Krigin, polynominal basis or radial basis functions. In this paper, sparse-grid interpolation will be used, as introduced in [10].

5 ROBUST DESIGN APPROACH

The fuzzy arithmetical analysis of the crumple-zone structure is performed with three fuzzy-valued parameters as mentioned above. Instead of a perfect frontal crash, a quasi-frontal crash with a wall angle of $\alpha = 5°$ is simulated, to enforce a more assymmetric behavior of the crumple-zone deformation.

The influence of the fuzzy input parameters on the maximal absolute value of the acceleration and the maximal absorbed energy is to be investigated. The acceleration is measured at the mounting point. The translation and rotation of the mounting point are restricted, so that only a translation along the driving axis is allowed. The acceleration serves as a loading measure for the occupants of the car. Because LS-DYNA does not provide direct access to the plastic deformation energy, the internal energy is chosen as a measure for the energy absorption. It also takes into account the elastic deformation energy.

For the surrogate model, an adaptive sparse-grid model with 2000 evaluation points is constructed. The normalized, fuzzy-valued results of the maximal acceleration and internal energy are shown in Figure 3. The results show a good conformity between the Transformation Method and the sampling approach for an equally chosen number of evaluations. For the sampling approach, a combined random and pattern-based scheme is chosen which is able to recover outliers of the solution (see Figure 3b). The two-dimensional fuzzy output is shown in Figure 4 on a 256×256 grid. The result of the Transformation Method leads to an overestimation and a too conservative result in the multivariate case. With the used sampling approach, the dependencies of the two output parameters are evident. Observe that for the two-dimensional ouput significantly more evaluations are needed to obtain an acceptable resolution.

To deduce an appropriate measure to rate the robustness of the model in respect of fuzzy-valued uncertainty, the fuzzy-valued output and input are to be put in relation to each other. Such a robustness measure can be introduced by

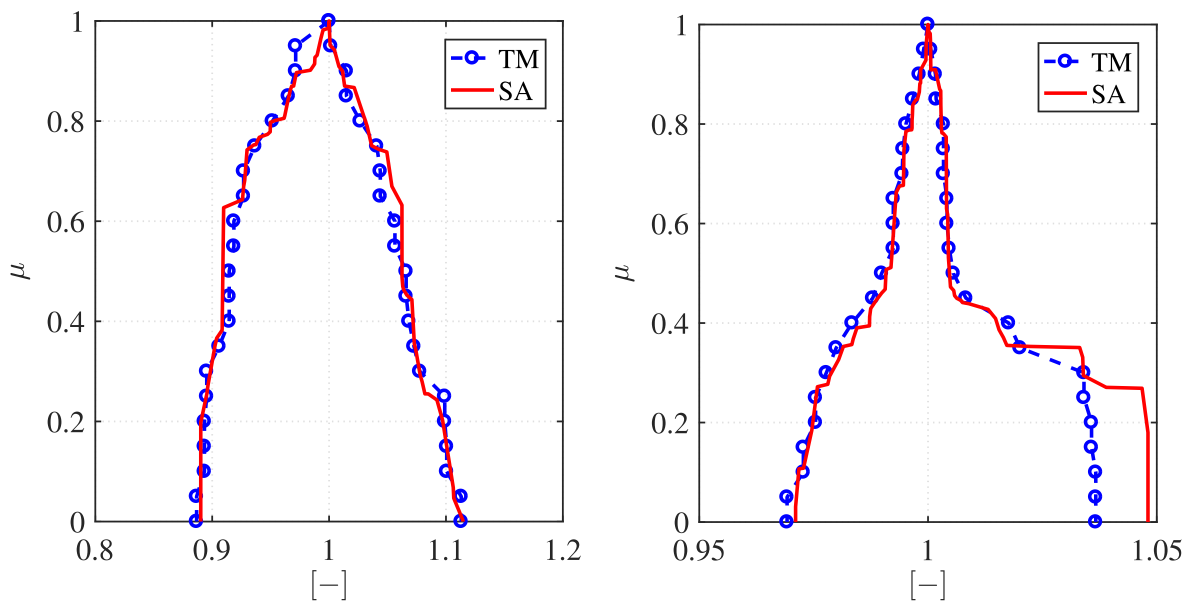
$$\rho\left(\tilde{\boldsymbol{q}}, \tilde{\boldsymbol{p}}\right) = \frac{\int\limits_{\boldsymbol{z} \in \boldsymbol{Z} \neq \emptyset} \mu_{\tilde{\boldsymbol{q}}} \, \mathrm{d}\boldsymbol{z}}{\int\limits_{\boldsymbol{z} \in \boldsymbol{Z} \neq \emptyset} \mathrm{d}\boldsymbol{z}} \cdot \frac{\int\limits_{\boldsymbol{x} \in \boldsymbol{X} \neq \emptyset} \mathrm{d}\boldsymbol{x}}{\int\limits_{\boldsymbol{x} \in \boldsymbol{X} \neq \emptyset} \mu_{\tilde{\boldsymbol{p}}} \, \mathrm{d}\boldsymbol{x}} \,. \tag{7}$$

It refers to the (relative) cardinality of fuzzy sets as described in [7] and describes the sensitivity of the dependent fuzzy output variables with respect to the fuzzy-valued input parameters. For the considered crumple-zone structure, the robustness measure results in $\rho = 0.0188$. Observe that the normalization is redundant in case of already normalized fuzzy sets. Also, only measures with equal dimensions in their input and output domain are comparable.

In order to obtain a robust optimal design, not only the absorbed energy of the crumple-zone structure has to be maximized and the maximal acceleration has to be minimized, but also the introduced robustness measure needs to be minimized. The problem can be reformulated as a multi-objective optimization problem. There exist multiple concepts to obtain an optimal solution in the presence of conflicting objectives. A short overview of gradient-free methods is given in [11]. One way of attaining a formulation for multi-objective optimization is the weighted-sum method. By this means, the different objectives are aggregated into one scalar optimization problem (see e.g. [12]). Some of the drawbacks of this method are outlined in [13]. The problem is formulated according to

$$\begin{aligned}
\underset{\boldsymbol{x}}{\text{minimize}} \quad & F(\boldsymbol{x}) = \left(\sum_{k=1}^{l} (\lambda_k f_k(\boldsymbol{x}))^p \right)^{\frac{1}{p}}, \ \lambda_k \in [0,1], \ p \in \mathbb{N} \\
\text{subject to} \quad & g_j(\boldsymbol{x}) \leq 0, \ j = 1, \ldots, m, \\
& x_i^l \leq x_i \leq x_i^u, \ i = 1, \ldots, n, \\
& \sum_{k=1}^{l} \lambda_k = 1 ,
\end{aligned} \tag{8}$$

with the single, reformulated objective functions $f_k(\boldsymbol{x})$, the weighting factors λ_k, and $p = 1$. A generalization of the weighted-sum approach is formulated as a weighted $\mathcal{L}_p$-norm problem. The weights, and therefore, the preferences of the designer are determined a priori.



(a) Normalized acceleration $\tilde{q}_{\bar{a}}$ (b) Normalized internal energy $\tilde{q}_{\bar{\pi}}$

Figure 3: Fuzzy-valued output parameters for the Transformation Method (TM) and the sampling approach (SA).

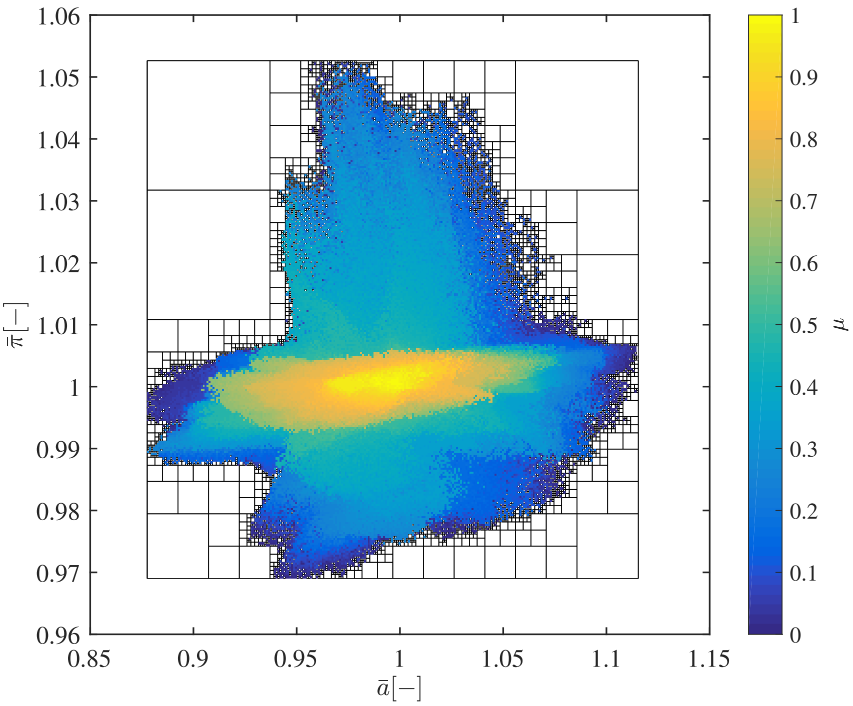


Figure 4: Contour plot of the decomposed, two-dimensional fuzzy output $\tilde{\boldsymbol{q}}$ of the normalized internal energy $\bar{\pi}$ and the normalized acceleration $\bar{a}$.

6 CONCLUSIONS AND OUTLOOK

In this paper, a fuzzy arithmetical approach to robust design, regarding uncertainties of primarily epistemic type, is performed on an exemplary application, namely the crumple-zone structure of a Ford Taurus. The uncertainties arising from design parameters which are still to be determined are modeled in terms of fuzzy numbers. Explicitely, the sheet thicknesses of three different parts of the crumple-zone structure are treated as design parameters. In order to perform a fuzzy arithmetical analysis, two different methods are outlined and discussed. A surrogate model, based on sparse-grid interpolation, is used to significantly reduce the computation time. As a result, the fuzzy-valued acceleration and internal energy of the system are shown. With the sampling approach of fuzzy arithmetic, a two-dimensional ouput is constructed, showing the dependencies of the output parameters, and thus, obtaining a tighter bound of the fuzzy-valued output than e.g. the Transformation Method. The decomposition of the output domain is performed using the quadtree decomposition. Using the presented methods to perform a fuzzy arithmetical analysis, a new robustness criterion for the multivariate case can be deduced, which represents an appropriate measure for the sensitivity of the system output with respect to the fuzzy-valued input parameters. By this means, a multi-objective optimization problem for robust optimal design based on fuzzy arithmetic is formulated.

Apparently, the presented paper only shows the principle approach for robust design. In the next step, the actual optimization of the crumple-zone structure is to be performed. For this purpose, appropriate design parameters, e.g. material properties, sheet thickness or geometry, need to be chosen. Since the optimization needs additional model evaluations, it can be included in the sparse-grid surrogate model. The addition of supplementary parts, as e.g. the engine mount, provides more accurate results. Using model order reduction techniques, the full-scale model is manageable in terms of computation time. Finally, regression on sparse grids can be used to obtain a more suitable surrogate model of the crumple-zone structure.

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