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OPERATIONAL MODAL ANALYSIS OF BRODIE TOWER USING A BAYESIAN APPROACH

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Abstract. *This paper presents a full-scale ambient vibration test and modal identification of Brodie Tower, an eight-storey office building at the University of Liverpool. Five triaxial force-balance accelerometers were deployed. The measurement scheme comprised seven setups covering four locations on each floor, acquiring twenty minutes of ambient data for each setup. The modal properties of the building are identified using a Bayesian fast Fourier transform modal identification method incorporating multiple setup data, which accounts for the variability and the quality of data among different setups. Besides providing the estimates of modal parameters through the posterior (i.e., given data) most probable values, this method can also quantitatively assess their identification precision through the posterior covariance. The identified modal properties and their accuracy in different setups are compared and discussed. The identified global mode shapes are also compared with those obtained by global least square method. The data quality in each setup and how they affect the identification results are investigated.*

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1 INTRODUCTION

Operational Modal Analysis (OMA) allows one to identify the modal properties (e.g., natural frequencies, damping ratios and mode shapes) of a structure from ambient 'output-only' data. Modal properties are often among the first few quantities to be identified in a structural vibration project. They are often examined to provide information for structural model updating, structural modification, damage detection, or more generally, structural health monitoring [1–4].

In field vibration tests, one common situation is to measure a large number of DOFs (degrees of freedom) with a limited number of sensors. In order to identify the 'global' mode shape comprising a potentially large number of DOFs, a multiple-setup scheme is required where sensors are 'roved' to different DOFs in different setups to cover all DOFs of interest [5–7]. Conventionally, the modal parameters are identified individually using the data from each setup. The 'representative' natural frequencies and damping ratios may be taken as the averaged values from different setups. The global mode shapes need to be assembled (or 'glued') from the partial ones that only cover the DOFs measured in individual setups. Techniques such as the post separate estimation rescaling method [8] and the global least square (LS) method [9] can be used for assembling the global mode shapes. For good quality data in all setups, reported cases show that different methods yield practically the same results. In general, the quality of the global mode shape depends critically on whether the identified mode shapes of different setups match at the reference DOFs, which should not be taken for granted. Recently, identification methods capable of incorporating data directly from multiple setups have been developed based on stochastic subspace identification method [6] and Bayesian fast Fourier transform (FFT) approach [10]. Reported cases show that incorporating data from multiple setups allows one to have reasonable identification results even when the data quality in some setups is low, for which methods based on analysing data in individual setups give poor or questionable results.

Field OMA data has a variety of complications that are difficult to replicate numerically by synthetic data, especially for multiple setup data where environmental (e.g., excitation) and human (e.g., sensor alignment) factors can change over different setups. Investigation with field data is therefore indispensable for OMA research with multiple setups. As a contribution along this line, this paper presents an ambient vibration test of the Brodie Tower at the University of Liverpool. The modal properties of the building are identified using a recently developed fast Bayesian FFT method incorporating multiple setup data [10]. The identified modal properties and their uncertainties in different setups are compared and discussed. The data quality in each setup and how it affects the identification results are investigated. The identified global mode shapes are also investigated by comparing the results with those assembled by the global LS method.

2 AMBEINT VIBRATION TEST OF BRODIE TOWER

The Brodie Tower is an eight-storey concrete building located in the main campus of the University of Liverpool, UK, see Figure 1. It has a 'T' shape floor plan measuring 25 m (length) × 28 m (width) and approximately 25 m high. It hosts the civil engineering department, with a heavy structure laboratory in the basement.

Figure 1: Overview of Brodie Tower.

An ambient vibration test with multiple setups was performed to determine the modal properties of the Brodie Tower, aiming at a mode shape that covers the T-shape of the floor plan in different storeys. Figure 2 shows a set of equipment used at each measurement location. Five triaxial force-balance accelerometers were deployed. The sensors have a noise level of approximately $0.1\mu g/\sqrt{\mathrm{Hz}}$ for frequencies above 1 Hz. To obtain synchronised data among all the sensors, high precision clocks were used during the test. These clocks can provide accurate timing for the sensors so that their data can be logged locally without interconnection but still in a synchronous manner. Before the test, each clock was first synchronised using a GPS receiver. After that, the GPS receiver was removed and the clock was able to continuously provide practically synchronised time stamping for the sensor within the frequency range of interest for one or two days.

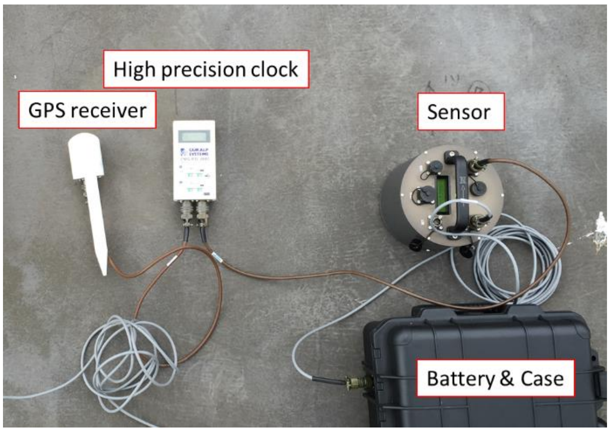

Figure 2: Test equipment per location.

Figure 3 shows the sensor location and setup plan. Four locations forming a T-shape were measured on each floor. With seven floors this gives 7×4=28 locations. Sensor 1 was used as a reference and its location remained the same in all setups. The remaining four sensors were roved from the top floor to the bottom, covering all locations in seven setups. The data in each setup was recorded for 20 minutes at a sampling rate of 50 Hz. The transition from one setup to another took about 5 minutes. Synchronising clocks in advance took approximately 20 minutes. The whole test lasted for approximately 3.25 hours from 1:30 pm to 4:45 pm.

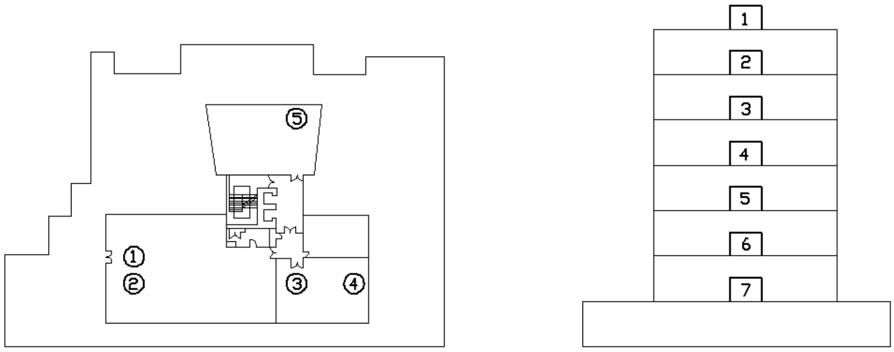

Figure 3: Sensor location (left) and setup scheme (right, setup number indicated).

3 BAYESIAN MODAL IDENTIFICATION WITH MULTIPLE SETUP DATA

The data obtained from the field test in the last section is analysed using a recently developed Bayesian modal identification method that is capable of incorporating data from different setups directly [10]. The theory is summarised in this section. Consider ambient vibration data from n_s setups, each covering a possibly different set of DOFs. Let $\boldsymbol{\theta}$ denote a set of modal parameters of a well-separated mode, which includes natural frequencies $\{f_i\}$, damping ratios $\{\zeta_i\}$, the spectral density of modal excitations $\{S_i\}$, the spectral density of prediction errors $\{S_{ei}\}$ and partial mode shapes $\{\boldsymbol{\varphi}_i \in R^{n_i \times 1}\}$ (i.e., covering the DOFs in a particular setup only), where n_i is the number of measured DOFs in setup i. Here, the subscript i denotes that the quantity refers to the one in the i^{th} setup. Let $\mathbf{L}_i \in R^{n_i \times n}$ be the selection matrix, where n is the total number of measured DOFs. The (j,k) entry of $\mathbf{L}_i$ equals to 1 for DOF k measured at the j^{th} channel in setup i, and zero for the remaining entries. The partial mode shape $\boldsymbol{\varphi}_i$ is related to the global mode shape $\boldsymbol{\Phi} \in R^{n \times 1}$ by:

$$\boldsymbol{\varphi}_i = \mathbf{L}_i \boldsymbol{\Phi} \qquad (i = 1,...,n_s) \tag{1}$$

The measured acceleration data is modelled as

$$\hat{\ddot{\mathbf{x}}}_j = \ddot{\mathbf{x}}_j + \boldsymbol{\varepsilon}_j \qquad (j = 1,..., N) \tag{2}$$

where $\ddot{\mathbf{x}}_j$ denotes the theoretical modal acceleration that depends on $\boldsymbol{\theta}$; $\boldsymbol{\varepsilon}_j$ is the prediction error; N is the number of samples per channel. The scaled FFT of the data is defined as

$$\mathcal{F}_k = \sqrt{(2\Delta t / N)} \sum_{j=1}^{N} \hat{\ddot{\mathbf{x}}}_j \exp\{-2\pi \mathbf{i}[(k-1)(j-1)/N]\} \qquad (k = 1,....,N) \tag{3}$$

where Δt is the time interval and $\mathbf{i}^2 = -1$. Let $\{\mathcal{F}_k\}$ denote the collection of $\mathcal{F}_k$ over a selected band around the mode of interest. For a small Δt and large N, it can be shown that $\{\mathcal{F}_k\}$ are asymptotically independent and jointly 'circularly complex Gaussian' [11]. Applying Bayes' theorem and assuming a non-informative prior distribution for $\boldsymbol{\theta}$, the posterior probability density function is proportional to the likelihood function, i.e.,

$$p(\boldsymbol{\theta} | \{\mathbf{Z}_k\}) \propto p(\{\mathbf{Z}_k\} | \boldsymbol{\theta}) \tag{4}$$

The most probable value (MPV) of the modal parameters can be determined by maximising $p(\{\mathbf{Z}_k\}|\boldsymbol{\theta})$, or equivalently, minimising its negative logarithm of the likelihood function (NLLF) with respect to $\boldsymbol{\theta}$. As the NLLF depends on $\boldsymbol{\theta}$ in a general nonlinear manner, analytical solution for the MPV has not been sought. Rather, it can be shown that the MPV of the global mode shape can be determined analytically in terms of other parameters. This allows the MPV of the full setup of modal parameters to be determined by an iterative algorithm, where parameters of different groups are updated until convergence.

4 DATA ANALYSIS

Data analysis and modal identification are presented in this section. In section 4.1, singular value (SV) spectrum is first examined to locate potential modes. Modal identification results are presented in section 4.2. Section 4.3 investigates the identified global mode shapes by comparing the results with those assembled by the global LS method.

4.1 The SV spectrum

The SV spectrum is often used to discover potential modes in frequency domain. It is a plot of the eigenvalues of power spectra density matrix of data with frequency. Lines that describe the shape of dynamic amplification in the SV spectrum indicate potential modes. Figure 4 shows the root SV spectrum of the data in setup 2. Six potential modes below 10 Hz are highlighted. The bar below each mode indicates the band whose FFT will be used for identifying the mode; the circle indicates the initial guess of natural frequency.

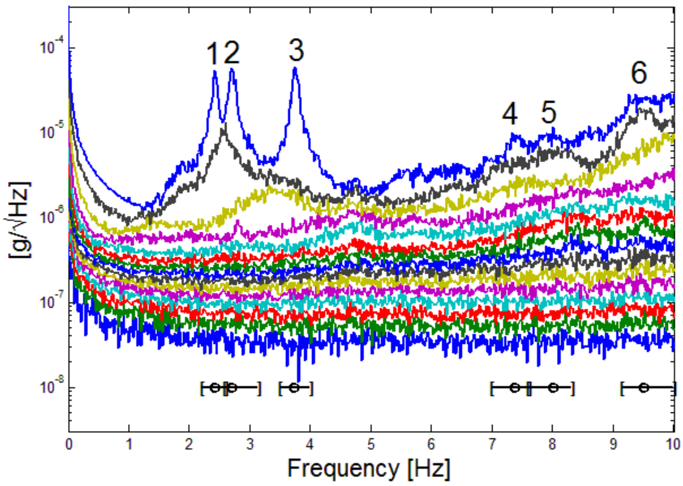

Figure 4: The root singular value spectrum of the data in setup 2.

4.2 The identified modal properties

Figure 5 shows the identified natural frequencies and damping ratios in different setups. The blue circles represent the MPVs of the modal parameters and the error bars cover +/- 2 posterior standard deviation. The sample mean and sample coefficient of variation (c.o.v. = sample standard deviation / sample mean) of the modal parameters among different setups are shown in Table 1. The results show that the identified natural frequencies and damping ratios vary among the setups, but the variability of the natural frequencies is relatively small compared to the damping ratios.

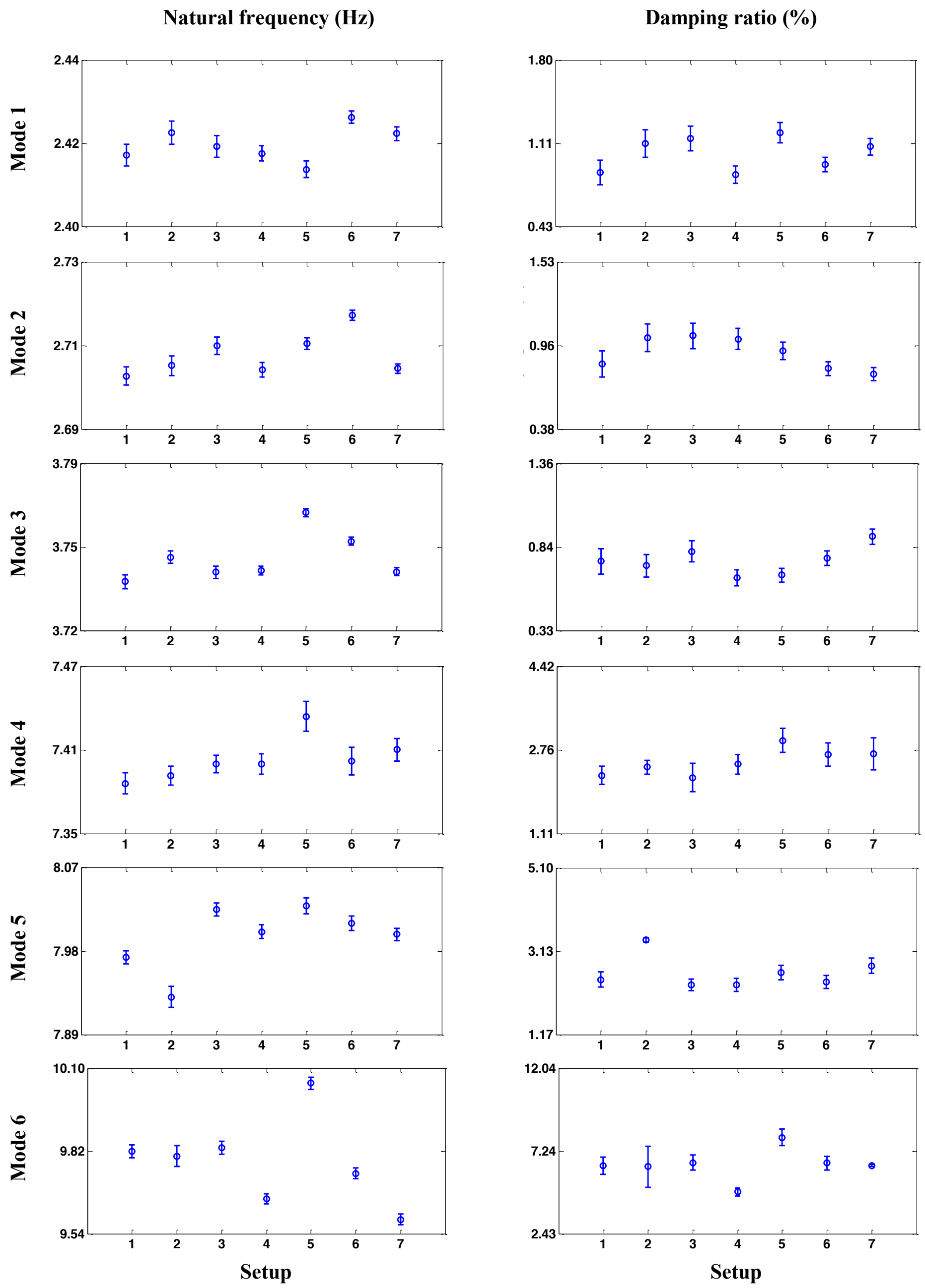

Figure 5: The identified natural frequencies and damping ratios among the setups, the error bars cover +/- 2 standard deviation.

		Mode					
		1	2	3	4	5	6
Natural Frequency	mean (Hz)	2.422	2.707	3.750	7.405	7.995	9.782
	c.o.v. (%)	0.16	0.2	0.25	0.22	0.42	1.5
Damping Ratio	mean (%)	1.031	0.905	0.754	2.525	2.624	6.439
	c.o.v. (%)	14	13	12	11	15	14

Table 1: The sample mean and sample c.o.v. of the natural frequencies and damping ratios among the setups.

Based on the identification results, the modal signal-to-noise (s/n) ratio can be calculated, which reflects how well-excited of a mode relative to the prediction error level. It is given by the ratio of the spectral density of modal excitation to the spectral density of prediction error at the resonance peak [12], i.e.,

$$\gamma_i = S_i / 4S_{ei}\zeta_i^2 \tag{5}$$

where γ_i is the modal s/n ratio of the setup i.

Figure 6 shows the modal s/n ratio of mode 1 to 6 among different setups, where different colour of '*' represents the modal s/n ratios in different setups. It can be seen that the modal s/n ratio of the first three modes are very high (over 500). The modal s/n ratio of mode 4 and 5 are moderate (30~100), while mode 6 has the lowest modal s/n ratio, in the order of 10. The level of modal s/n ratio also can be predicted from the SV spectrum. The first three peaks in the SV spectrum (Figure 4) suggest that the modal s/n ratios are relatively high, while the same is not true for the mode 4 to 6.

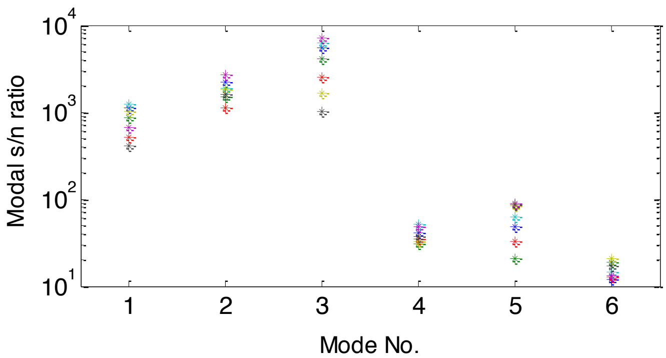

Figure 6: The modal s/n ratio among different setups of the mode 1 to 6.

4.3 The global mode shapes

Figure 7 and Figure 8 show the global mode shapes obtained by the Bayesian method incorporating multiple setups and the global LS method, respectively. The sample mean of the natural frequencies and damping ratios are also shown in these figures. The modal assurance criterion (MAC) values between the global mode shapes obtained by the two methods are listed in Table 2.

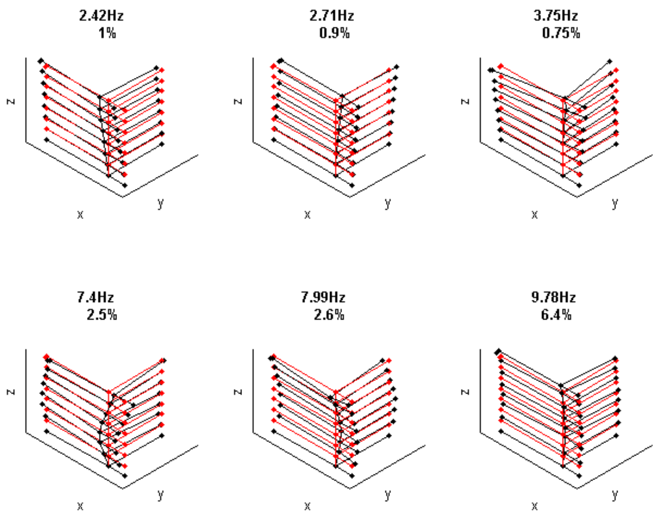

Figure 7: The global mode shapes obtained by the Bayesian method incorporating multiple setups.

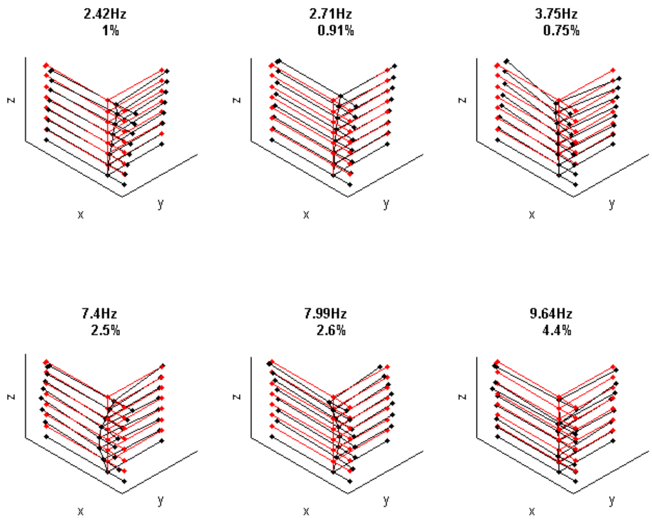

Figure 8: The global mode shapes assembled by the global LS method.

Mode	1	2	3	4	5	6
MAC	0.9999	1.0000	0.9999	0.9999	0.9999	0.8244

Table 2: The MAC values between the global mode shapes obtained by the Bayesian method incorporating multiple setups and the global LS method.

It can be seen that the first five global mode shapes obtained by these methods are very close to each other and their MAC values are almost equal to 1. The mode 1 and 2 are translational modes in x and y direction, respectively. The mode 3 is a torsional mode with its torsional centre at the left side of the T-shape connection. The mode 4 and 5 are second harmonics of the translational modes in the x and y direction, respectively.

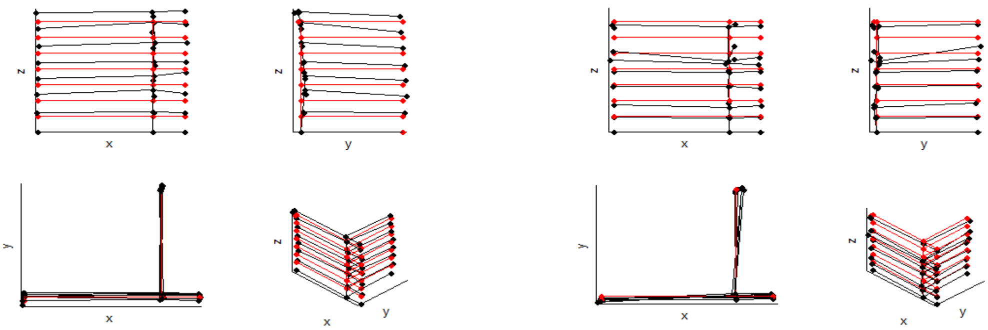

Figure 9: Multiple views of the global mode shape of mode 6 obtained by the Bayesian method incorporating multiple setups (left) and the global LS method (right).

For the mode 6, the global mode shape obtained by the Bayesian method incorporating multiple setups is different from the one assembled by the global LS method. The MAC value between the two methods is 0.8244. Figure 9 shows a detailed plot of the mode shape obtained by these methods. The one obtained by the Bayesian method incorporating multiple setups is a combination of a vertical mode and a second translational mode in the y direction. The one assembled by the global LS method is not physically plausible. The result obtained by the Bayesian method incorporating multiple setups is more reasonable, which demonstrates the potential benefit of incorporating information directly from multiple setup data.

5 CONCLUSIONS

This paper investigates the quality of identification results based on field test data using the Bayesian modal identification method incorporating multiple setup data. A full-scale ambient vibration test of an eight-storey building has been presented. The identified modal parameters and their c.o.v.s in different setups are compared and discussed. The identified global mode shapes are investigated by comparing the results with those assembled by the global LS method.

Based on the identification results, it can be seen that when the modal s/n ratio is high or moderate (e.g., > 100), the global mode shapes obtained by the Bayesian method incorporating multiple setups and the global LS method are very close to each other. When the modal s/n ratio is low (e.g., ~10), the global LS method may give questionable results, while the Bayesian method incorporating multiple setups still gives reasonable results.

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