

MODEL UPDATING OF A NONLINEAR EXPERIMENTAL VEHICLE USING SUBSTRUCTURING AND UNSCENTED KALMAN FILTERING

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Abstract. *This study establishes a computational framework for nonlinear finite element (FE) model updating of large-scale structures, through the exploration of two case studies pertaining to a spring-mass-damper chain model and to a laboratory vehicle with nonlinear suspensions. The proposed approach combines a substructuring model reduction approach with a near real-time system identification scheme, namely the Unscented Kalman Filter (UKF). The former aims at isolating and locally updating individual structural subsystems of a large-scale structure, while the latter, in contrast to other alternatives (e.g. the Extended Kalman Filter), offers a number of advantages in treating nonlinear systems, such as a derivative free calculation and a capacity for handling higher order nonlinearities. To this end, after formulating a detailed large-scale FE model for the vehicle frame substructure, a lumped model is adopted for the description of the nonlinear suspensions. Accordingly, a joint state and parameter estimation (JS&PE) problem is formulated on the basis of the lumped model. The proposed framework uses acceleration measurements from a limited number of sensors attached on the structure and a UKF observer for fusing these with a nonlinear FE substructure model, resulting into a JS&PE problem. The results indicate the validity of the proposed framework and motivate further implementation in large-scale structural systems with nonlinear components.*

1 INTRODUCTION

A structural system may be often formulated as an assembly of distinct linear and nonlinear substructures [1]. For instance, a vehicle structure consists of the body frame, which is designed to behave linearly, and the four suspension-wheel substructures, which typically exhibit nonlinear behavior and are subjected to external excitation. In order to simulate the dynamic behavior of such systems it is important to develop a high fidelity finite element (FE) model, reducing the discrepancies between analytical predictions and the real structure. To this end, a model updating procedure need be put in place in order to fine-tune and adjust the parameters of numerical models on the basis of experimental measurements. A main challenge involved in the updating procedures pertains to the oftentimes large number of degrees of freedom (DOFs) involved. To tackle with this problem, appropriate substructuring methods in either the time or the frequency domain have been developed and are commonly employed [2-4].

Furthermore, large-scale nonlinear FE models are further characterized by uncertainties, which are usually represented as unknown model parameters (e.g. individual flexibility, material properties, etc.) [5]. The main objective of this study is to establish a computational framework for identifying and adjusting these parameters, while estimating the structural states, in a problem that is referred to as joint state and parameter estimation (JS&PE) [6-7]. To achieve this, the Unscented Kalman Filter (UKF) is utilized [8], due to its efficient performance of in real-time nonlinear system identification problems [9-13].

Under this setting, this study proposes a multi-stage optimization process for the nonlinear model updating of large-scale uncertain structures, which is based on the integration of substructuring and the UKF. To this end, a nonlinear and uncertain structural system is decomposed into linear and nonlinear uncertain substructures and the kinematic and dynamic constraints along the boundaries are established. The UKF is then applied to the uncertain and/or nonlinear substructure(s), resulting in the estimation of both the unknown structural states and parameters. The acceleration time histories of each local substructure are imported as base excitations in the linear substructures, in order to calculate the dynamic response of the latter. Finally, the calculated acceleration time histories at the measurement locations of the linear substructure are compared with the experimental measurements, in order to quantify the discrepancy between analytical and experimental models and define a response residual. These residuals are minimized to acquire a best fit model between the estimated quantities and those identified from the experiments.

The organization of this paper is as follows. In the following section, a brief review of UKF formulation for state and parameter estimation is presented. Then, the effectiveness and accuracy of the developed computational framework is demonstrated by presenting numerical results obtained for two selected examples. The study concludes by presenting a summary of the results.

2 REVIEW OF THE UNSCENTED KALMAN FILTER FORMULATION FOR STATE AND PARAMETER ESTIMATION

A broad range of mechanical systems may be composed into individual structural components that are linearly deformable and interconnected by local elements of purely nonlinear behavior. In this case, the corresponding equation of motion is

$$\mathbf{M}\ddot{\mathbf{z}}(t) + \mathbf{C}\dot{\mathbf{z}}(t) + \mathbf{K}\mathbf{z}(t) + \mathbf{h}(\mathbf{z}, \dot{\mathbf{z}}) = \mathbf{F}(t) \quad (1)$$

where the vector $\mathbf{z}(t)$ corresponds to the displacement of the system, \mathbf{M} , \mathbf{C} and \mathbf{K} are the mass, damping and stiffness matrices, respectively, $\mathbf{h}(\mathbf{z}, \dot{\mathbf{z}})$ includes the nonlinear forces imposed by

the interconnecting elements, herein assumed to be “smooth”, and $\mathbf{F}(t)$ is the vector of externally applied forces. The prediction of the response of systems represented by Eq.(1) is a difficult task, since in most practical cases the number of the equations of motion is quite large and the nonlinearities are dominant. As a result, such systems can only be studied by applying special numerical methodologies. In many cases, the underlying computations are facilitated after applying appropriate model reduction methodologies that lead to a significant “compression” of the original deformable coordinates, without affecting considerably the accuracy of the results.

The KF is a Bayesian approximation technique for state estimation using noise-corrupted measurements from a subset of the structural degrees of freedom [14]. The KF assumes availability of system matrices, as well as the presence of process and measurement noise, and in its discrete-time version is represented by a state-space model of the form

$$\begin{aligned}\mathbf{x}_{k+1} &= \mathbf{A}\mathbf{x}_k + \mathbf{B}\mathbf{u}_k + \mathbf{w}_k \\ \mathbf{y}_k &= \mathbf{C}\mathbf{x}_k + \mathbf{D}\mathbf{u}_k + \mathbf{v}_k\end{aligned}\quad (2)$$

where $\mathbf{A} \in \mathbf{R}^{n \times n}$ is the state matrix, $\mathbf{B} \in \mathbf{R}^{n \times l}$ is the input matrix, $\mathbf{C} \in \mathbf{R}^{m \times n}$ is the output matrix, $\mathbf{D} \in \mathbf{R}^{m \times l}$ is the feedforward matrix, $\mathbf{u} \in \mathbf{R}^{l \times 1}$ is the control vector, $\mathbf{x} \in \mathbf{R}^{n \times 1}$ is the state vector, comprising the system displacements and velocities, $\mathbf{x} = [\mathbf{z}, \dot{\mathbf{z}}]^T$, $\mathbf{y} \in \mathbf{R}^{m \times 1}$ is the observation vector and $k = 1, 2, \dots$ are the discrete-time steps. The stochastic nature of the KF is introduced by the process and measurement noise vectors $\mathbf{w} \in \mathbf{R}^{n \times 1}$ and $\mathbf{v} \in \mathbf{R}^{m \times 1}$, respectively, assumed to belong to the $N(\mathbf{0}, \mathbf{Q}_k)$ and $N(\mathbf{0}, \mathbf{R}_k)$ Gaussian distributions, respectively.

The effective performance of the KF depends on (a) how accurately the process model can track the actual system; (b) the assumption of additive, independent white Gaussian noise; and (c) the accuracy of the process and measurement noise processes. For large modeling uncertainties and/or unknown structural parameters, the KF is generally amenable to inconsistencies, or even fails at providing state estimates. To this end, the use of an alternate Bayesian filter is proposed herein, namely, the Unscented Kalman Filter (UKF), which is capable of joint parameter and state identification.

In handling linear systems of the type of Eq.(2), where the system properties, i.e., the elements of matrix \mathbf{A} , are considered as unknowns, a joint state and parameter formulation is adopted. This demands an augmentation of the regular state vector \mathbf{x} , in order to include those properties of the system that are considered as unknowns and which can be gathered in a parameter vector $\boldsymbol{\theta}$. The augmented state vector is defined as $\bar{\mathbf{x}} = [\mathbf{x}, \boldsymbol{\theta}]^T$. The resulting system is of nonlinear nature since it comprises bilinear products of the components \mathbf{x} & $\boldsymbol{\theta}$ of $\bar{\mathbf{x}}$. The UKF is chosen herein in place of the widely used nonlinear filter alternative, the extended KF (EKF) [15, 16], since it is able to overcome some significant shortcomings of the latter when dealing with higher order nonlinearities and noise contamination. The UKF models the state as a Gaussian random variable whose distribution can be approximated by a carefully chosen set of deterministic points, namely the sigma points. These points capture the prior mean and covariance of the state and when propagated through the nonlinear function, provide an improved posterior estimate of the transformed state. This process is known as the unscented transformation (UT) [8]. The process and observation equations are in this case reformulated for the general case of a nonlinear system as,

$$\begin{aligned}\bar{\mathbf{x}}_{k+1} &= \mathbf{f}(\mathbf{x}_k, \mathbf{u}_k) + \bar{\mathbf{w}}_k \\ \mathbf{y}_k &= \mathbf{h}(\mathbf{x}_k, \mathbf{u}_k) + \mathbf{v}_k\end{aligned}\quad (3)$$

where $\mathbf{f}(\cdot)$ and $\mathbf{h}(\cdot)$ are generally nonlinear functions and $\bar{\mathbf{w}}_k$ is defined as previously. The main steps of the UKF are summarized in Tab.1.

<p>Initialize:</p> <ol style="list-style-type: none"> 1. Set initial values for the augmented state vector mean and covariance: $\bar{\mathbf{x}}_0$ and $\bar{\mathbf{P}}_0$ 2. Set the parameters of the UKF (n_x the size of the augmented state vector) $a = 1, b = 2, \kappa = 0$ $\lambda = \alpha^2(n_x + \kappa) - n_x, c = \alpha^2(n_x + \kappa)$ $W_m^0 = \frac{\lambda}{(n_x + \lambda)}, W_m^i = \frac{\lambda}{2(n_x + \lambda)}, i = 1, \dots, 2n_x$ $W_c^0 = \frac{\lambda}{(n_x + \lambda)} + (1 - \alpha^2 + b), W_c^i = W_m^i, i = 1, \dots, 2n_x$ $\boldsymbol{\mu}_x = [W_m^0 \dots W_m^{2n_x}]^T$ $\mathbf{M}_x = (\mathbf{I} - [\boldsymbol{\mu}_x \dots \boldsymbol{\mu}_x]) \times \text{diag}(W_c^0 \dots W_c^{2n_x})(\mathbf{I} - [\boldsymbol{\mu}_x \dots \boldsymbol{\mu}_x])^T$
<p>Update:</p> <ol style="list-style-type: none"> 1. Calculate Sigma points: $\mathbf{X}_k^- = [\bar{\mathbf{x}}_k^- \dots \bar{\mathbf{x}}_k^-] + \sqrt{c} [\mathbf{0} \sqrt{\bar{\mathbf{P}}_k^-} \dots - \sqrt{\bar{\mathbf{P}}_k^-}]$ 2. Propagate sigma points through the output equation: $\mathbf{Y}_k^- = \mathbf{h}(\mathbf{X}_k^-, k)$ 3. Calculate output mean and covariance: $\hat{\mathbf{y}}_k = \mathbf{Y}_k^- \boldsymbol{\mu}_x, \mathbf{P}_k^{yy} = \mathbf{Y}_k^- \mathbf{M}_x \mathbf{Y}_k^{-T} + \mathbf{R}$ 4. Calculate cross covariance between state and output: $\mathbf{P}_k^{xy} = \mathbf{Z}_k^- \mathbf{M}_x \mathbf{D}_k^{-T}$ 5. Calculate state gain: $\mathbf{P}_k^{yy} \mathbf{K}_k = \mathbf{P}_k^{xy}$ 6. Update state mean and covariance: $\mathbf{x}_k = \bar{\mathbf{x}}_k^- + \mathbf{K}_k(\mathbf{y}_k - \hat{\mathbf{y}}_k)$
<p>Predict:</p> <ol style="list-style-type: none"> 1. Calculate Sigma points: $\mathbf{X}_k = [\mathbf{x}_k \dots \mathbf{x}_k] + \sqrt{c} [\mathbf{0} \sqrt{\mathbf{P}_k} \dots - \sqrt{\mathbf{P}_k}]$ 2. Propagate sigma points through the state equation: $\hat{\mathbf{X}}_k = \mathbf{f}(\mathbf{X}_k, k)$ 3. Predict state mean and covariance for $k + 1$: $\bar{\mathbf{x}}_{k+1}^- = \hat{\mathbf{X}}_k \boldsymbol{\mu}_x, \mathbf{P}_{k+1}^- = \hat{\mathbf{X}}_k \mathbf{M}_x \hat{\mathbf{X}}_k^T + \mathbf{Q}$

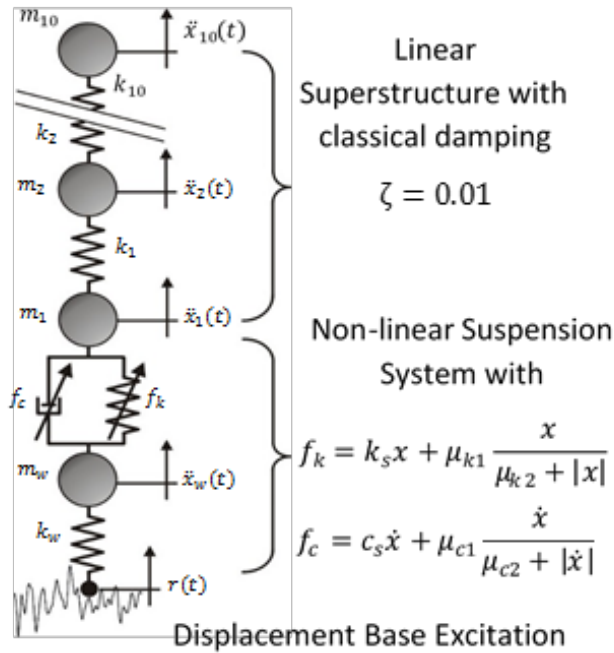
Table 1: The general scheme of the UKF algorithm for joint state and parameter estimation

3 NUMERICAL APPLICATION TO VEHICLE-LIKE CASE STUDIES

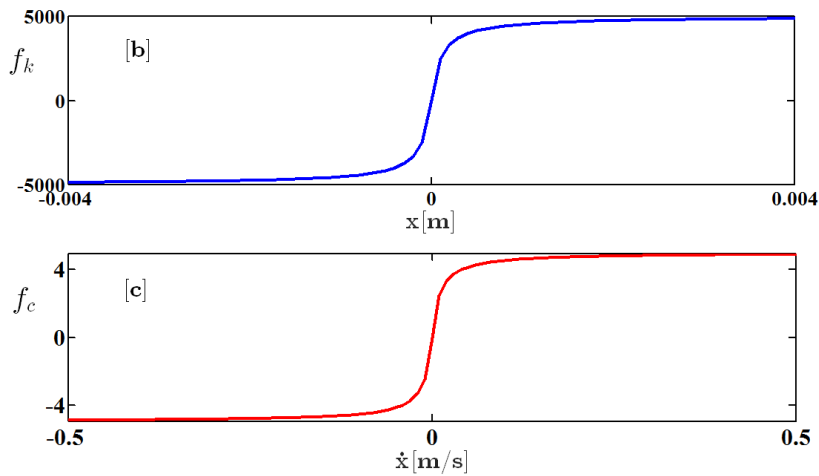
The UKF outlined in the previous section is now validated and assessed in two numerical case studies. The first pertains to a spring-mass-damper system, while the second corresponds to a complex laboratory model of a vehicle.

3.1 A spring-mass-damper model

Figure 1a displays a spring-mass chain-like mechanical model with 11-DOFs, which comprises two substructures. The lower one contains the first DOF, of mass m_w , which is connected to the ground through a linear spring of stiffness k_w and to the first mass (m_1) of the upper substructure via a nonlinear spring-damper, in which the associated restoring and damping forces are given by (see also Fig.1b) respectively, with $x = x_1 - x_w$.



(a) 11-DOF spring-mass-damper model



(b) The nonlinear restoring and damping forces.

Figure 1: First case study.

$$f_k = k_s x + \mu_{k1} \frac{x}{\mu_{k2} + |x|} \quad \text{and} \quad f_c = c_s \dot{x} + \mu_{c1} \frac{\dot{x}}{\mu_{c2} + |\dot{x}|}, \quad (4)$$

The upper substructure includes the remaining 10 DOFs and it is assumed linear and proportionally damped, with 1% modal damping ratio in every mode. Due to the presence of the interconnecting nonlinear spring-damper in the lower substructure, the equations of motion for this example are strongly nonlinear, since the values of the nonlinear terms in Eq.(4) are comparable to the ones of the linear terms. To this end, the simulation of the structure is succeeded using a variant of the Newmark's method [2]. A random displacement base excitation $r(t)$ is used as an input to the system and the calculated acceleration responses of a subset of the DOFs (see below), noise-corrupted with 15% measurement error, are considered as the available output measurements. In addition, it is assumed that the values μ_{k1} and μ_{c1} of the nonlinear restoring and damping forces of the lower substructure are unknown, along with the structural states, e.g. the DOF displacements and velocities.

Under this setting, the applied computational framework consists of the following steps:

1. The UKF is applied to the lower (nonlinear) substructure, in order to estimate the acceleration at the boundary between the two substructures.
2. The UKF-estimated acceleration time history is subsequently used as a base excitation to the upper substructure, which is simulated separately, in order to obtain the dynamic response of the trailing DOFs.
3. The calculated acceleration time histories of steps 1-2 are compared to their counterparts from the simulation of the total structure.

In order to obtain an insight about the optimal number and location of the “sensors”, several trials are executed revealing that four (4) sensors are capable of rendering the UKF effective. It is nevertheless important that the corresponding locations are distributed along the structure, instead of being concentrated around a specific area. Indicatively, the locations «3-5-7-11» were found to be a good choice.

Figures 2-3 display indicative results of the applied method. Both unknown parameters converge to their actual values (Fig.2), with the one of the restoring force reaching quite rapidly. This efficiency is also reflected to the estimation quality of the states, as for example the displacement of the 4th DOF (Fig.3), verifying the efficiency of the devised JS&PE scheme.

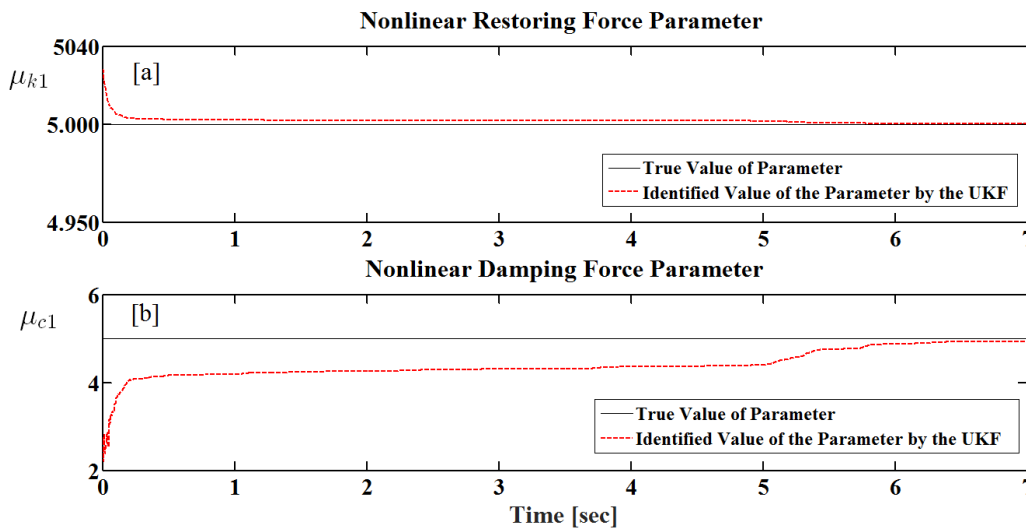
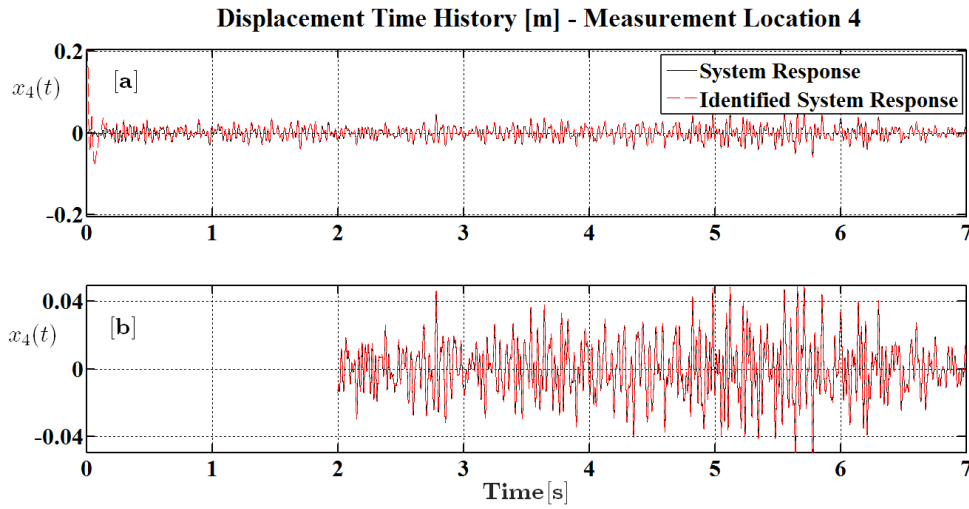


Figure 2: Convergence of the nonlinear parameters (a) μ_{k1} , (b) μ_{c1} .

Figure 3: 4th DOF's displacement (a) total timeframe, (b) after 2s.

3.2 Small scale vehicle-like frame structure

The proposed computational framework is now applied to the laboratory structure of Fig.4, that has been designed for small-scale simulations of vehicle frames [3,4]. The frame structure is characterized by predominantly linear response and high modal density and it is supported to four “suspension” systems with strongly nonlinear behavior. They consist of a lower set of linear discrete spring-damper units, connected to concentrated masses that simulate the wheelsets, as well as of an upper set of a nonlinear discrete spring-damper units connected to the frame that simulate the action of the suspension. The measurement points, indicated by 1-4, correspond to connection points between the frame and its supporting structures, while points 5-7 correspond to available measurements along the frame.

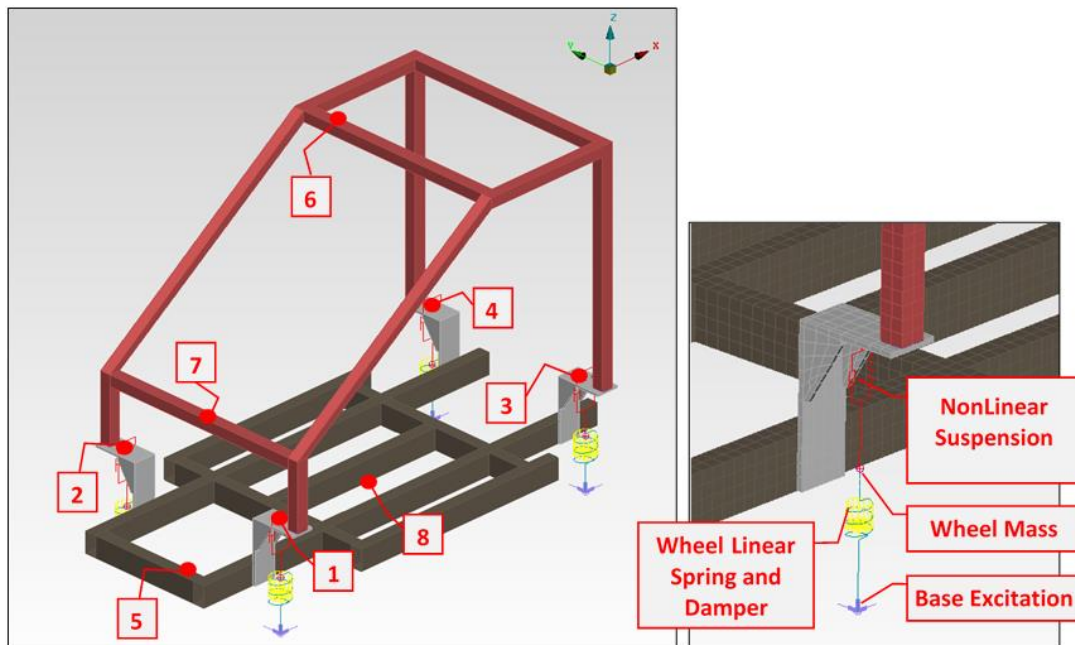


Figure 4: Small scale vehicle model with frame and nonlinear supports.

Based on the geometric details and the material properties of the structure, a detailed FE model of the vehicle frame is created, consisting of 45,564 DOFs. The nonlinear restoring and damping forces in the suspensions assume the earlier form, and the same computational steps are applied herein as well. That is, a nonlinear transient response analysis of the full vehicle model (frame and supports) is performed first, by applying four different transient displacement base excitation histories to the wheel subsystems in the vertical direction. The model is then solved by using the same direct integration scheme as in the previous example and the acceleration histories at the selected locations 1-7 are calculated. Again, the values μ_{k1} and μ_{c1} of the nonlinear restoring and damping forces of the suspension subsystem which connected to the boundary location 2 are considered as unknown, along with the displacements and velocities.

The results are expanded over Figs.5-6 and confirm the efficiency of the proposed UKF scheme. Both unknown nonlinear parameters converge rapidly to their true values, while the estimated displacements at the measurement locations 5, 6, 7 and 8 result almost identical to their counterparts, which have been calculated from the simulation of the full structure.

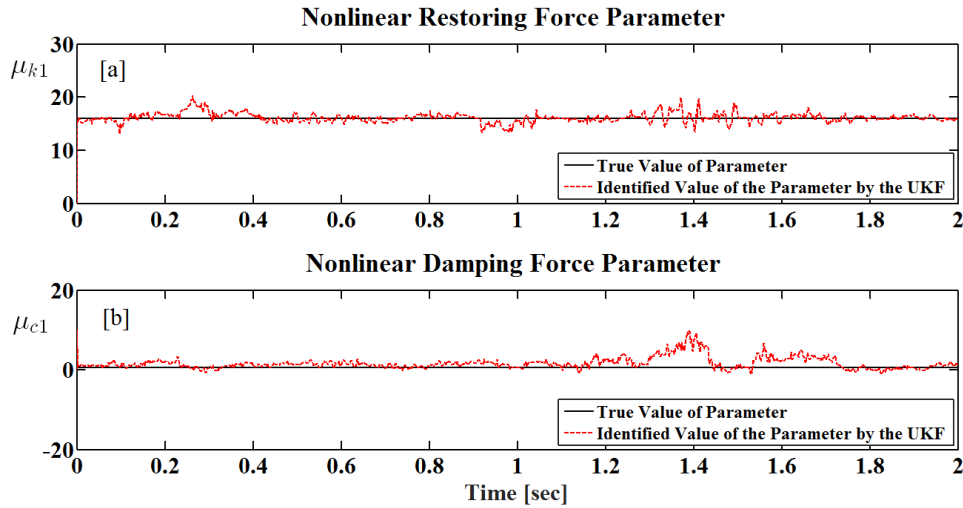


Figure 5: Convergence of the nonlinear parameters of suspension subsystem 2 (a) μ_{k1} , (b) μ_{c1} .

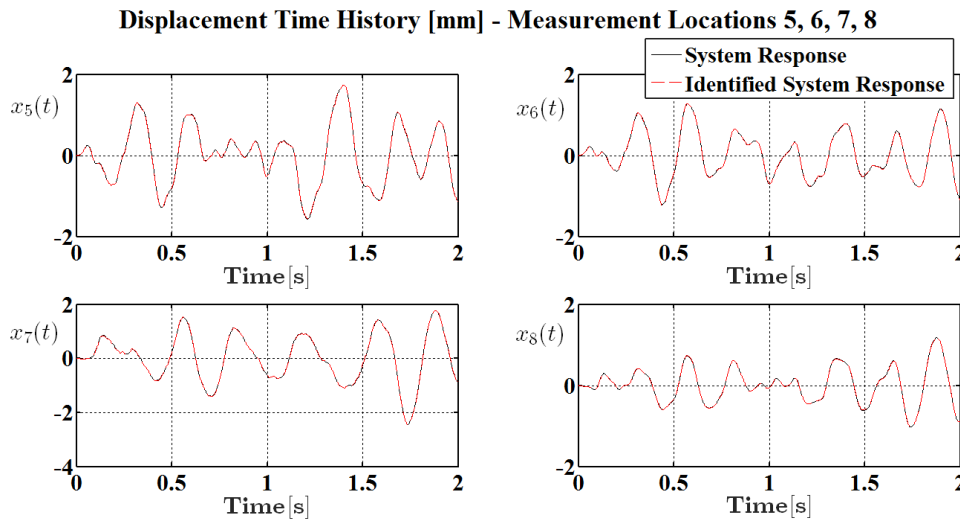


Figure 6: UKF state estimation of displacement time histories in measurement locations 5, 6, 7 and 8.

4 SUMMARY

This work outlines a computational framework for the effective finite element (FE) model updating of large-scale nonlinear uncertain structures. The proposed approach combines a substructuring model reduction stage with a near real-time system identification scheme, the UKF. The nonlinear observer is employed herein for the JS&PE of the nonlinear substructure, which is successfully handled using acceleration measurements in a limited number of DOFs. The application of the proposed scheme to the selected numerical case studies verifies its efficacy, encouraging the further investigation of its performance in more complicated structures.

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