Available online at www.eccomasproceedia.org
Eccomas Proceedia UNCECOMP (2021) 49-62

ECCOMAS Proceedia

UNCECOMP 2021
4th ECCOMAS Thematic Conference on
Uncertainty Quantification in Computational Sciences and Engineering
M. Papadrakakis, V. Papadopoulos, G. Stefanou (eds.)
Streamed from Athens, Greece, 28 -30 June 2021

# STRUCTURAL RELIABILITY ESTIMATION OF STEEL MAST EXHIBITING RANDOM MECHANICAL AND ENVIRONMENTAL PARAMETERS

**R. Bredow[1], M.Kamiński[2]**

[1] PhD Candidate, Department of Structural Mechanics, Łódź University of Technology, POLAND
al. Politechniki 6, Łódź 90-924
e-mail: rafal.bredow@dokt.p.lodz.pl

[2] Professor, Department of Structural Mechanics, Łódź University of Technology, POLAND
al.Politechniki 6, Łódź 90-924
e-mail: marcin.kaminski@p.lodz.pl

---

**Abstract.** *The aim of this work is to study an influence of environmental and mechanical uncertainties on reliability assessment of some steel mast subjected to the given dynamic wind spectrum. Some exemplary steel guyed mast structure has been tested including geometrical non-linearities inherent in its dynamic response history spectra for the ultimate and serviceability limit states. Numerical solution of dynamic excitation problem has been obtained using the Finite Element Method system ROBOT. Further research, which has been performed in computer algebra system MAPLE 2019, included sensitivity study regarding an order of polynomial approximation of structural response functions and also the resulting structural probabilistic characteristics presented as the functions of the input uncertainties. The Weighted Least Squares Method with triangular weight functions has been applied to recover some structural response polynomials. Probabilistic analysis has been performed with the use of the Iterative Stochastic Perturbation Technique, where accuracy of this method has been compared with the Monte-Carlo simulation and also with the semi-analytical approach. A coincidence in this comparison for the first two probabilistic moments of structural response has been discussed in this paper accordingly.*

**Keywords:** generalized stochastic perturbation technique, Stochastic Finite Element Method, reliability analysis, structural mechanics.

---

ISSN:2623-3339 © 2021 The Authors. Published by Eccomas Proceedia.
Peer-review under responsibility of the organizing committee of UNCECOMP 2021.
doi: 10.7712/120221.8021.18961

## 1 INTRODUCTION

This work contains a study of the influence of environmental and mechanical uncertainties on reliability index of some guyed steel mast subjected to the given dynamic wind spectrum. This excitation has been defined using relatively short period spectrum following some experimental measurements. A structure of this mast has a height equal to 198.0 m and its shaft has been designed with the use of S235J2 steel in form of three-walled lattice with side width equal to 1.30 m. Leg members have been modelled as round pipes with diameter of 168.3 mm with the cross-section wall thickness adjusted to the ultimate limit state conditions. The mast face lacing elements have been designed as round pipes of the diameter 63.5 mm and their wall thickness has been designed quite similarly. Mast guys have been attached to the shaft at the altitudes equal 60.0 m, 120.0 m and 180.0 m from the ground level and they are introduced with the inclination angle equal to 45 degrees. A spiral single strand steel rope 1x37 with the diameter of 32.0 mm has been applied with the mean strength equal to 1960.0 MPa. The mean elasticity modulus of the cables has been assumed as 150.0 GPa, whereas elasticity modulus of the mast shaft elements has been adopted as 210.0 GPa. An initial tension for the mast guys has been provided by prestressing equivalent to 11.0 cm, 22.0 cm and 31.0 cm, correspondingly for the consecutive attachment levels with ascending order starting from the bottom.

Numerical model prepared in the Finite Element Method system ROBOT consists of 903 finite elements. Mast shaft has been modelled by 2 node bar elements described by linear shape functions and 6 degrees of freedom at each end – mast structure consists of 894 of such elements. Mast guys has been modelled as cable finite elements implemented in this system according to the small-sag theory. This furtherly contributes to assumptions such as the equilibrium of the cable is being found considering constant tensile force of the cable along its length.

## 2 UNCERTAINTY ANALYSIS

Several environmental and mechanical uncertainties have been taken into account for computer analysis of the dynamic response spectra in the most fragile elements of the mast. They have been defined as Gaussian variables with the given expectations and some interval of coefficient of variation. Both types of uncertainties and their implementation methods into numerical analysis have been discussed here as the functions of the approximating polynomial order and the input uncertainty level. The final graphs attached below include only these parameters, which appeared to be decisive for the mast reliability analysis.

### 2.1 Environmental uncertainties

The first environmental uncertainty is the external temperature applied to the mast structure. Two ranges of temperature load applied uniformly to the entire structure have been taken into consideration, namely from -10°C to +40°C, and also from -50°C until ±0°C. These two case studies of external temperature have been additionally discretized into 11 sub-cases with an increase equal to °5°C. The second environmental uncertainty has been described by the uncertain wind velocity distribution. This wind load has been modelled according to the Eurocode 1 guidelines for towers, chimneys and masts including an effect of the local wind gusts [1]. A dynamic analysis of the wind influence on this structure has been performed for a time interval of 10 minutes according to the spectrum included in Fig. 1.

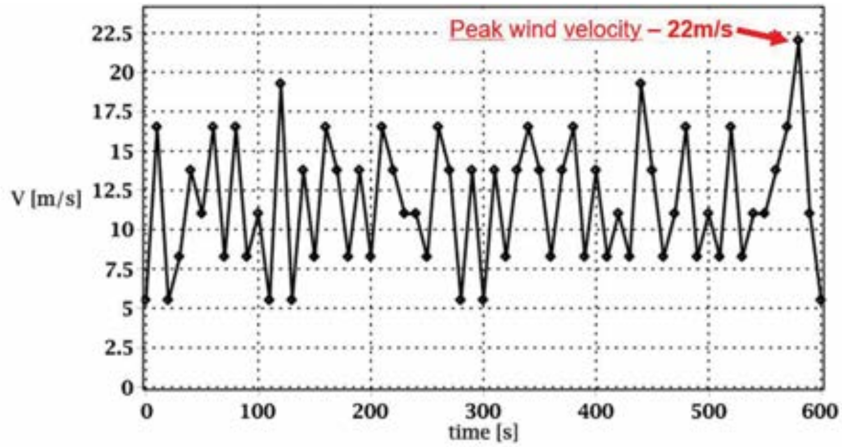


Figure 1: Wind velocity fluctuations in 10-minute interval

The uncertainty description of dynamic wind action upon this structure has been performed similarly to the temperature load case. It means that 11 types of dynamic wind load histories have been analyzed, each of them described by a different multiplication factor applied to the spectrum presented in Fig.1. Values of peak wind velocity taken into account are specifically presented and discussed further in section 3.2. Wind action expressed as pressure applied to the mast consists of some mean load and supplementary patch load simulating additional wind gusts along certain parts of the mast according to [1]. An example of the wind load distribution applied to the mast shaft, which consists of both load types, has been presented in Fig. 2.

a) b)

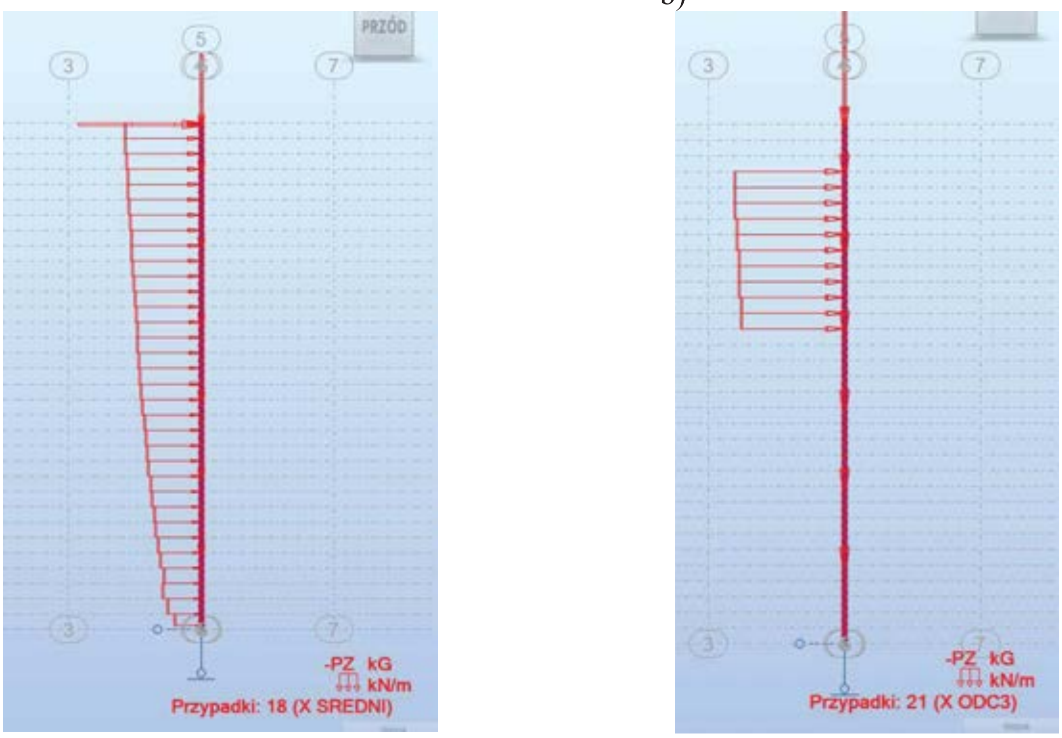


Figure 2: Wind load distribution along the mast height: a) mean load, b) patch load

### 2.2 Mechanical uncertainties

Mechanical uncertainties considered in this analysis are - the elasticity modulus of mast guys and, independently, elasticity modulus of mast shaft. In both cases 11 representative values of elasticity modulus have been taken into account. These representative test values of elasticity moduli have been calculated based on expected value, which refers directly to Eurocodes namely 210.0 GPa for mast shaft elements and 150.0 GPa for mast guys respectively. Multiplication factors for these representative values have been introduced as 0.90, 0.92, 0.94, 0.96, 0.98, 1.00, 1.02, 1.04, 1.06, 1.08, 1.10, consecutively.

### 2.3 State variables

State variables for the steel guyed mast structure have been chosen according to the Eurocode guidelines [2], [3], which indicates that verification of the Ultimate Limit States (ULS) and Serviceability Limit States (SLS) in probabilistic context is sufficient for its reliability. Analyzed state variables of the greatest interest represent both ULS and SLS states and they refer to the stress-state of main leg element, stress-state of face lacing element, global horizontal displacement of the top of mast shaft and also extreme twisting of the shaft. First two obviously have been taken upon with respect to ULS and the remaining two – for the SLS.

## 3 NUMERICAL SOLUTION

### 3.1 Dynamic response spectra

Numerical solution has been obtained via simulations performed in the system ROBOT using non-linear dynamic analysis procedure based on the Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm. Integration of equations of motion induced by the wind spectrum has been performed with the use of Hilber-Hughes-Taylor (HHT) solver [4]. An accuracy of the HHT method for geometrically non-linear guyed mast structure has been studied by a contrast with the Newmark solver [5], which has been presented in Figure 3.

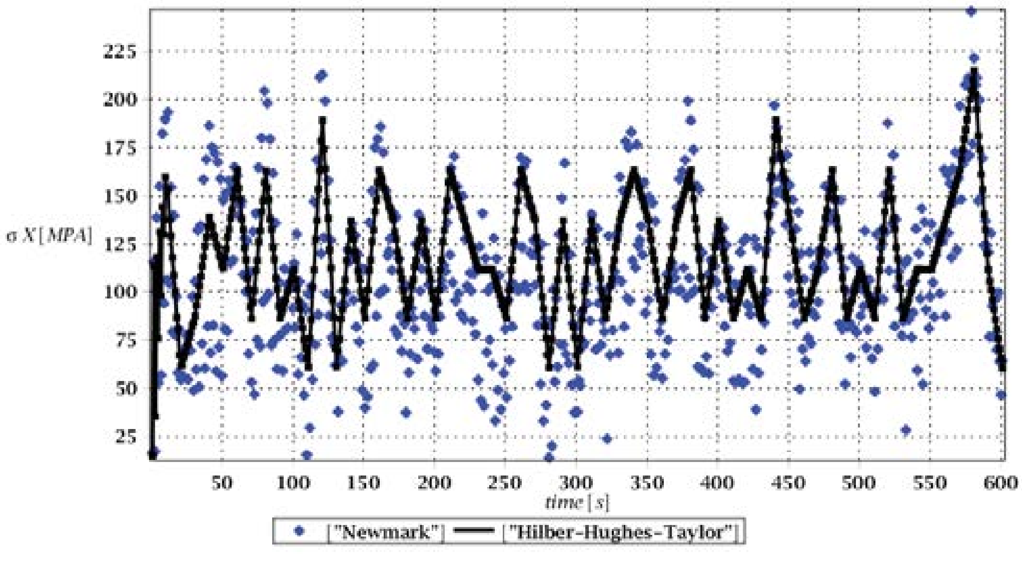


Figure 3: Response history for the stress in main leg computed using the HHT and the Newmark methods

The HHT solver is based upon specific time discretization, where the structural displacements and velocities in the given time step are described using displacements, velocities and accelerations in the previous time step as

$$\begin{cases} x_{i+1} = x_i + \Delta t \cdot \dot{x}_i + (1/2-\beta)\cdot(\Delta t)^2 \cdot \ddot{x}_i \\ \dot{x}_{i+1} = \dot{x}_i + (1-\gamma)\cdot \Delta t \cdot \ddot{x}_i \end{cases} \tag{1}$$

Time step of the method has been set here as 0.10 s and the output response results have been saved every tenth time steps (at every 1.0 s). The mast dynamic response has been recalculated for several realizations of each random parameter about its expected value, so that 176 series of the Stochastic Finite Element Method computations have been performed resulting in 105,600 discrete results of the principal state variables.

### 3.2 Structural Response Function approximation

The Weighted Least Squares Method (WLSM) has been selected in order to perform the Structural Response Function (SRF) approximation in the form of polynomials of order varying from the 5$^{th}$ until the 10$^{th}$ one [6]. This has been completed for each state variable in the time step corresponding to the extreme values in the limit states. Let us note that the uniform, triangular and Dirac weight functions distributions have been initially considered for this approximation procedure. Triangular weight functions exhibited the best accuracy for the provided series of dynamic response data with respect to least square error minimization and also overfitting phenomenon minimization criterium, so that they have been applied in further SFEM analysis. The prescribed weights for each uncertain parameter have been given in such a way that the greatest weight corresponds to the expected value. Referring to Table 1, one can notice that weights for WLSM approximation of the temperature load case should be (1,2,3,4,5,6,7,8,9,8,7), while for the peak wind are proposed as (1,2,3,4,5,6,5,4,3,2,1). The discrete values of each uncertain parameters have been presented in Table 1.

| | Input uncertain parameter | | | | |
|---|---|---|---|---|---|
| No. | Temperature load 1 °C | Temperature load 2 °C | Peak wind velocity m s$^{-1}$ | Elasticity modulus – guys GPa | Elasticity modulus – shaft GPa |
| 1 | -10.00 | -50.00 | 19.80 | 135.0 | 189.0 |
| 2 | -5.00 | -45.00 | 20.24 | 138.0 | 193.2 |
| 3 | 0.00 | <u>-40.00</u> | 20.68 | 141.0 | 197.4 |
| 4 | 5.00 | -35.00. | 21.12 | 144.0 | 201.6 |
| 5 | 10.00 | -30.00 | 21.56 | 147.0 | 205.8 |
| 6 | 15.00 | -25.00 | <u>22.00</u> | <u>150.0</u> | <u>210.0</u> |
| 7 | 20.00 | -20.00. | 22.44 | 153.0 | 214.2 |
| 8 | 25.00 | -15.00 | 22.88 | 156.0 | 218.4 |
| 9 | <u>30.00</u> | -10.00 | 23.32 | 159.0 | 222.6 |
| 10 | 35.00 | -5.00 | 23.76 | 162.0 | 226.8 |
| 11 | 40.00 | 0.00 | 24.20 | 165.0 | 231.0 |

Table 1: Discretization of the two uncertain parameters (expected values underlined)

Each data of the response history has been approximated by $5^{th}$ to $10^{th}$ order polynomials. Finally, 4 uncertain parameters, 4 state variables and 600 time steps have been analyzed and the WLSM approximation by 6 different polynomials has been performed. So that general database consists of 57,600 polynomials obtained solely by finding the solution with the HHT solver. For a brevity of this work presentation, the SRF representing extreme value of some state variables have been taken into account only. It has been assumed that this moment in dynamic analysis would be considered as the most dangerous from structural engineering perspective and its SRF should be serve in further mast reliability index estimation.

## 4 PROBABILISTIC ANALYSIS

Probabilistic analysis has been performed with the Stochastic Perturbation Technique (SPT), and independently using Monte-Carlo simulations (MCS) and semi-analytical method (SAM) also [7], [8]. The results obtained by SPT are marked with asterisk, these coming from the MCS - with a cross, while these obtained by SAM - with a diagonal box. The main study has been performed to verify how the order of approximation would affect the resulting basic probabilistic characteristics and, finally, reliability index determination. A coincidence in-between three different probabilistic methods has been also investigated. Each SRF has been independently approximated as a function of some uncertain parameter. To pursue the abovementioned investigation it has been assumed that the SRF are some functions of a random variable with unknown standard deviation. Then, the SRFs have been presented as a result related directly to the coefficient of variation of this particular variable.

Expected values of normal stresses in the mast leg are presented in Fig. 4 below as the functions of the input coefficient of variation of Young modulus of the guys and also of the response polynomial order. As it is demonstrated, an increase of the input uncertainty leads each time to a decrease of this expectation. Further, one may notice that the resulting extreme expectation seems to be quite sensitive to the chosen approximation order, especially for larger values of the coefficient $\alpha$. Higher orders of this approximation make these variations more remarkable. The resulting coefficients of variation of these stresses in addition to a wind pressure coefficient of variation have been presented further in Fig. 5. An interrelation in-between these coefficients is almost linear and the impact of the polynomial approximation order is definitely smaller than in Fig. 4. On the other hand, the resulting coefficients of variation of these stresses with respect to Young modulus of the mast guys shown in Fig. 6 presented more nonlinear interrelation to coefficient $\alpha$. Increase of parameter $\alpha$ in such case results in non-linear increase of output coefficient of variation.

The expected values of the normal stresses in face lacing elements are contained in Fig 7 as the functions of the input coefficient of variation of Young modulus of the guys and also of the response polynomial order. Fifth and sixth order polynomials in this case presented opposite monotonicity to higher order polynomials which can be observed for values of parameter $\alpha$ greater than 0.10. Once again one may notice that obtained expectations are sensitive to polynomial order for larger values of abovementioned parameter $\alpha$. The resulting coefficient of variation of this stresses has been presented in Fig. 8. In this case an interrelation in-between input and output variations is expotentially related. Some additional inaccuracy in between SPT and SAM has been observed for tenth order polynomial approximation. It is seen that this randomness is the largest one, which confirms a fundamental role of the Ultimate Limit State for this structure safety.

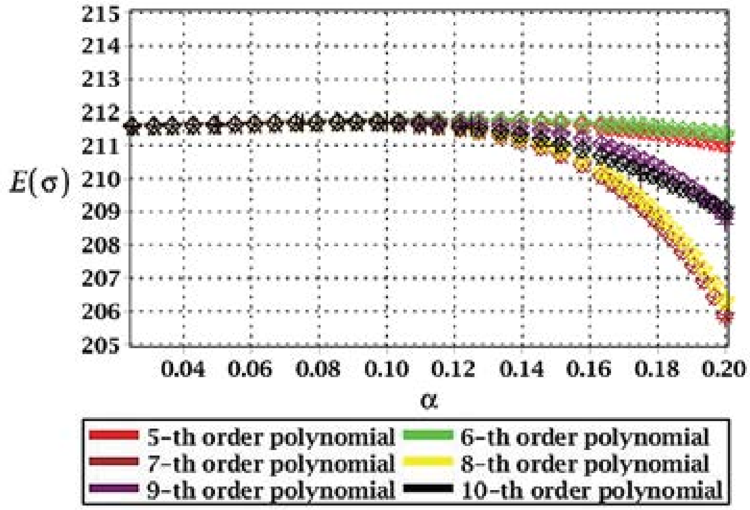


Figure 4: Expected value of response function describing stress in main leg in reference to guys elasticity modulus coefficient of variation

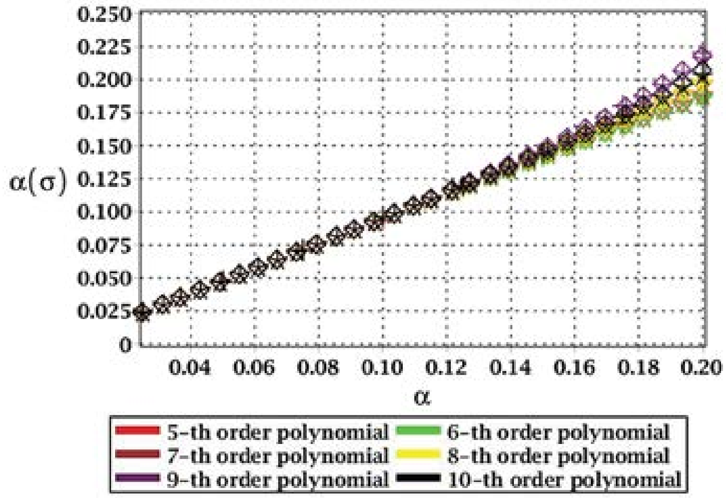


Figure 5: COV of response function describing stress in main leg in reference to wind load coefficient of variation

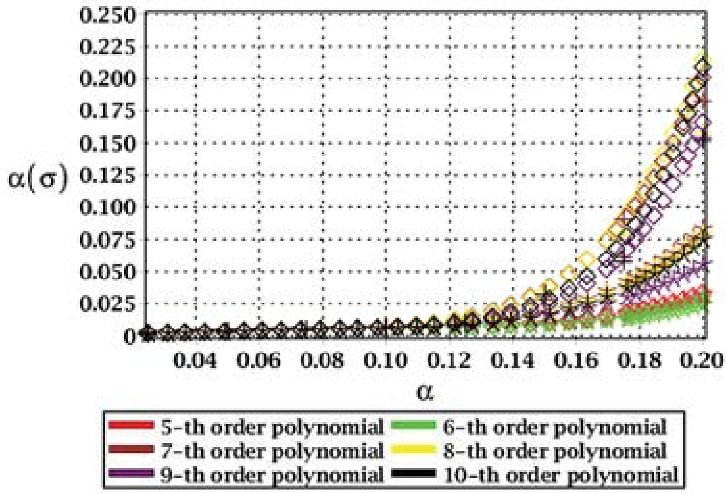


Figure 6: COV of response function describing stress in main leg in reference to guys elasticity modulus coefficient of variation

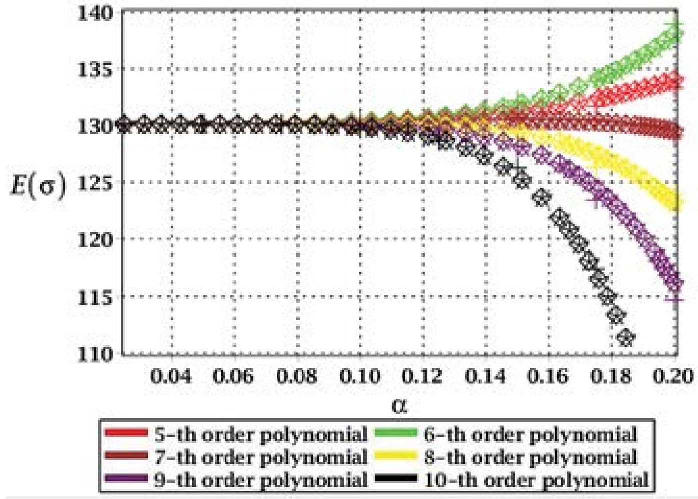


Figure 7: Expected value of response function describing stress in face lacing in reference to guys elasticity modulus coefficient of variation

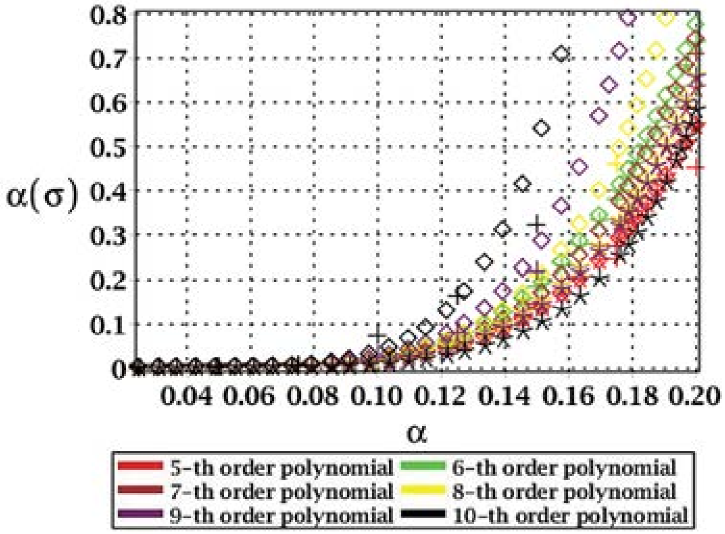


Figure 8: COV of response function describing stress in face lacing in reference to guys elasticity modulus coefficient of variation

Expected values of global horizontal displacement are shown in Fig. 9 below as the functions of the input coefficient of variation of Young modulus of the guys and also of the response polynomial order. As it was demonstrated, an increase of the input uncertainty leads each time to a increase of this expectation except for tenth order polynomial approximation. Further, one may notice that the resulting extreme expectation seems to be quite sensitive to the chosen approximation order, especially for larger values of the coefficient α. The resulting coefficients of variation of these stresses related to guys Young modulus coefficient of variation have been presented further in Fig. 10. An interrelation in-between these coefficients is expotentially monotonous. The best approximations of these coefficients of variations in-between all three probabilistic techniques has been observed for fifth, sixth and seventh order of polynomial approximation.

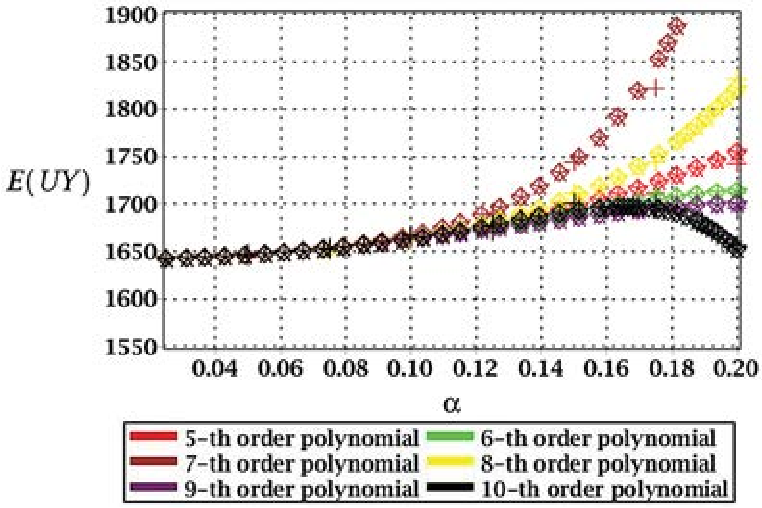


Figure 9: Expected value of response function in reference to guys elasticity modulus coefficient of variation

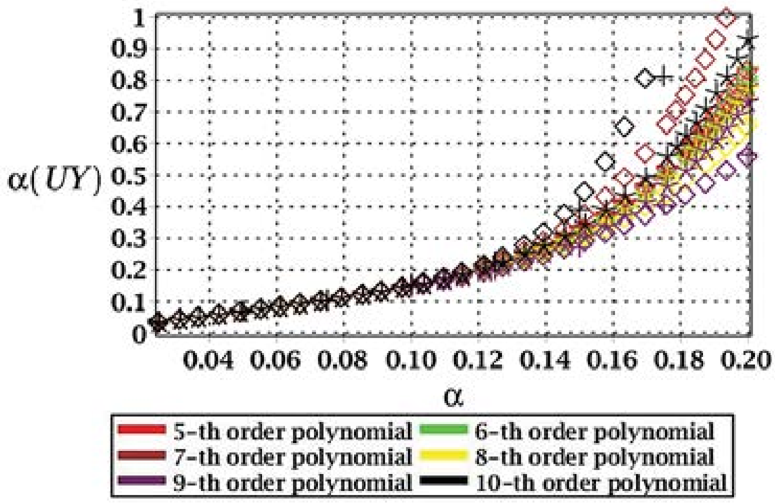


Figure 10: COV of response function in reference to guys elasticity modulus coefficient of variation

Expectations of a twist of the mast shaft are collected in Fig. 11 below as the functions of the input coefficient of variation of Young modulus of mast shaft and also of the response polynomial order. As it is demonstrated, an increase of the input uncertainty leads to a decrease of this expectation except for fifth order polynomial approximation where minor increase is observed. Further, one may notice that the resulting extreme expectation seems to be quite sensitive to the chosen approximation order, especially for larger values of the coefficient α. The resulting coefficients of variation of these stresses related to mast shaft Young modulus coefficient of variation have been presented further in Fig. 12. An interrelation in-between these coefficients is nonlinearly monotonous. The best approximations of these coefficients of variations in-between all three probabilistic techniques has been observed for fifth, sixth, seventh and eighth order of polynomial approximation.

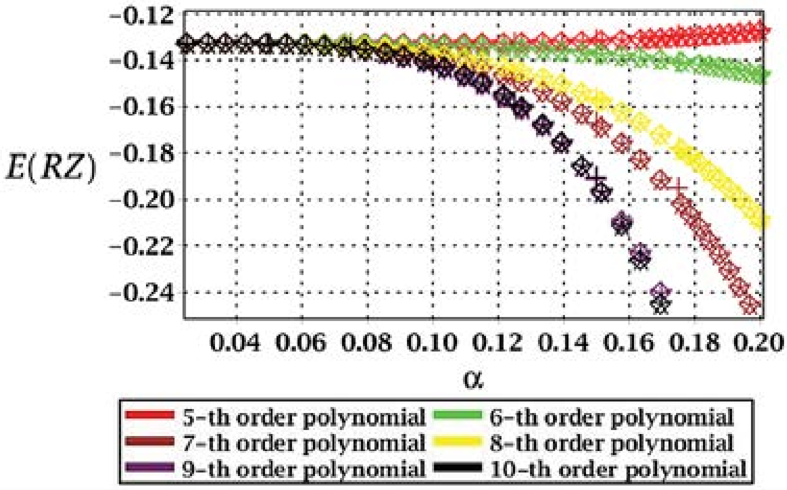


Figure 11: Expected value of response function in reference to shaft elasticity modulus coefficient of variation

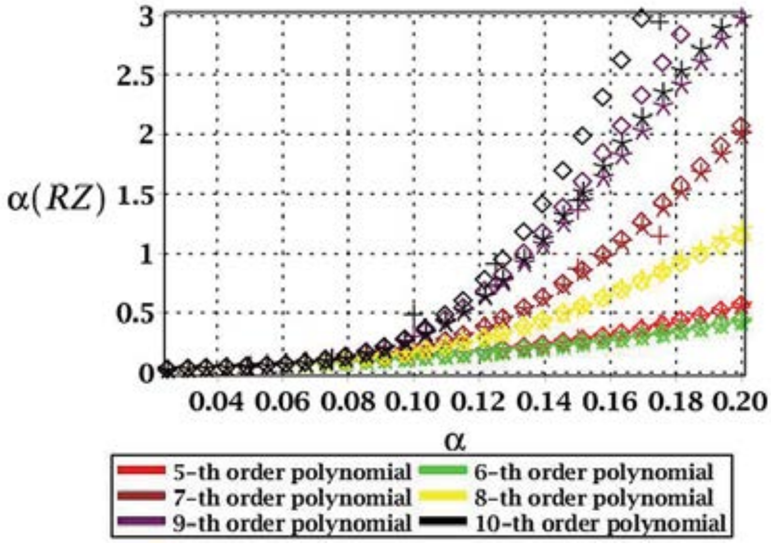


Figure 12: COV of response function in reference to shaft elasticity modulus coefficient of variation

## 5 RELIABILITY ASSESSMENT

Eurocode 0 statements regarding reliability of structures it should be noted that structures such as guyed steel masts undoubtedly belong to the highest reliability class RC3 thus their reliability assessment should be available at hand. The Authors propose in this paper to express reliability of the mast structure by the Cornell theory exposed in Eurocode 0. According to [2] the minimum reliability index value regarding ULS for RC3 class is equal to 4.3. For SLS the Eurocode 0 does not provide target reliability values with respect to the reversible SLS states and their arbitrary assumption must comply with the expectations of the investor as well as allow safe exploitation of the equipment attached to the structure.

The reliability index has been calculated as a function of coefficient of variation of the given input parameter and results of such investigation has been presented for most significant random parameters with respect to considered state variable under assessment. Significance of chosen random parameters once again has been dictated by the sensitivity of the mast structure which has been visualized in probabilistic response spectra presented in chapter 4. From Figs. 13-16 can be deduced that all proposed polynomial approximations produces similar outcome in form of reliability responses related to input parameter α. From all uncertainties taken into account it can be noticed that the mast structure exhibits the greatest sensitivity towards uncertain Young modulus of guys and towards wind pressure. This observation can be expressed directly through determination and presentation of the reliability index as for Figs.14-15 reliability drops below some arbitrary level of acceptance at relatively small values of input α parameter. When acceptance level is set as reliability index equal to 2.0 for example, then the mean wind pressure and Young modulus should be described using random distributions with a coefficient of variation smaller than 0.08.

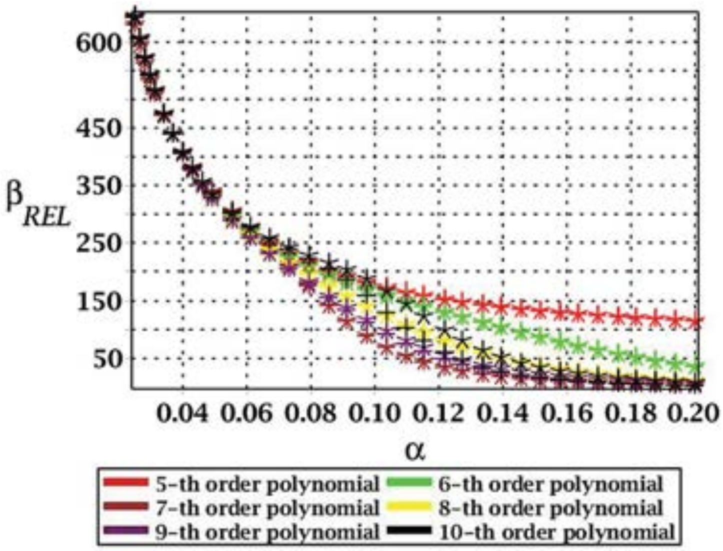


Figure 13: Reliability index in function of input COV of shaft elasticity modulus with respect to ULS state of stress in face lacing elements

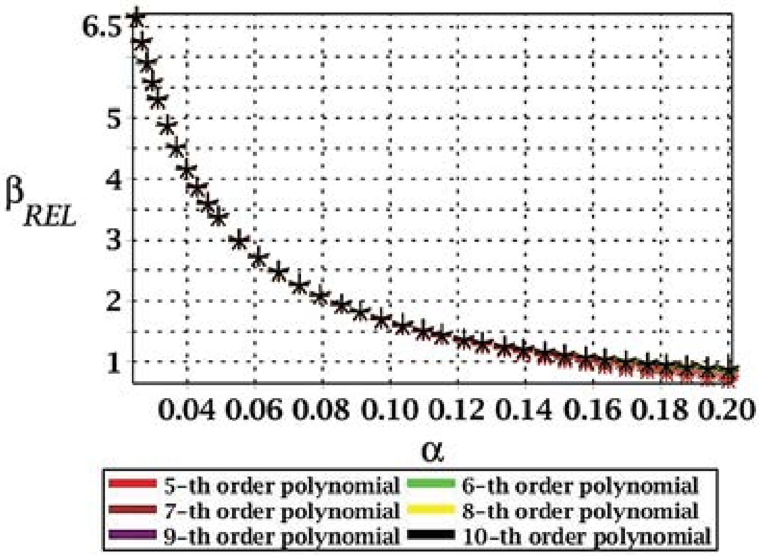


Figure 14: Reliability index in function of input COV of wind velocity with respect to SLS state of global horizontal displacement

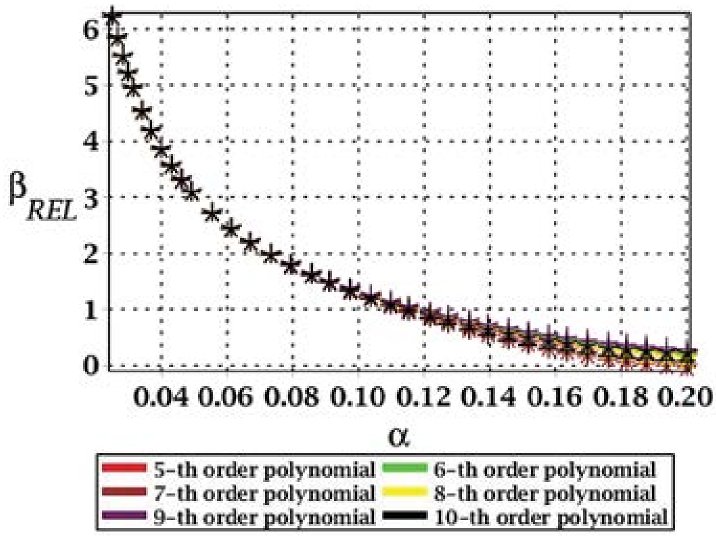


Figure 15: Reliability index in function of input COV of guys elasticity modulus with respect to SLS state of global horizontal displacement

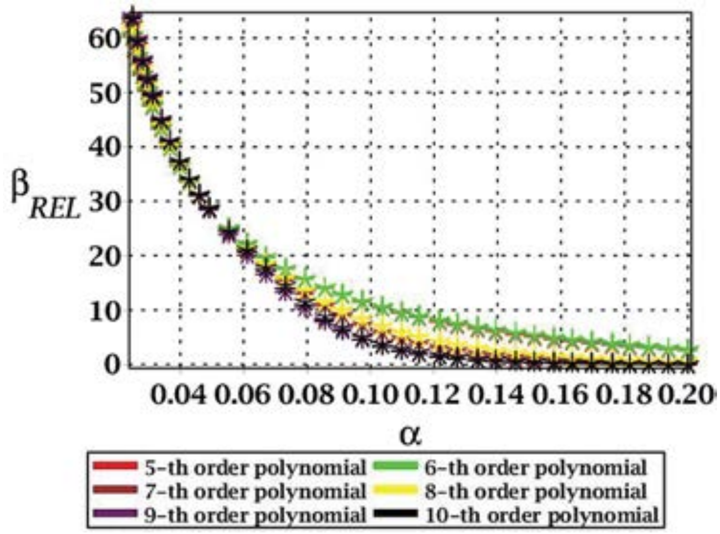


Figure 16: Reliability index in function of input COV of shaft elasticity modulus with respect to SLS state of twist of mast shaft

## 6 CONCLUDING REMARKS

The Iterative Stochastic Perturbation Technique shows quite satisfactory coincidence in comparison with both Monte-Carlo simulation estimators and also with the semi-analytical method moments and it requires also remarkably less computational time and effort. This is very important considering the fact that nonlinear geometrical effects are considered during dynamic excitation of the structure, which undoubtedly is large scale civil engineering structure. More interestingly, higher order polynomials have returned neither more stable nor predictable random responses. Different orders of polynomials used in WLSM show exquisite accuracy when the input coefficient of variation is smaller than about 0.10 – 0.12. When the input COV is above this value, the differences in-between various orders of polynomials become more significant and even some of them show opposite monotonicity than the others. Some differences in between the SPT and referential techniques are observed above this uncertainty level. Nevertheless, material uncertainties or these associated with the dead loads are well known and undergo smaller deviations than this limit in civil engineering. In such case it has been proven that an order of the polynomial used in reliability index assessment can be arbitrarily assumed and may result in quite satisfactory outcome. On the other hand, when uncertainties taken into account exhibit larger deviations such as wind velocity history or some other environmental phenomenon, the technique presented in this paper might be used to firstly verify and choose proper other of polynomial or other function in such manner that it would perform satisfactory coincidence in comparison with referential techniques. Such a function then might be used in non-stationary reliability assessment of the structure exhibiting some nonlinearities and subjected at the same time to the dynamic excitation.

One can notice that the given guyed mast structure exhibits different sensitivities with respect to the chosen uncertainty sources. What is more, the structural sensitivity can be associated with the certain state variable under consideration. In example let us imagine considering the twist of mast shaft – one can assume that the general structural stiffness of the mast towards twist could be directly associated with the stiffness of mast shaft itself. Obtained results confirms that the uncertainty in mast shaft elasticity modulus contributes in the greatest manner to its dynamic response to the wind excitation. The output COV of structure response function correlated with shaft elasticity modulus increases most significantly with increasing parameter $\alpha$ of elasticity modulus of mast shaft. Similar observations can be made with respect to the other state variables. From this analysis it can be furtherly concluded that for both ULS states (stress analysis in both main legs and the face lacing elements) the wind velocity is the most significant random parameter and for the SLS state expressed by horizontal displacement the mast structure exhibits the greatest sensitivity to elasticity of mast guys. What is more the temperature load is the least significant uncertain parameter considered in this paper with respect to all four state variables taken into account.

Sensitivity analysis and reliability index estimation are strictly related with each other as it may be observed from in this paper. Reliability index reaches the lowest values for the uncertain parameter to which the structure is the most sensitive – regarding limit state under consideration. In summary for both ULS associated with stress the structure presents the greatest sensitivity towards uncertainty expressed in wind velocity. Then for SLS associated with horizontal displacement the structure exhibits significant sensitivity with respect to both wind velocity and elasticity of mast guys and finally for SLS associated with structural rotation, the structure exhibits the greatest sensitivity towards elasticity modulus of mast shaft.

From another perspective it can be also noticed that various orders of approximating polynomials usually lead to similar results of probabilistic computer analysis and also, in consequence, to similar reliability index values.

## REFERENCES

[1] *Eurocode 1: Actions on structures – Part 1-4: General actions – Wind actions* – European Committee for Standardization, Brussels, 2010

[2] *Eurocode 0: Basis of structural design* – European Committee for Standardization, Brussels, 2002

[3] *Eurocode 3: Design of steel structures – Part 3-1: Towers, masts and chimneys – Towers*

*and masts* – European Committee for Standardization, Brussels, 2006

[4] Hilber, H.M., Hughes, T.J.R. & Taylor, R.L., *Improved Numerical Dissipation for Time Integration Algorithms in Structural Dynamics,* Earthquake Engineering & Structural Dynamics, 1997, 5, 282-292

[5] Newmark, N.M., *A method of computation for structural dynamics*, Journal of the Engineering Mechanics Division, 1959, 85 (EM3): 67-94

[6] Kamiński, M., Bredow, R., *Uncertainty Analysis for Overhead Powerlines by the Generalized Stochastic Perturbation Technique*, Journal of Aerospace Engineering, DOI: 10.1061/(ASCE)AS.1943-5525.0001270, 2021, (in press).

[7] Kamiński M.. *The Stochastic Perturbation Method for Computational Mechanics* – Chichester, Wiley.

[8] Bredow. R., Kamiński, M., *Computer analysis of dynamic reliability of some concrete beam structure exhibiting random damping*, Int. J. of Applied Mechanics and Engineering, 2021, vol. 26, No.1, pp.45-64