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# QUANTIFYING THE EPISTEMIC UNCERTAINTY LINKED TO THE DEVELOPMENT PROCESS OF AN EARLY-STAGE COMPONENT FOR CRASHWORTHINESS

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**Abstract.** *We propose in this work a new method to quantify the epistemic uncertainty linked to the early-stages of the development process of non-linear dynamic systems. It is here applied to the development of the crashworthiness of the front of a new car. More specifically, we investigate the effects on a component of the parts around it. The uncertainty derives from the many unknowns that drive the design of a component to change. Nevertheless, it is quantifiable thanks to the solution space methodology. It is then propagated through a simplified Finite Elements environment that represents the front of a newly developed car. A model quantifies the uncertainty by exploiting the force ranges computed in the solution space methodology. However, the model inherits a high number of variables, making the propagation of the uncertainty problematic. To solve the dimensionality issue we propose a new uncertainty model that couples the variables together. Experiments on early-stage designs of components are used to express the dependencies. They are designed to translate the corridors of solution space into new force ranges that capture the crash behavior of a component. Therefore, from an initial number of possibly 29 independent variables, we end up with only 6; one per component making up the crash sub-assembly of the front of the car. The proposed method yields results similar to those yielded by the model with 29 variables. This shows that we effectively solved the dimensionality problem without losing information.*

*Our method estimates the effects on a component of its neighboring systems. This allows a developer to quickly identify in the early-stage phases of development the designs that would not work well in the final product. Thus, the method reduces the chances of costly re-design in a late-stage phase of the development.*

**Keywords:** Crashworthiness, Solution Space, Uncertainty Quantification, Simplified Modelling

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## 1 INTRODUCTION

The design of a component changes a lot between the initial and final configuration of a new car. Because of this epistemic uncertainty, concurrent development of a whole subassembly of a car is still difficult. The findings in the field of systems engineering support the process by identifying sets of components that can be developed independently of each other. Methods like the solution space approach decouple the components by providing a set of requirements for each one [1]. More specifically, if the requirements are fulfilled, the method allows to develop each part independently, so that the design targets are reached in the sub-assembly. The method, however, is thought for the early stages of development. When it comes to the actual engineering of a component, new changes arise when the final sub-assembly is tested all together. Therefore, concurrent engineering still cannot be used from the beginning to the end of the development process.

The problems of developing components in parallel rise when the components themselves are assembled. Consider the study case of this paper: the bonnet area of a car and the components here responsible for dissipating the kinetic energy during a crash. Thanks to the solution space method, these components can be developed independently. Their geometry and, consequently, how they deform, determines how much energy each component dissipates. However, the deformation mode changes when testing a component singularly or in the assembly. The differences are due to the interactions that come into play between different components. Since the solution space method identifies the good design on the basis of the energy they are dissipating, a good one can become bad when tested in the final sub-assembly. This shift causes a layer of epistemic uncertainty around the behavior a developer expects from the design.

If, on the one hand, concurrent engineering reduces the time to market of a product, on the other hand, fixing a design becomes more difficult due to the uncertainty on the expected behavior. The method presented in [2] provides a way to use the information in the solution space method to quantify the uncertainty of an early-stage design in the field of crashworthiness. It evaluates the variability to be expected from each component, so that it can then be cascaded on an early-stage design by means of a simplified Finite Elements (FE) model —the model is called Geometry Space Finite Elements Model (GSFEM). Thanks to the GSFEM, the method provides the developers with an insight on how the behavior of their new design is affected by other components. This insight is provided already in the early stages. However, the uncertainty model in [2] inherits a high number of independent variables from the solution space method. The number ranges from a minimum of 22 to a maximum of 29. Therefore, the approach previously proposed suffers from a dimensionality problem: the number of samples required for an appropriate uncertainty propagation and quantification is too big. Here, we tackle the dimensionality issue by replacing the uncertainty model with one based on data measured on actual components.

To solve the dimensionality problem, we look at how the components deform and how they relate to the solution space methodology. In the study case of our paper, the solution space model has 31 variables, grouped into 7 components. In other words, each component is divided into a set of sections, each associated to an independent variable. According to [3], the sections define the set-up of the solution space model (Fig. 1). When it comes to the physical problem, the sections belonging to the same component are not independent. We exploit these physical dependencies to reduce the dimensions of the uncertainty problem. Therefore, we propose a way to link the variables belonging to the same component. In our example, they are reduced to 6.

Reducing the number of variables in the uncertainty model yields the following main benefits:

1. The usability of the whole method increases;
2. Engineers have a usable way of identifying bad designs already in the early-stages of development;
3. The GSFEM becomes exploitable for studies on the interactions between components; these studies are outside the scope of this paper.

The paper is structured in 4 sections: 1) an overview of the current state of art concerning the solution space method, GSFEM and useful uncertainty methods; 2) the methodology to build the new uncertainty model; 3) the results and a critical reflection on them; 4) the conclusion.

## 2 LITERATURE OVERVIEW

Let us start by giving an overview on the three main topics addressed in this paper: 1) the solution space method, 2) the simplified FE model, and 3) how to model the epistemic uncertainty. Regarding the first topic we will give a brief overview of the method. The simplified FE model is then described in more details, while for the last topic we will present different models to support the choice of using a B-splines based one.

### 2.1 Solution Space

The solution space method belongs to the field of systems engineering [1, 4]. Its purpose is to cascade high-level requirements down on the component-level that developers prefer. This can be done in two ways, called direct and indirect method [5, 6]. We focus on the first approach.

The direct approach is based on an analytical representation of the product to be developed. In the crashworthiness panorama, it means representing an impact by basic physical relations. The analytical model considered in [2] captures the frontal impact against a rigid barrier with full overlap. Therefore, the method defines a low-fidelity model that captures how each component behaves during the crash. Through this representation, a range is identified, so that each component can vary independently of the others and still guarantee the crashworthiness of the car —these ranges define the so-called *feasible space*. They are physically defined by a set of forces that are plotted against the deformation length (Fig. 1), called *corridors* [7]. In other words, the different behaviors allowed for a component —the possible way it can absorb the kinetic energy— are represented in the direct approach by a range of forces that need to be measured on the component over a predetermined displacement.

As shown in the figure, the corridors of the components are divided into several sections. Dividing each component into several sections provides the output of the method with a certain level of resolution. It gives the engineers a more usable low-level requirement formulation. For example, component 1 is divided into 9 sections. Therefore, 9 variables are needed to compute its set of forces, one per section. The corridors are nothing else than a force-interval assigned to each section of the component. The criteria for an acceptable balance between the right level of resolution and computational effort is found in [3]. In our application, the best compromise yields 31 independent variables, and, thus, the dimensionality problem for cascading the uncertainty.

In the analytical model, the structure of the car and how it behaves during the impact is combined with the high-level requirements defined for the whole car —minimum energy absorption,

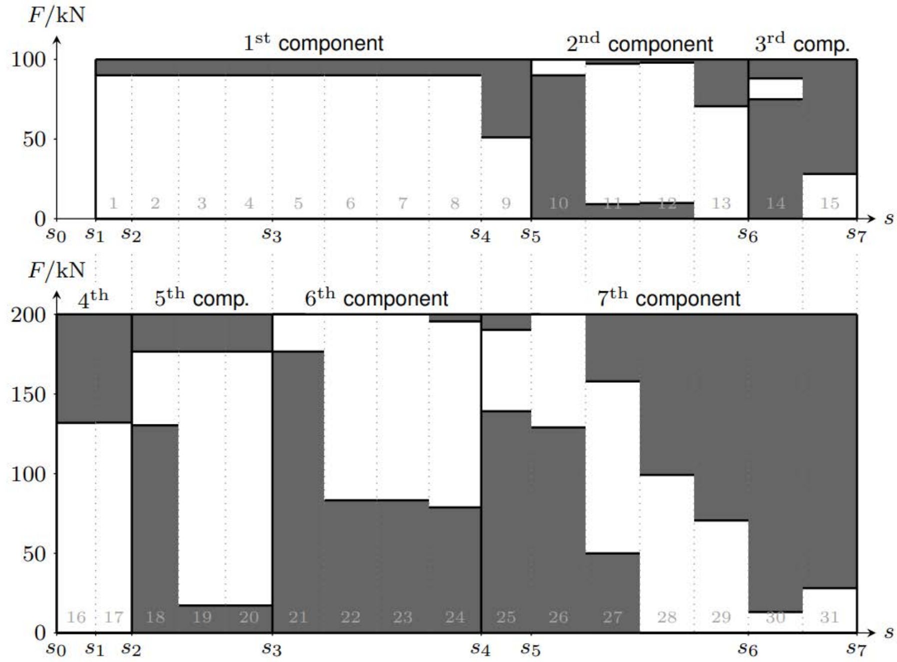



Figure 1: Corridors resulted from applying the solution space method [7]

maximum allowable deceleration, and order of deformation [8]— to define the feasible space. These are linked by the relation $E = F \cdot s$, where $E$ is the energy to dissipate, $F$ the absorbed force and $s$ the deformation. The car structure and how it behaves is derived from the deformation space model [5, 9]. However, this model represents only the total deformation of the crash structure. In other words, it captures in a very simplified way the relations between the components and how much each contributes to the total deformation. The deformation space model also defines the resolution of the analytical model by drawing the number of sections. However, to obtain the simplified model, one first needs to describe the structure of the car. It is realized in the Geometry Space Model (GSM).

The GSM describes the structure of a car in a simplified way, to propagate it to the deformation space model, and consequently to the analytical model. Such information provides the description of how much each component deforms, and the relations between different components [5, 9]. Since the Geometry Space Finite Elements Model (GSFEM) is based on the GSM, we describe the GSM more in details here. The first step to build the model is to identify the components that constitute the crash structure. Our study case is about the 2017 Honda Accord model [10]. Its crash structure is made up of 7 components, highlighted in Fig. 2 on the left. Notice that they are separated in two distinct load paths. Therefore, the GSM is composed of two loadpaths: the upper one of 3 components; the lower one of 4 components. The second step consists of determining the deformation length for each component. It can be measured or evaluated mathematically [3]. The GSM is complete once the information of the crash structure is merged with that of the deformation (Fig. 2 on the right). Hence, a car is represented by a simplified structure where each component is spatially located and fully described in its deformable and undeformable parts. The first spans as much as the deformation length. The second one is as long as the remaining part of the component. The simplified representation of the crash structure is used not only to define the deformation space model, but also to model the GSFEM.

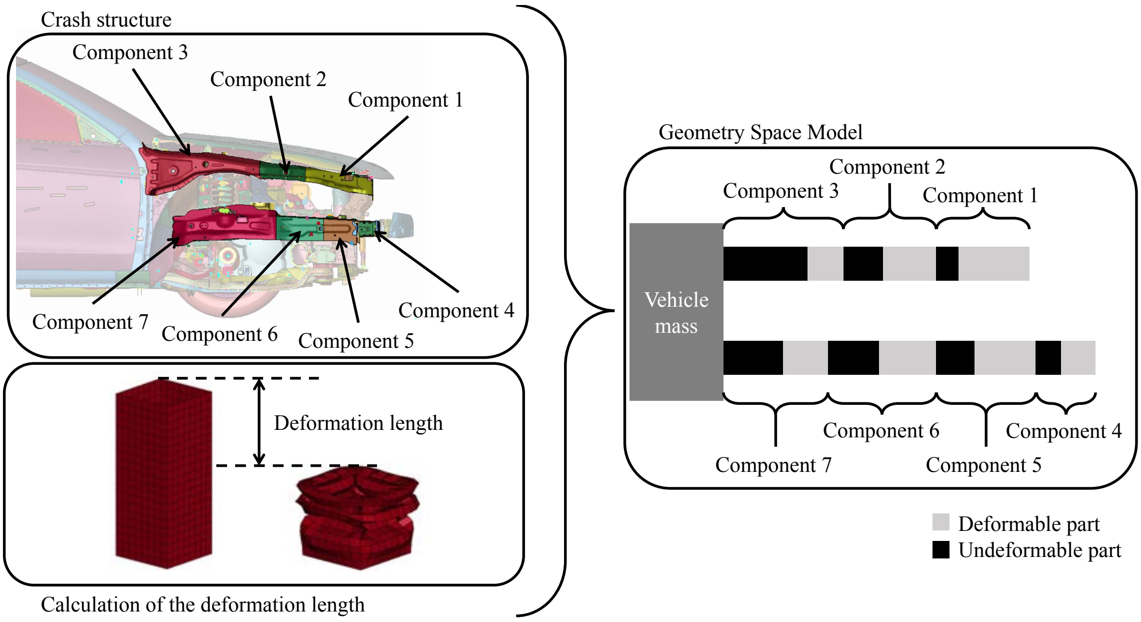



Figure 2: Process of building the Geometry Space Model: left top corner represents the step describing the crash structure; left bottom corner represents the calculations for the deformation length; on the right the final Geometry Space Model is shown

## 2.2 Geometry Space Finite Elements Model

The Geometry Space Finite Elements Model (GSFEM) takes advantage of the simplified modeling approach of the GSM to represent the corridors in an FE environment, such as LS-Dyna. It is built to be cheap to run, so that it can be evaluated a high number of times. It is so to allow using the model to cascade the uncertainty derived from the solution space method [2].

To ensure a low computational effort of the GSFEM, one-dimensional elements are used to represent the components defined in the GSM. Each component is made up of two parts, one corresponding to the deformable part of the GSM, and one to the undeformable part (Fig. 3). The first is modeled so that a stepped force-deformation curve can be imposed, defining its plastic deformation. The curve is created to match the stepped trait of the corridors, like the red one shown in Fig 3. The second part of the elements is modeled as a very stiff spring that acts as a rigid body. The use of very stiff springs for these parts avoids numerical instabilities caused by one-dimensional rigid bodies. Each component defined in the GSM corresponds in the GSFEM to a 1D element. Therefore, for the properties of the GSM, the crash structure of the vehicle is fully described by a simplified FE model made of one-dimensional components.

In the GSFEM, full meshes can be combined with the one-dimensional elements to cascade the uncertainty. The full mesh is used for the newly developed component that an engineer wants to assess the expected behavior of. The other components are used to represent the variables of the uncertainty model in the FE environment. The independent variables define the curves of the deformable parts of the 1D elements. Each step corresponds to one uncertainty variable. The one-dimensional structure and the full mesh are connected rigidly. The connection distributes the force of the 1D elements on the ends of the full mesh. As a last remark, notice that an extra one-dimensional element is used to represent other forms of energy dissipation that occur during a crash. For this reason, the extra load path in the deformation space model was already introduced in literature [7].

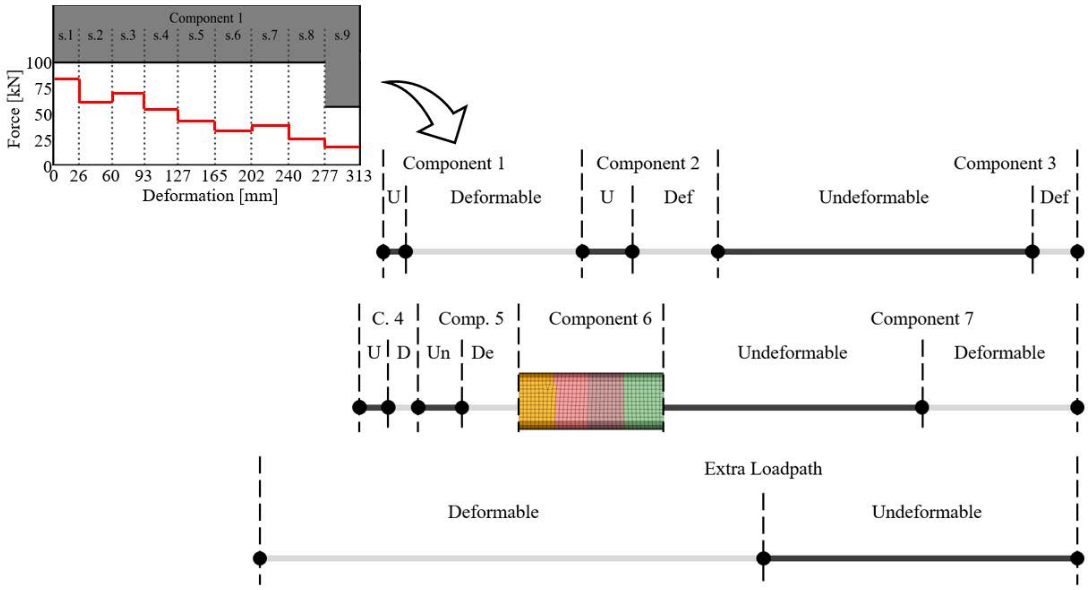



Figure 3: Geometry Space Finite Elements Model and the type of stepped curve it accepts

## 2.3 Uncertainty models

Last but not least, we look at the model used to quantify the uncertainty related to the early design of a component. In [2], the uncertainty model is not very elaborated: all variables linked to the 1D components are considered to be independent, thus a variable also in the uncertainty model. This yields in our case a 27-dimensional space. Each variable can vary between the upper and lower limit of the corridors of the solution space method. Given the non-linearity of crash scenarios, no criteria was set to define a quantity that can help to describe the bounds of the interval obtained when cascading the uncertainty. Therefore, a Latin-Hypercube sampling strategy was used to propagate the uncertainty and assess a variability in the behavior of the component —to characterize the behavior, the force-deformation curve is measured. Although the approach is not efficient, it showed that the GSFEM can be used to propagate uncertainty and it can be used to evaluate a high number of samples in a short amount of time.

To increase the effectiveness of the whole method, the dimensionality of the problem must be reduced. To do so, we can exploit the fact that the sections within one component are not physically independent. In other words, the behavior suggested by the solution space method where each section is independent of the others is a behavior not reflected in a real crash. Within one component, the independence between sections can only be achieved mathematically, but not physically. These interrelations are now used to couple together the variables of the uncertainty model within a component. Methods like those in [11, 12] couple the variables by defining a subspace where they are dependent. The correlation is mathematically expressed iteratively between two variables at a time. The information we use is, however, contained in continuous time-signals. Thus, to use the said methods, we would have to reduce the force-deformation curves to a set of scalar values, making the methods not suitable for us.

We need a methodology that is capable of handling continuous signals. In [13, 14], the uncertainty intervals of space-dependent variables is represented by response surfaces. They expand over the entire area of an FE model and define the upper and lower limit of the variables assigned to a specific region of the FE model. The approach was then modified to handle one dimensional time-based responses [15]. The variability of the signals is represented here by

B-splines. We find this suitable for our application since we are also looking to characterize the variability of signals. Moreover, the use of B-splines allows us a high degree of control over the key-points of the curves. They better relate the curves to the corridors. However, the final uncertainty realization differs from [15] because we consider signals of measured forces in the deformation domain and not in the time domain.

## 3 METHODOLOGY

The usability of the Geometry Space Finite Elements Model (GSFEM) is currently limited by the uncertainty model that was used in [2]. It can have between 22 and 29 independent variables, rendering the model unusable due to its dimensions. Therefore, we propose here a different uncertainty model, where the variables are linked together by the information gained from actual components. We propose here to build a model on the data collected from FE simulated drop-tower tests. The tests are run on newly developed components or on components with similar positioning and function in similar cars. The data collected provides information on how the sections are connected, so that the behavior that is imposed to the 1D elements better resembles that of the actual components. In this section, we also address how the new uncertainty model interacts with the GSFEM. Describing the interaction is necessary because the output of the uncertainty is no longer a stepped curve. Thus, it cannot be the input of the GSFEM. Let us now look at the details.

### 3.1 Creating the uncertainty model

The first step we take to build the new uncertainty model is generating the required data. Because the corridors define a force range in the deformation domain, we build the uncertainty model in the same domain. Therefore, for each component, we want to describe the behavior by measuring the force it absorbs in relation to its deformation. To quantify the limits of the variability, we look at the energy each component dissipates. The dissipated energy ($E$) is linked to the force ($F$) and deformation ($s$) by the same expression introduced in Section 2.1: $E = F \cdot s$. Having defined this energy-based criterion, we can look for the force-deformation signal corresponding to the upper limit and the one corresponding to the lower limit. Each limit can be defined by either the corridors of the solution space method or by manufacturing constraints. Therefore, both constraints must be evaluated. The most conservative must be chosen. In other words, the upper bound ($E^U_{\text{limit}}$) is set to the minimum between the maximum dissipated energy defined by the solution space method ($E^{\max}_{\text{solspace}}$), and the maximum dissipated energy defined by the manufacturability ($E^{\max}_{\text{manufacturability}}$). Meanwhile, the lower bound ($E^L_{limit}$) equals the maximum between the minimum dissipated energy calculated with the solution space method ($E^{\min}_{\text{solspace}}$), and with the manufacturability constraints ($E^{\min}_{\text{manufacturability}}$).

$$\begin{aligned} E^U_{\text{limit}} &= \min\left(E^{\max}_{\text{solspace}}, E^{\max}_{\text{manufacturability}}\right) \\ E^L_{\text{limit}} &= \max\left(E^{\min}_{\text{solspace}}, E^{\min}_{\text{manufacturability}}\right). \end{aligned} \tag{1}$$

The energy dissipated by a component according to the solution space method is directly linked to the force ranges represented by the corridors. The minimum corresponds to the product of the lower limit of the corridors multiplied by the deformation length of a component. Similarly, the maximum corresponds to the product of the upper limits multiplied by the same deformation length. For example, component 4 (see Figs. 1 and 2) has two sections: the bounds of the first section are $F^1_{Lower} = 0$ and $F^1_{Upper} = 65.930$ kN, while those of the second section are $F^2_{Lower} = 0$ and $F^2_{Upper} = 66.015$ kN. Therefore, the minimum energy absorbable is:

$$E_{Minimum} = F^1_{Lower} s^1 + F^2_{Lower} s^2 = 0. \tag{2}$$

Meanwhile, the maximum absorbable energy for component 4 according to the solution space method is:

$$E_{Maximum} = F^1_{Upper} s^1 + F^2_{Upper} s^2 = 65.930 \text{ kN} \cdot 28 \text{ mm} + 66.015 \text{ kN} \cdot 26 \text{ mm} = 3,562 \text{ J}. \tag{3}$$

The FE simulation to measure the force-deformation curve can then be tailored so that the component dissipates the energy defined by the corridors and that it deforms over the total length set in the GSM. Notice that the deformation set in the GSM cannot be met in a FE simulation because of its non-linearities. Therefore, the constraint on the deformation is relaxed: the one measured must be within $\pm 10\%$ of the length fixed in the GSM.

The limits of manufacturability are related to the thickness of the metal sheets: the shell of a component cannot be thinner than $0.7$ mm or thicker than $4.5$ mm. This constraint affects the energy-based criterion, because of the deformation length constraint set in the GSM. Regardless of the geometry, a component must deform over the length defined in the GSM. Supposing a fixed energy a component dissipates the component deforms less than the fixed length when the shell thickness increases. On the contrary, when the shell thickness decreases, the deformation of the component becomes larger. Therefore, at the manufacturability limits, the simulations are tailored so that the component deforms of the pre-set length.

To be able to sample the model, the upper and lower bounds must be mathematically represented by a function. For this, we propose using B-splines. The choice is motivated by the level of control we have over the key-points of the curve. These are strategically placed in the center, at the beginning, and at the end of each section of the solution space model (the green dots in Fig. 4). Once the locations of the key points are fixed, the B-splines representing the bounds are trained on the data measured with the FE simulations (gray curves in Fig. 4). We train a set of parameters for the upper bound, resulting in the blue curve (Fig. 4), and a different one for the lower bound, resulting in the red curve (Fig. 4). We now have described an interval derived from the data measured on the actual components. Notice that the B-spline manages to capture the behavior of the components during a crash. Moreover, they also smooth the measured signal. They filter the noise introduced by the FE simulation. Therefore, we consider this new uncertainty model to successfully link the sections belonging to one component.

### 3.2 Sampling strategy for cascading uncertainty

Lastly, we need to address how the uncertainty model we just described interacts with the GSFEM. The latter accepts only stepped curves; therefore, sampling involves not only generating a B-spline in between the upper and lower bounds, but also transforming it into a stepped curve.

Generating a B-spline in between the upper and lower bound is achieved by changing the B-spline coefficients at the key-points. The new curve is generated using SciPy [16]. As a sampling strategy, a Latin-Hypercube one is used again. We are aware that the technique here used to propagate the uncertainty is incomplete. With the approach taken, we assess only whether or not the uncertainty model yields an interesting outcome. Like in [2], we postpone the formulation of a criterion to define a quantity to demark the bounds of the output. In other words, we focus on the uncertainty model linked to the 1D elements rather than its propagation.

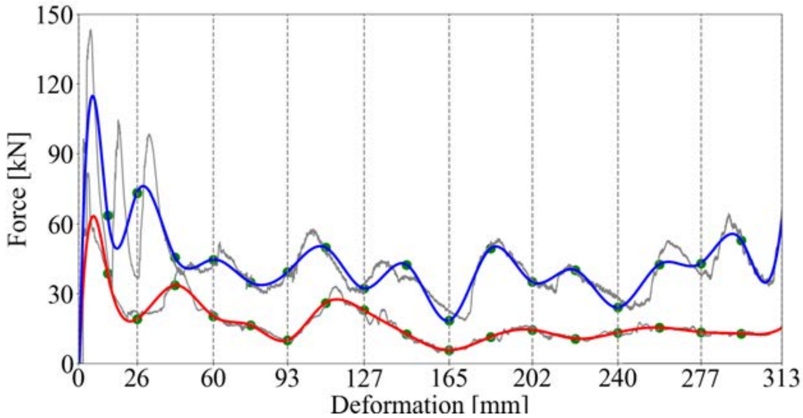



Figure 4: Splines representing the uncertainty model bounds (upper limit in blue, lower limit in red) of one component and the signal they were trained on (in gray)

Each sample is composed of a set of curves —one per 1D component— that need to be given as input to the GSFEM. However, the curves must be in a stepped form. Each step is as long as the corresponding corridor, as explained in Section 2.2. Therefore, the sampled B-spline is first divided into the same sections of the deformation space model. Over each section the curve is approximated to its mean value. An example of what the final curve looks like is shown in Fig. 5.

Let us finally apply the proposed methodology to the example of [2]. Of the 7 components showed in Fig. 2, we fix an early-stage design for component 6 and evaluate how the variation of the other components affect its behavior. We use the GSFEM showed in Fig. 3. The uncertainty model we just proposed has only 6 variables instead of 27: one per component. Each variable defines the position of the B-spline to sample in relation to the bounds. They can vary between 1 and 0, where 1 corresponds to the upper bound and 0 to the lower one. We run 1000 samples, placed according to a Latin-Hypercube strategy. To assess the effect of changing the other component, we measure the force-deformation curve on component 6. We define as positive result if the uncertainty model produces effects similar to those measured in [2].

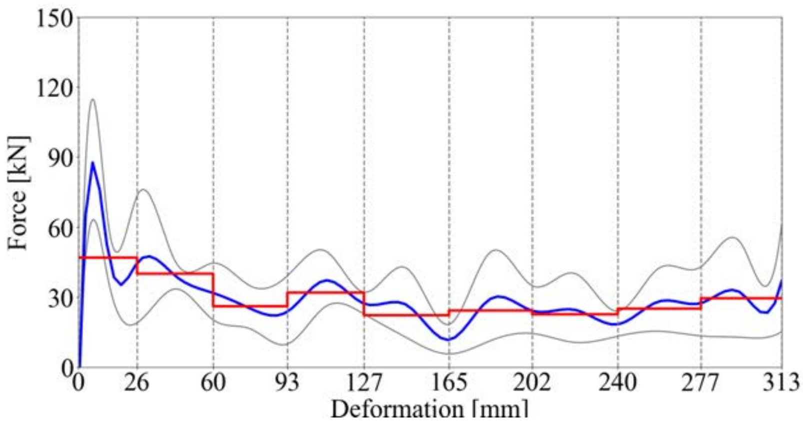


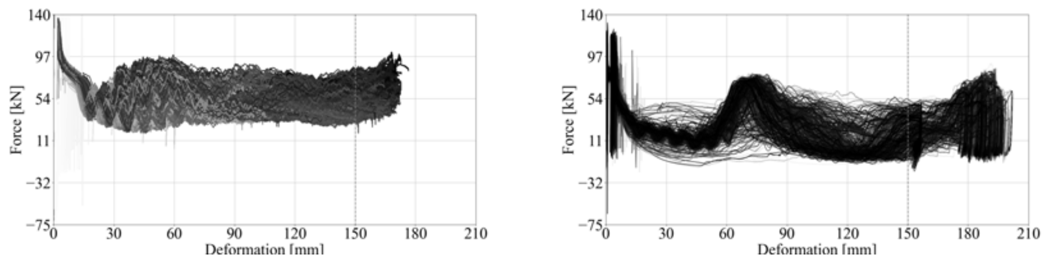

Figure 5: Example of a stepped curve (in red) compatible with GSFEM derived from a sample spline (in blue), in gray the bounds of the uncertainty model of a component

## 4 RESULTS AND DISCUSSION

The 1000 samples yield a positive result: the measured curves are similar to those in [2] as shown in Fig. 6. The two results are not compatible for a proper comparison because the sample space domain is different. While the results in Fig. 6a come from a 27-dimensional sample space, those in Fig. 6b derive from a 6-dimensional space. However, both measure a deformation pattern typical of the component tested. However, the interval in Fig. 6b is smaller because the intervals of the uncertainty model are smaller. The figure also highlights that the new model marks a clear pattern on the performance of component 6. This is one of the advantages of the new uncertainty model. Using real data excludes behaviors that are physically not acceptable. Therefore, the new interval is to be considered more precise. The new model not only shows the effects of the other components on the sixth one, but also tells us that component 6 maintains its general behavior in the final assembly.

On the left side of Fig. 6b one can notice negative spikes. They are caused by the noise introduced by the 1D elements. We did not define any damping factor to reduce the noise of the 1D elements. Imposing a damping factor alters the energy each element absorbs. On the right side, the dashed vertical line represents the target deformation set by the GSM. As the figure shows, this target is not met. However, to get a better understanding of the matter, we can plot the total deformation length of component 6 against the number of samples (Fig. 7). About 54% of the total samples yield a deformation length greater than the one defined in the GSM —the samples marked in red. The remaining samples, marked in blue, fulfil the deformation requirement.

The variations of the other components affect the behavior of a newly designed component. The effect is already exemplified by the force measured on component 6, as already assessed in [2]. The effects can also be seen on the measured deformation length: when the component deforms more than the length set in the GSM, it is dissipating an excessive amount of energy. Further investigation is needed to understand whether it would compromise the crashworthiness or not. However, it is out of the scope of this paper. With the results presented here, we show that the new uncertainty model can yield effects similar to those of the model presented in [2].




(a) (b)

Figure 6: Force-deformation curves measured on component 6 of the GSFEM: a) from the 10,000 samples run in [2]; b) from 1000 different samples run with the new uncertainty model; the vertical gray dashed line marks the deformation target defined in the GSM

## 5 CONCLUSION

In this paper we take the method introduced in [2] and change the uncertainty model to make the entire methodology more usable. We propose a new model to solve the dimensionality

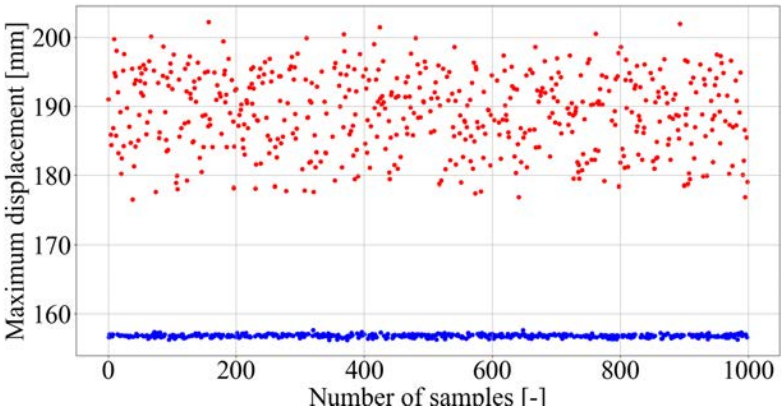



Figure 7: Deformations measured over the 1000 samples: in blue the samples that fulfil the deformation length set in the GSM; in red the samples that do not

problem. It increases the usability of the Geometry Space Finite Elements Model (GSFEM): it provides similar insights to [2] on newly-designed components, with a lower number of simulations. The new model has only 6 dimensions instead of 27. To do this, we used the force signals measured from an already developed component to link the variables belonging to a component. The fact that the results presented in Section 4 are similar to those in [2] shows that the method proposed here is effective. Not only does it successfully reduce the number of variables in the uncertainty model, but it also exploits more realistic information. The GSFEM is effectively rendered more usable. It opens the road to further studies of the interactions between components. In addition, it provides developers a more efficient way of assessing how their design is affected by other components in the early-stage of development.

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