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IMPRECISE PROBABILITIES AS AN ANSWER TO THE INDETERMINACY INHERENT TO MECHANICAL TOLERANCES

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Abstract. *In mechanical design, tolerances are defined to constrain the acceptable deviations of a systems features, and to satisfy a set of quality and functional requirements. This is generally done statistically, by estimating probabilities of failure. In regard to the approaches developed for the modeling of mechanical assemblies and the standardized ways of representing tolerances, there is a lack in approaches to simulate and estimate the full scope of effects of a given tolerance choice. This is due to the fact that tolerances are used to define the acceptable geometric space for the defects, but leave an indeterminacy concerning their distribution and nature. In practice, designers make assumptions, which can lead to either over-, or under- estimate the probability of failure. In this paper, work has been done to put in evidence the effect of the ambiguity in the graphical language of tolerances on the results obtained in tolerance analysis, and proposes the usage of the imprecise probability framework to complement these studies and obtain more realistic results.*

Keywords: Imprecise Probabilities, Statistical Tolerance Analysis, Monte-Carlo Simulation, Deviation Domain, Failure Probabilities

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1 INTRODUCTION

Every manufacturing process involves uncertainties, affecting the product which will present defects and deviations compared to the design. Being unavoidable, designers indicate the maximal authorized deviation of its geometric elements to comply to a set of functional and quality requirements. In an industrial setting where reliability and repeatability is key, it is necessary to estimate the rate of failures per million of parts made, as this parameter is often indicated as a process requirement [14].

A variety of tolerance types exist, all constraining the geometrical features of a solid mechanical part (points / lines / surfaces) differently, and defining a geometrical hypothetical space in which the defect is allowed to exist. This information is conveyed from the design stage to the manufacturing entities, graphically on the design plans, using the standardized Geometrical Product Specification (GPS) system [13]. The tolerances specifications have been developed to maintain a coherence between their definition, the metrological considerations and the manufacturing methods.

Considerable research efforts have been put into the simulation of over-constrained mechanisms with geometrical defects and gaps [6,10], on the development of spatial mathematical models representing the deviation space [2, 17], and on statistical methods to either determine the ideal tolerance values [11], or to assess the effect of a specific tolerance choice on the metric of interest [9]. All these considerations are part of the more general field of tolerance analysis.

Tolerance analysis addresses various issues :

- The construction of a mathematical model of a mechanical system with defects. This model can mostly be seen as an indicator function, that specifies if for a set of defects the system complies to the requirements.
- The choice of a probabilistic model for the deviations, simulating the random defects that will be expected from the manufacturing process. Usually, assumptions concerning the distribution of the defects within the tolerance space are made, for instance assuming some quality level of the machine, centered defects etc.
- The estimation of the effect of these defects on the quantity of interest - i.e. the rate of defect assemblies, usually in the form of a Monte-Carlo simulation, or more refined failure probability estimation methods [3]

While significant progress has been made in addressing those, limitations remain regarding the simulation of the defects. Tolerances only define the localization of the defects, but provide no information on distribution and nature of the defects within this space. This is even more problematic when the modeled geometric element (also called feature) presents multiple degrees of freedom and is modeled using a combination of elementary displacements [19]. In the context of tolerance analysis and defect simulation, a degree of freedom of a geometric element represents the different ways this element can deviate from the nominal geometry, for example with a distance or angular defect.

When statistically simulating those defects, as each degree of freedom is driven by a separate random variable, there is no unique way of choosing the right distributions for them. This epistemic indeterminacy in the link between the tolerances and the probabilistic distributions is usually blurred in the simulation through arbitrary allocations, even if some approaches try to include some sources of epistemic uncertainty [5] .

This allocation problem is depicted in Fig. 1, where for a center-line having one localization tolerance, the line admits two degrees of freedom, one translation and one rotation. One could assume the defects to be of either type or a combination of both, but the tolerance itself gives no indication on its nature.

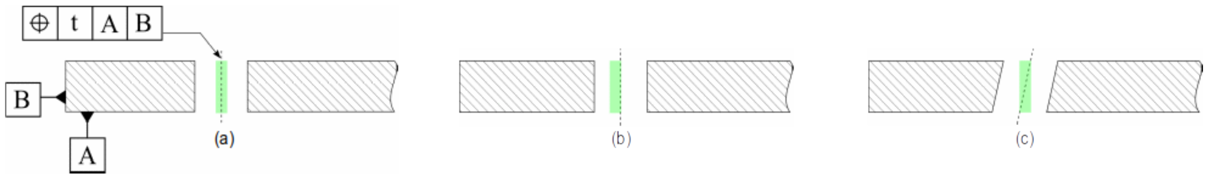

Figure 1: Simplified 2D Computer aided design (CAD) model with a tolerance applied on the localization of a center-line of a drill hole (a). This feature presents two degrees of freedom, as it can be translated (b) or rotated (c) while still respecting the valid tolerance zone (in green).

There is usually no hindsight concerning the distribution of defects within the tolerance space, and the designers usually give indications on the capability of the process, by constraining the probability of the defect to be outside of the acceptable localization, but not by constraining the nature of the defect itself. An usual approach consists in distributing the defects equally between the degrees of freedom, disregarding other possible cases. This choice can in turn have an effect on the results of the simulations, and to a potential over- or under-estimation of the rates of failure.

To deal with problems involving epistemic uncertainties, methods based on imprecise probabilities have gained in popularity [4,8], with numerous approaches based on interval theory [1], P-Boxes [7], fuzzy set theory [15] and active Bayesian approaches [16]. These approaches allow to include the lack of knowledge on the model into the analysis, so that this imprecision is propagated to the results, to encompass the full range of possible outcomes, without arbitrariness.

This paper explores the use of imprecise probabilities in mechanical tolerance analysis problems, when no other information than the tolerance and the expected machine capability is at hand. By doing so, a more realistic understanding of the effects of the tolerance on the system can be obtained, and the analysis can have meaning sooner in the design stage, without needing knowledge about the potential manufacturing process and the defects it will generate. A case study will be presented to show the variability induced by the imprecision in the probabilistic model, and to discuss the potential benefits of imprecise probabilities in tolerance analysis and mechanical design.

2 USUAL APPROACH AND LIMITATIONS

In this section, the different steps of tolerance analysis will be presented, by giving first an indication on the framework and the hypothesis in use, and then by showing the limitations in the probabilistic modelling of geometric defects.

2.1 Framework for tolerance analysis, and hypothesis

Framework Designers indicate the tolerance type, the value, and the quality level to expect from the manufacturing process. No indication is given regarding the expected distribution of the defect, as it is case dependent, time dependent, cannot be controlled easily, influenced by numerous uncertainty sources and can only be known a posteriori.

In face of all different modelling methods of mechanical systems developed, they will not be detailed here. Extensive work has been done in that field [3, 6, 12], and the model can be regarded as a function taking as an input a set of defects and returning if the system complies to a set of requirements. In Equation (1), M is a model of an assembly, X a vector of defects, and Y either an indicator function (returning 0 or 1) or the output of interest of the model as a scalar. The vector X can be re-expressed to show the different features and degrees of freedom. Let $i \in [1, N_f]$ be the index of the feature, and N_f the number of features, and $j \in [1, d_i]$ the index of the degree of freedom of the feature i. Each $X_{i,j}$ represents a variable controlling one unique degree of freedom.

$$Y = M(X) = M(X_{1,1}, \ldots, X_{1,d_1}, \ \ldots, \quad X_{i,1}, \ldots, X_{i,d_i} \ \ldots, \quad X_{N_f,1}, \ldots, X_{N_f,d_{N_f}}) \quad (1)$$

Focus will be put on the link between the mechanical tolerances definition, and the construction of the probabilistic model for simulating the defects, using a simple two dimensional example.

Hypothesis In most tolerance analysis approaches, form defects are neglected, and only position and orientation defects are modeled, implying that only the contour points of the features are considered as contact points [6]. Additionally, the effect of rotations is considered using the small displacement hypothesis [18], which enables the effect of rotations to be modeled as a first order Taylor expansion, where $\sin(\theta) \simeq \theta$ and $\tan(\theta) \simeq \theta$. The number of degrees of freedom (DOF) of a toleranced feature depends on the nature of the feature, and at most it has 6 degrees of freedom: 3 translations and 3 rotations.

2.2 Probabilistic modeling of defects, mathematical representation and limitations

Choosing a probabilistic distribution. When using the probabilistic approach, each DOF of a feature is driven by a different random variable. For sake of simplicity, all distributions are considered normal and centered on 0. This type of approximation is often made in tolerancing problems and can be justified using the central limit theorem as it is usually in the context of mass production. In this approach, the standard deviation of the normal distribution is chosen based on the Tolerance Interval (TI) and process capability (C_m) to ensure that only a small percentage of parts fall outside of the specified tolerance [10].

$$\sigma = \frac{t}{6 \times C_m} \tag{2}$$

As an example, in the one dimensional case, where a line has a tolerance interval of value t on its length T then it's real length X is expressed simply as expressed in Equation (3)

$$X = T \pm \frac{t}{2} \tag{3}$$

With a process capability set to $C_m = 1$, using this approach, the standard deviation of the normal distribution modelling the defect is expressed in Equation (4)

$$\sigma = t/6 \tag{4}$$

In that case, less then 0.27% of parts will be out of tolerance. The process capability is usually estimated on production lines as a quality metric [9], but it can also be used to simulate less capable processes, or to accentuate the effects presented here, see Equation (2).

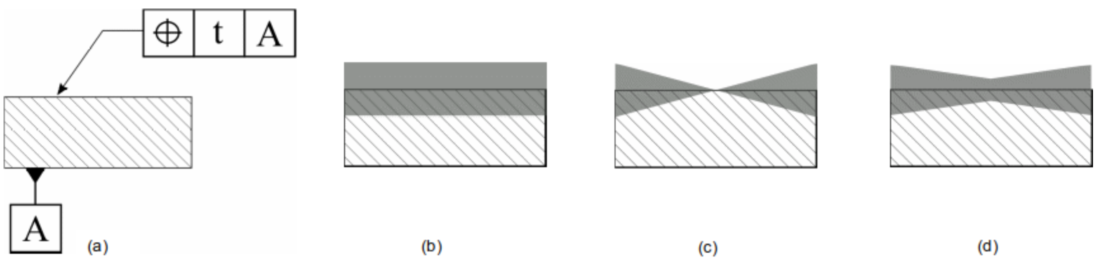

Figure 2: 2D CAD model with a location tolerance applied on the top surface (a). The gray area represent the zone in which the tolerance has to exist (b) ; different ways of filling this space are possible, as with only orientation defects (c) ; or as a combination of both position and orientation defects

Deviation domains Depicted in Fig. 2, a rectangular part has a localization tolerance applied on its upper surface. This tolerance constrains the acceptable deviation of the upper surface relative to its nominal position. In this example, there are only two degrees of freedom, one translation and rotation. For the part to comply with the tolerance requirements, it is sufficient to observe if the two upper corner points are within the tolerance zone, as shown in gray in Fig. 2 (b). This idea is simply expressed in Equation (5), where the combined effect of rotation and rotation has to be within the tolerance interval. With the small displacement hypothesis, the effect of the rotation is approximated with a first order expression, keeping equations linear.

$$-\frac{t}{2} \leq u \pm \frac{l}{2}\theta \leq \frac{t}{2} \tag{5}$$

In 2D, all possible defect configurations allowed by the tolerance can be represented. This can be done using a deviation domain [2], which represents the hypothetical space of the elementary defects, as shown in Fig. 3. The extreme values for the position and rotation defects are chosen considering each degree of freedom individually, so that at that value the deviated surface is at the exact limit of the tolerance space.

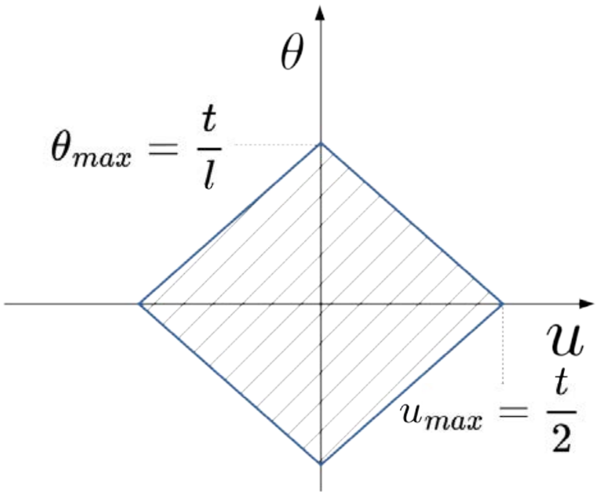

Figure 3: Deviation domain of the toleranced surface Fig. 2. The hatched area represents the acceptable domain for the defects, and this space is called $\mathcal{D}$.

Probabilistic modelling of defects From the definition of the deviation domain and the quality requirements, the approach can be generalized, by constraining the total variance of the defects, as expressed in Equation (4). Then, for each degree of freedom taken individually, its standard deviation for the random variable modelling can be expressed using the extreme values found Fig. 3.

$$\text{Var}\left(u \pm \frac{l}{2}\right) = \left(\frac{t}{6 \cdot C_m}\right)^2 \tag{6}$$

$$\sigma_u = \frac{t}{6 \cdot C_m} \ , \quad \sigma_\theta = \frac{t}{l \cdot 6 \cdot C_m} \tag{7}$$

As the effects originating from the rotation and translation are both linear, the total defect can be modeled as their linear combination. A linear combination of identical independent normal laws is a normal law, its variance is equal to the original one, as shown below.

Let $X_1, X_2, ..., X_n$ be independent random variables with a standard normal distribution, i.e. $X_i \sim N(0,1)$ for $i = 1, 2, ..., n$. Let $\lambda_1, \lambda_2, ..., \lambda_n$ be positive real numbers such that $\sum_{i=1}^{n} \lambda_i = 1$, and define the random variable Z as a linear combination of $X_1, X_2, ..., X_n$, as expressed in Equation (8)

$$Z = \sum_{i=1}^{n} \lambda_i X_i \tag{8}$$

It is demonstrated here that the standard deviation of Z is not greater than the standard deviation of each X_i, in Equation (9)

$$Var(Z) = Var\left(\sum_{i=1}^{n} \lambda_i X_i\right) = \sum_{i=1}^{n} \lambda_i^2 Var(X_i) = \sum_{i=1}^{n} \lambda_i^2 \leq \sum_{i=1}^{n} \lambda_i = 1 \tag{9}$$

where the last inequality follows from the fact that λ_i are positive and sum up to 1. Therefore, in Equation (10)

$$\sigma_Z = \sqrt{Var(Z)} \leq \sqrt{1} = 1 \tag{10}$$

which means that the standard deviation of Z is not greater than that of the X_i, and that the probability of being out of the deviation domain $\mathcal{D}$ does not increase, justifying this statistical modeling of defects.

In the usual case, designers arbitrarily choose an allocation for the defects based on their knowledge, uniformly allocate the defects between the DOFs, or use a set of basic combinations as for example $\lambda \in [0.0, 0.5, 1.0]$ in this case.

The objective of tolerance analysis is to evaluate the effects of the tolerance choice on the system's behavior and performance, and to identify the critical tolerances that need to be controlled to meet the system's functional and quality requirements. The traditional statistical tolerance analysis approach supposes that for a given tolerance, the deviations follow a specific distribution within the deviation domain. However in practice, the distribution of the deviations is an epistemic unknown, and making arbitrary choices concerning them can affect the accuracy or validity of the tolerance analysis.

In the next section, we will discuss how the framework of imprecise probabilities can serve to reason about the indeterminacy in the allocation of the defects, and how this approach can generate more accurate tolerance analysis results, based on the linear combination of defects presented here.

3 MODELLING THE INDETEMINACY IN MECHANICAL TOLERANCES WITH IMPRECISE PROBABILITIES

Imprecise probabilities offer tools that deal with problems where the probabilities cannot be known with certainty, and that reason on the effects this imprecision has on the results of those problems. The aim is to include the lack of information into the study, so that the results are as precise as can be, without guessing.

Various methods as interval propagation, P-Boxes or fuzzy probabilities deal with these imprecisions, and try to give bounds for the output rather then a crisp value. In tolerance analysis where a coefficient is imprecise, in the sense that it can take any value without one being more favorable then another, imprecise probabilities give the right scope to deal with it. This imprecise coefficient represents the fact that the manufacturing process does not yet exist, and that any process respecting the localization and the capability is valid. We can re-express Equation (1) and introduce the imprecision in the form of λ coefficients, as expressed in Equation (11)

$$Y = M(X) = M(\lambda_{1,1} \cdot X_{1,1}, \ldots, \lambda_{1,d_1} \cdot X_{1,d_1}, \ldots, \lambda_{N_f,1} \cdot X_{N_f,1}, \ldots, \lambda_{N_f,d_{N_f}} \cdot X_{N_f,d_{N_f}})$$
$$0 \leq \lambda_{i,j} \leq 1, \quad \sum_{j=1}^{d_i} \lambda_{i,j} = 1 \tag{11}$$

It is observed in the statement above, that the total defect is the linear combination of elementary ones. By adding this imprecision a new epistemic dimension is also added. In tolerancing, for a feature of dimension $d \in [1;6]$, the inclusion of this epistemic uncertainty adds $d-1$ new dimensions, that have to be treated separately.

A naive but proven approach is the double loop, using two designs of experiments (DOE), with the first loop fixing the values of the epistemic uncertainty, and the second sampling over the stochastic uncertainty to estimate a probability of failure or the distribution of the output. The goal of this approach is to give the upper and lower value for the output of the model, as it is sure that the response of the system has to be in these bounds as long as quality requirements (process capabilities) are met.

4 APPLICATIVE EXAMPLE

In this section, the proposed methodology will be applied on a minimal but sufficient example, exposing the potential benefits of using imprecise probabilities in tolerancing problems, as well as showing the influence of this imprecision on the tolerance choices.

The assembly is composed of a rectangular pocket and a rectangular part, the symmetries allowing for two dimensional modeling. The rectangular part is slightly shorter then the pocket, to maintain a functional gap. The assembly condition is for this gap to be positive. Tolerances are only defined for the functional distances having an effect on the gap and therefore on the functionality of the assembly.

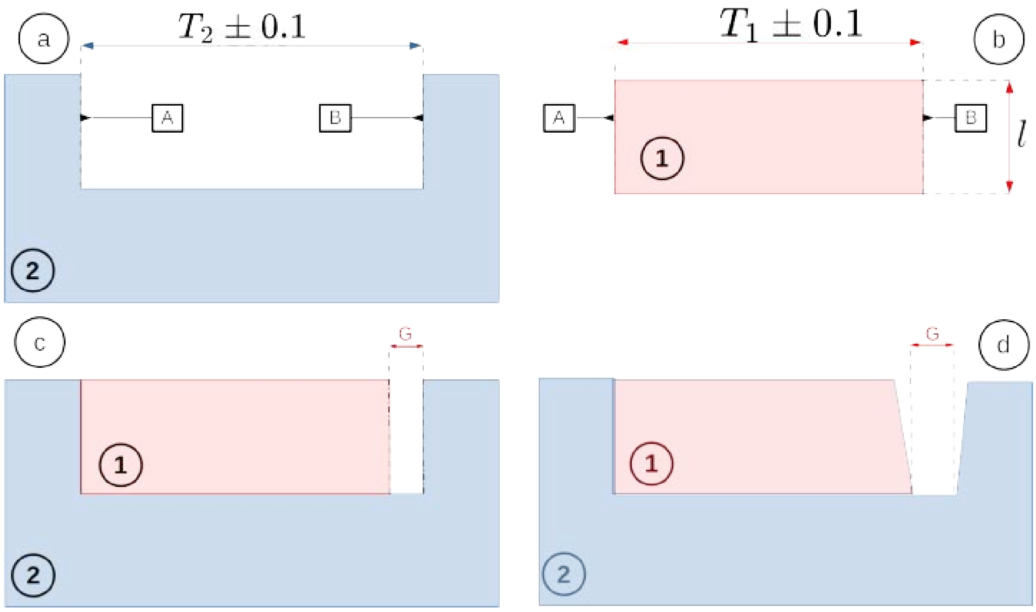

Figure 4: Schematic view of the pocket (a), the rectangular part (b), the nominal assembly (c) , and the assembly with defects (d). The gap is the minimal distance between the parts.

The tolerances are defined for the distance between the surface A and B, A being the datum reference frame. This frame is supposed perfect for sake of simplicity, and contact is always guarantied between the two datum frames, surface $1A$ and $2A$. Defects will only be modeled for the surfaces B. Tolerances have been allocated equally between the parts and chosen using the worst case approach, so that defects within the tolerance zone always ensure mountability. The values for the nominal gap and the tolerance are as followed, $G_n = 0.2mm$ and $t = \frac{G_n}{2} = 0.1mm$

Using the same parameterization than in Section 2.2, we construct the mathematical model of the assembly with defects and express the gap.
The displacement of the upper and lower corner X_i of part 1 is given by Equation (12). Where u_i and θ_i denote the defects associated with translation and rotation respectively.

$$\begin{aligned} \Delta X_{i,Q} &= u_i \pm \frac{l}{2}\theta_i \ , \quad Q \in [upper, lower] \ , \quad i \in [1,2] \\ X_{i,Q} &= T_i + \Delta X_{i,Q} = T_i + u_i \pm \frac{l}{2}\theta_i \end{aligned} \tag{12}$$

For part 2 the equations are analogous, except that the normal of the surface B is opposed to that of part 1. From there we write the expression of the lower and upper gap. The final gap of the part is the minimum of the two as seen in Equation (13).

$$G_Q = \tilde{X}_{2,Q} - \tilde{X}_{1,Q} \quad Q \in [upper, lower]$$
$$G_{tot} = \min(G_{upper}, G_{lower}) \tag{13}$$

Once the mathematical model of the assembly constructed, the model for the deviations has to be chosen. As detailed in Section 2.2, we employ a statistical method allowing for a few percentage of parts out of tolerance, using the quality metric and the tolerance interval. To make the effect of the allocation of defects more visible, a process capability is chosen such that 5% of parts are out of tolerance.

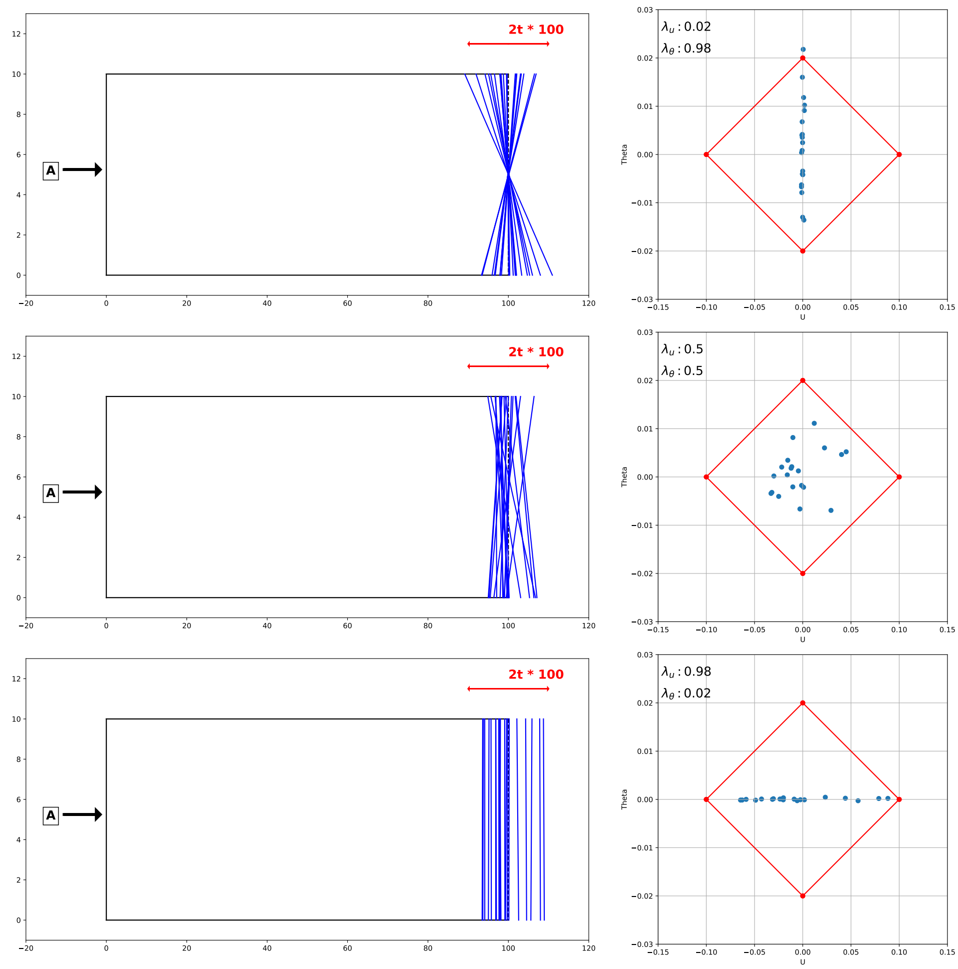

Figure 5: Different allocations of the defect, on the left it's the geometric effect on the part, on the right the it shows the elementary defects plotted in the deviation domain. Top : $\lambda_u = 1, \lambda_\theta = 0$, middle : $\lambda_u = 0, \lambda_\theta = 1$, bottom : $\lambda_u = 0.5, \lambda_\theta = 0.5$

We can define the following probability distributions for each DOF of the feature in Equation (14), where u_i and θ_i are realizations of the following distributions.

$$
\begin{aligned}
U_i &\sim N(0, \sigma_{u,i}),\ i \in [1,2] \\
\Theta_i &\sim N(0, \sigma_{\theta,i}),\ i \in [1,2]
\end{aligned}
\tag{14}
$$

As expressed in Equations (12)-(13), the total defect for each part is a linear combination of elementary defects and the final gap the minimal distance between the upper and lower parts. We can express the total distance for the upper (analogically for the lower) part as follows, with the additional introduction of the imprecise λ_i parameters, as expressed in Equation (15).

$$
\begin{aligned}
G_{upper} &= T_2 - \lambda_2 \cdot u_2 - \frac{l}{2}(1-\lambda_2)\cdot\theta_2 - T_1 - \lambda_1 \cdot u_1 + \frac{l}{2}(1-\lambda_1)\cdot\theta_1 \\
0 &\leq \lambda_{i,j} \leq 1, \quad \sum_{j=1}^{d_i} \lambda_{i,j} = 1
\end{aligned}
\tag{15}
$$

The λ_k represent the lack of knowledge on the probabilistic model, or the indeterminacy arising when trying to construct a probabilistic model from the tolerances. Depicted in Fig. 5, is a visual representation of the link between different allocations of defects, and the actual effect on the geometry of the model. For each allocation, the probability to be out of tolerance is constant, but the geometric space is filled in a different manner. For a part observed individually, this has no impact on its conformity, and each would pass metrological inspection similarly, the real impact is seen once in the context of the assembly of multiple parts.

When taking into account the imprecision in the defect allocation, it is not possible to find a crisp value for the probability of failure anymore, or to express the response of the system with a unique probabilistic model. Rather, only the upper and lower envelope of the response can be determined, in this case the gap between the two parts.

As the model is extremely cheap to evaluate, a naive double loop is used, with two separate design of experiments, one for the dimension of the epistemic uncertainty - i.e the allocation of defects, and one for the stochastic irreducible uncertainty corresponding to the natural variability in the fabrication process. The approach is to iterate over each point in the epistemic uncertainty dimension, to fix the λ values, and then to propagate the uncertainties as in the usual case as to estimate the probability function of the output for this fixed set of lambdas. By doing this over the whole space of λ the full scope of epistemic uncertainty in the response of the model can be obtained, in the form of a P-Box, where only the upper and lower envelope of the response is of interest, as seen in Fig. 6. Knowing this envelope we can also bound the probability of failure, as seen in Equation (16)

$$
0.01\% \leq p_f \leq 1.89\%
\tag{16}
$$

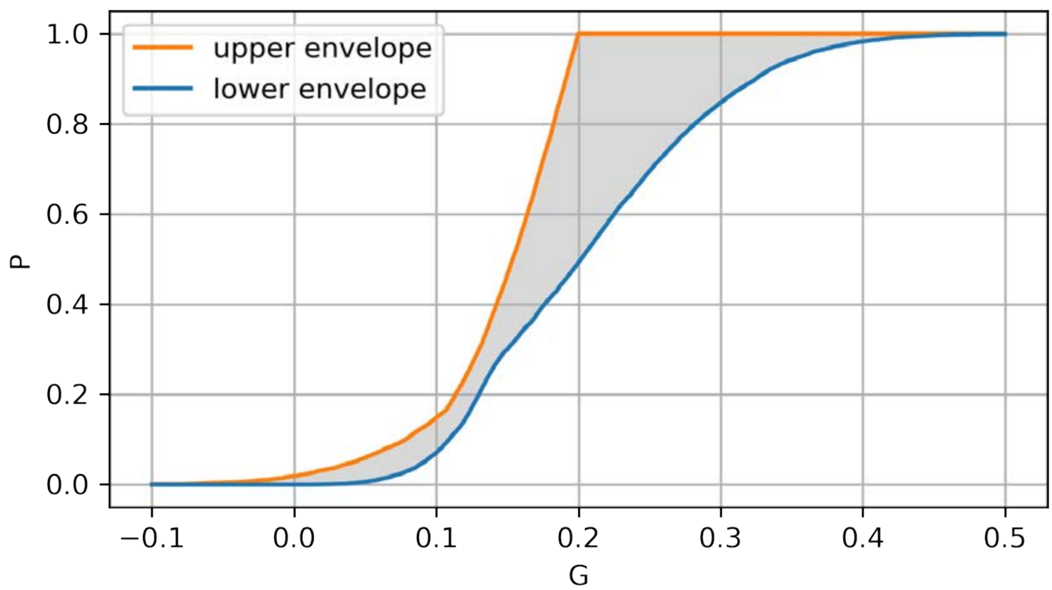

Figure 6: Probability Box obtained using a double loop approach over both the dimension of the imprecision and the space of defects, giving the probability of having a certain output gap.

5 CONCLUSIONS

This preliminary work on the use of imprecise probabilities in tolerancing problems shows that it is necessary to account for the ambiguity in the graphical tolerancing language, as even with this minimal example, the upper probability of failure is 200 times larger then the lower. Making the wrong modeling choices can lead to grossly under estimate this probability, and with this approach the full extent of the imprecision allowed by the tolerance choice can be estimated. As this ambiguity is a source of epistemic uncertainty, imprecise probabilities define a coherent framework to deal with them.

It should be noted that the allocation of defects is not the only potential source of epistemic uncertainty. Correlation between the degrees of freedom of a same feature is valid in this scope, and even non centered defects can be added, without ever leaving the tolerance interval and process capability requirements.

Limitations still subsist in the simulation of these results, as double loops are highly inefficient and not applicable to more complex systems as over-constrained mechanisms. In further work, the addition of new sources of epistemic uncertainty as well as the application of more efficient methods to determine the upper and lower bounds of failure will be explored. Additionally, in the context of tolerancing, it would be interesting to know which degree of freedom and defect has the worst impact on the probability of failure of the system, to guide the designers choices into defining the appropriate tolerances that would reduce the epistemic uncertainty in the probability of failure of the system.

Overall, this approach could help make more precise choices for the value and type of tolerances, thus leading to more robust design choices.

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