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INVESTIGATION OF A VALIDATION APPROACH CONSIDERING UNCERTAINTIES FOR COMPONENTS IN AUTOMOTIVE CRASH SIMULATION MODELS

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Abstract. *In the design of safe cars for crash, critical components should deform or fail in a certain manner. Uncertainties can influence the components, such they do not show the desired behavior. The design of such components can be improved via credible simulation models that are able to predict the effect of uncertainties. Our goal in this publication is to undertake the validation of crash simulation models in presence of uncertainties. Investigation of validation approaches for this application is challenging because crash experimental data are limited and inherently noisy. We use synthetic experimental results to address these challenges. We apply V&V 20 standard for the validation of a crash box model. Ultimately, we evaluate the validation results in terms of correctness and conservativeness; additionally, we examine some of the underlying assumptions in the validation approach.*

Keywords: Automotive crash, critical components, crash simulation, validation, uncertainties.

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1 INTRODUCTION

Traffic authorities design various safety standards, which vehicles should fulfill, to protect occupants and other persons involved in a crash event. Crashworthiness is the ability of a car to dissipate the impact energy of a crash and minimize the injury risk of the involved persons. It is evaluated by crash simulation models as well as physical tests. Critical components of a vehicle in crash, such as crash box, dissipate the maximum possible impact energy via plastic deformation. Uncertainties can affect the performance of the components such that they do not meet their design goals. Hence, it is essential to study the effect of uncertainties on the performance of the critical components during the design phase.

Crash simulation models are usually used during the design phase of vehicles, while crash tests are performed at the end of the design phase and shortly before series production. Uncertainties can cause variation in the results of crash tests. The credibility of simulation models to predict the effects of uncertainties can support engineers in developing countermeasures to prevent undesired behaviors during the design phase.

In literature related to the validation of impact and crash simulation models, the primary focus lies on understanding the physical phenomena involved in impact and consequently accounting for these phenomena in simulation models. Validation cases available in the literature are usually deterministic and involve simple geometries and conditions (see e.g. [1, 2, 3]). Eichmueller et al. [4] showed in a study that state-of-the-art simulation models are able to capture experimental results and their scatter if relevant details and uncertainties are considered in the model. More investigation is necessary for methods to consider uncertainties in the validation of crash models. Two challenges in the validation of crash simulation models are that experimental results are limited, due to the high cost and complexity of crash tests, and noisy, due to various sources of error and uncertainty which can be amplified by the dynamic nature of the problem.

Synthetic experimental results are beneficial for the investigation of validation methodologies. Stripling et al. [5] introduced the Method of Manufactured Universes (in short MMU). The goal of MMU is to validate a low-fidelity simulation model via synthetic experimental results that are generated from a high-fidelity simulation model. Here, the focus does not lie on the validation of the simulation model but rather on the validation method itself, and on the evaluation of assumptions of the validation approach. Whiting et al. [6] used MMU to compare different validation approaches for CFD simulations. MMU is also beneficial in the crash application field for coping with the noted challenges in the investigation of validation approaches.

Our final goal in this paper is to investigate an approach to consider uncertainties in the validation of the simulation model of components. We employ the V&V 20-2009 validation approach introduced by ASME [7]. Section 2 discusses the validation approach and generation of synthetic experimental results for the validation of a crash box model. We investigate two different implementations of V&V 20. Section 3 contains a thorough explanation of the validation procedure, validation results, and discussion of the results.

2 METHODOLOGY

This section contains an introduction to V&V 20 validation approach. We investigate the suitability of the validation approach for the crash application field using the MMU approach. This section also discusses how we employed the MMU in our investigation.

2.1 V&V 20 validation approach

The scope of ASME V&V 20-2009 [7] validation standard is to quantify the modeling error of a simulation model for a scalar System Response Quantity (in short SRQ) at a specific validation point. V&V 20 does not cover functional SRQs, e.g. curves, or prediction at new application points for which no experimental results are available. The approach considers uncertainties in experimental results, inputs of the simulation model, and numerical uncertainty. In the following, we review some of the main concepts and techniques of V&V 20.

A simulation model has various input parameters as variables. The behavior of the system can change based on values of the input variables, e.g. elastic or plastic deformation regions in solid mechanics. At a validation point, experiments and the simulation model have the same nominal input variables. Depending on the application, different validation points might be necessary to examine different system behaviors. Additionally, some inputs might be uncertain due to uncontrollable conditions of experiments or a lack of knowledge of the correct value of a parameter. At a given validation point, comparison Error, E, is the discrepancy between Simulation results, S, and experimental Data, D:

$$E = S - D. \tag{1}$$

The True value, T, of the considered SRQ is different from the experimental and simulation results because both results contain errors. Thus, the approach defines δ_S and δ_D, error in S and D respectively, as:

$$\begin{aligned} \delta_D &= D - T; \\ \delta_S &= S - T. \end{aligned} \tag{2}$$

The true value in nature in the equation above is unknown. Three sources of errors contribute to δ_S: a) modeling error, δ_{model}, resulting from simplified assumptions in the mathematical formulation of the physical phenomena, or simplifications in the simulation model itself, e.g. geometry, boundary conditions, material modeling, etc., b) numerical error, δ_{num}, originating from the fact that the numerical solution of the mathematical model is not exact, and c) parameter/input error, δ_{input}, caused by uncertainties in the input parameters of the simulation model. Simulation error, δ_s, is defined as a summation of the three error sources:

$$\delta_s = \delta_{model} + \delta_{num} + \delta_{input}. \tag{3}$$

The final goal of the V&V 20 is to estimate the modeling error, δ_{model}, within an uncertainty range. Restructuring Equations (1), (2), and (3) results in an expression to define δ_{model} as:

$$\delta_{model} = E - (\delta_{num} + \delta_{input} - \delta_D). \tag{4}$$

In Equation (4), the value of E can be defined once S and D are known. The exact values of the three error terms, in contrast, cannot be defined. Thus V&V 20 estimates the error terms via their standard uncertainties u_{num}, u_{input}, and u_D. Standard uncertainty is an estimation of the standard deviation of the parent distribution of each error term. Similarly, u_{val}, validation uncertainty, is an estimation of the standard deviation of the parent distribution of combined error sources on the right side of Equation (4). Quantification of validation uncertainty is the core of V&V 20. Knowing u_{val} and E, the interval of modeling error is:

$$E - u_{val} \leq \delta_{model} \leq E + u_{val}. \tag{5}$$

The validation uncertainty equals to the squared sum of individual standard uncertainties if the error sources are independent:

$$u_{val}^2 = u_{num}^2 + u_{input}^2 + u_D^2. \tag{6}$$

If common sources impact both input and experimental uncertainties, Equation (6) is not applicable. In such cases, it is necessary to eliminate the double contribution of the shared sources to the validation uncertainty.

V&V 20 standard [7] introduces different techniques for quantification of numerical, input, and experimental uncertainties. Numerical uncertainty is defined via the Grid Convergence Index (in short GCI). GCI quantifies the effect of mesh size on the numerical solution of the simulation model. For quantification of input uncertainty, the sensitivity coefficients method or Monte Carlo approach can be utilized. The sensitivity coefficients method is able to capture the linear effect of input uncertain parameters on output quantities. The Monte Carlo approach, on the other hand, captures also the nonlinear effect of parameters at the cost of a higher number of simulations. Methods such as Latin Hypercube Sampling (in short LHS) reduce the number of necessary simulations.

The estimation of experimental uncertainty depends on the measured experimental value, D, and the uncertainties that affect it. Is the experimental value measured directly, or is it defined via a set of measured variables and a data reduction equation? Do the uncertainties in experiments include only random error or systematic sources of error are also present in the experiments? Are the measured variables in the experiment independent, or do some variables correlate with each other? Answers to these questions define the method for quantification of the experimental uncertainty. If the experimental value is measured directly and only random uncertainties affect the experimental results, the experimental uncertainty is equal to the standard deviation of the experimental results.

The Monte Carlo approach can also be directly used for joint evaluation of the input and experimental uncertainties. If the experimental value, D, is measured directly, then we can write the comparison error in Equation (1) as:

$$E = S(x_1, x_2, .., x_n) - D. \tag{7}$$

Here, the comparison error is a function of uncertain input parameters, defined by x_i, and D. First, a probability distribution should be estimated for D. Usually, a distribution function is assumed for D due to limited experimental results. The approach also requires probability distributions of input uncertain parameters. Next, n number of samples are drawn in the joint parameter and experimental value space using LHS. For each sample i, the comparison error, E_i is evaluated. Mean and standard deviation of E define comparison error and the squared sum of input and experimental uncertainties:

$$\begin{aligned} E &= \tfrac{1}{n} \textstyle\sum_{i=1}^{n} E_i; \\ u_{input}^2 + u_D^2 &= \tfrac{1}{n-1} \textstyle\sum_{i=1}^{n} (E_i - E)^2. \end{aligned} \tag{8}$$

An alternative approach for evaluation of the validation uncertainty is to quantify u_{num}, u_{input}, and u_D separately. Consequently, the validation uncertainty can be defined via Equation (6), [8].

2.2 Manufactured universe and approximate model

The fundamental idea of MMU is to define a manufactured universe from which synthetic experimental results can be obtained. The manufactured universe is usually a high-fidelity simulation model. A simulation model, which is a low-fidelity model, is validated with synthetic experimental data and a validation method. The main focus does not lie on validation but rather on the assessment of the validation method.

We use synthetic experimental results to investigate the V&V 20 approach. The conclusions of our investigation should be applicable to real-world problems. Therefore, the high-fidelity model should capture the main phenomena present in crash events. Generally, uncertainties in crashworthiness fall into one of the following categories [9]:

1. Operational: Uncertainties in test conditions are in this category, such as impact velocity, impact angle, etc.

2. Manufacturing: Inherent variability in manufacturing causes uncertainties in the geometry and material of the manufactured components, such as imperfections in geometry, local variations in wall thickness, or material properties.

3. Modeling: Simplified assumptions in physics or simplifications in the simulation model.

We define the manufactured reality and approximate model such that the models include at least one uncertainty from each category.

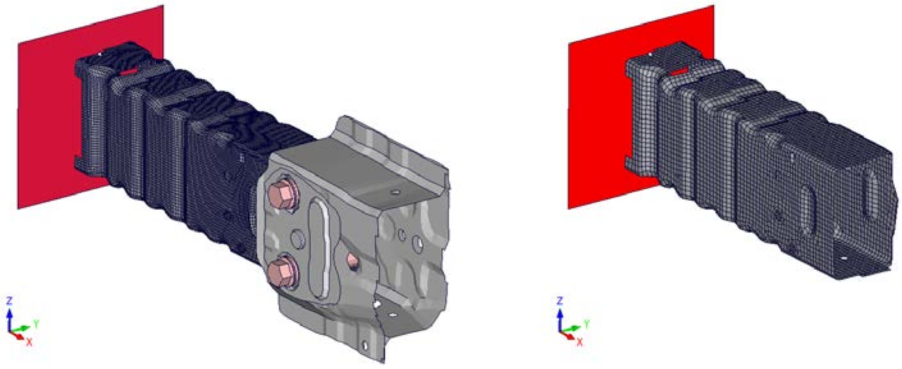
Figure 1 illustrates the manufactured reality and approximate model. The manufactured reality is a high-fidelity model of a crash box. It is fixed to a test rig and is impacted with a rigid wall with a defined initial velocity in X direction and defined mass. Following rules govern the manufactured universe and approximate model:

1. The crash box has a fine mesh in the high-fidelity model. A finer mesh improves the modeling of local effects in the component. Especially, mesh size influences the plasticity and failure behavior of the component. The crash box has a coarser mesh in the approximate model; the element edge length is two times larger. In industrial use cases, in which simulation models are typically bigger and more detailed, mesh size influences both the model accuracy and computational time, enhancing the former while deteriorating the latter. As a result, a compromise is necessary on the mesh size. Hence, it is logical to consider a coarse mesh in the approximate model.

2. Modeling of boundary conditions: The high-fidelity model incorporates detailed modeling of the test rig, including bolt connections and a part of the test rig. The test rig can have small elastic deformations as can be expected in real tests. However, the approximate model has a fixed boundary condition assumption and the deformation of boundaries is neglected.

3. In both models, a rigid barrier impacts the crash box with a mass of 150 kg and an initial velocity of 8 m/s. The impact velocity and impact angle around the Z-axis might vary from the nominal value in experiments. We assume that both uncertainties are known in the approximate model as well.

4. Manufacturing uncertainties: In the manufactured reality, the wall thickness of the crash box can vary due to manufacturing uncertainties. It is considered as an uncertain input parameter in the approximate model.



(a) Manufactured reality.

(b) Approximate model.

Figure 1: Illustration of simulation models used for the manufactured reality and the approximate model.

For validation, we select three SRQs as representatives of crash box behavior: mean force F_{mean}, maximum deformation $maxDef$, and force corresponding to the maximum deformation F_{maxDef}. Figure 2 shows the reaction force-deformation curve of a crash box. Mean Force is the area below the force-deformation curve, which equals to the amount of energy that the crash box absorbs, divided by its maximum deformation.

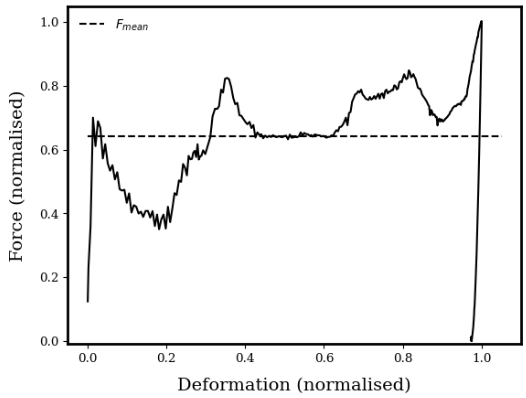


Figure 2: Force-deformation curve of a crash box and mean force.

3 RESULTS

The study case, presented in this section, serves as a validation example in presence of uncertainty. We show the procedure of quantifying model error while accounting for experimental and input uncertainties. Two approaches are used to quantify the modeling error: Joint sampling via the Monte Carlo approach and separate quantification of uncertainty sources.

For validation example, we generated 10 experimental results using the manufactured universe model. Uncertainties in experimental results, in general, can occur due to: experimental conditions, variations in experimental probes, or random measurement errors. We assume the first and second uncertainty sources in the experimental results. Table 1 shows the considered uncertainties and their corresponding ranges in which they vary.

Additionally, we assume all three uncertainties as uncertain input parameters in the approximate model. The distribution of each uncertain input parameter is known for the approximate

Table 1: Uncertain input parameters in the case-study.

Parameter	Distribution	Interval
Crash box wall thickness (mm)	uniform	$[2.25, 2.55]$
Impact velocity (m/s)	uniform	$[7.7, 8.3]$
Impact angle- Z direction (degrees)	uniform	$[-5, 5]$

model. However, the exact value of parameters in each experiment is not available.

3.1 Joint evaluation of input and experimental uncertainties

This section concerns with the evaluation of the modeling error following the Monte Carlo approach. Generally, the assumption of linearity, as in the sensitivity coefficients approach for propagation of uncertainties, is not correct for highly nonlinear impact problems involved in crash. Thus, the Monte Carlo approach is more suitable for the evaluation of uncertainties.

For validation, we consider the crash box mean force as SRQ. The validation steps are discussed in Section 2.1. First, a probability distribution should be estimated for experimental results, D. Figure 3 shows the histogram of the experimental results. Because the number of experiments is not enough to estimate a probability distribution, we assume the data has a normal distribution, and estimate only the parameters of the distribution.

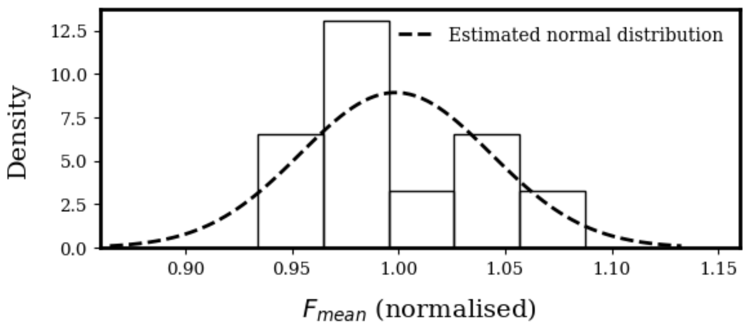
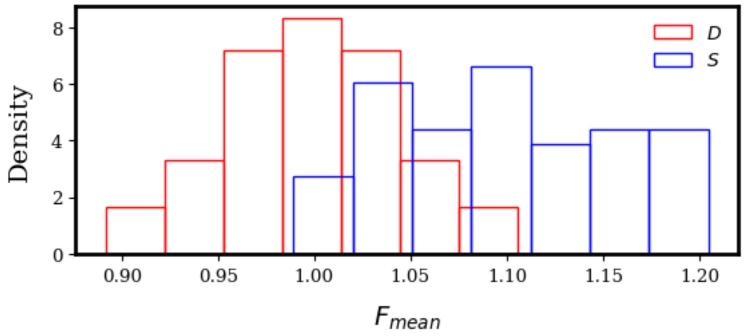


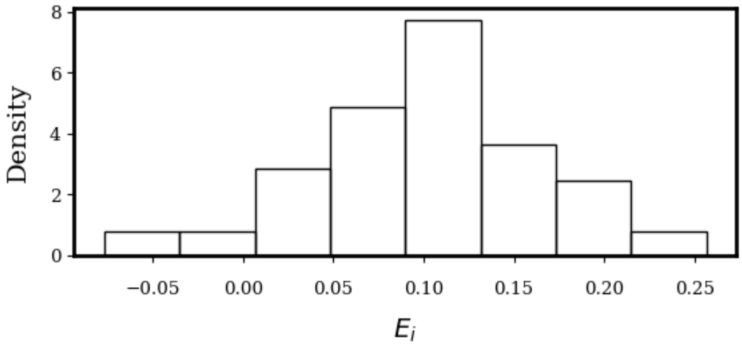
Figure 3: Histogram and estimated probability distribution of experimental results.

Next, the samples are generated using the input uncertain parameters with the corresponding distributions and the estimated probability distribution of D. Due to the high number of simulations required for Monte Carlo sampling, we use the LHS method. The number of samples is set to 60. The comparison error is calculated for each sample point, i, using the simulation result, S_i, and the corresponding experimental result, D_i. Figure. 4 illustrates the distributions of S_i, D_i, and E_i. The comparison error, E, and joint input and experimental uncertainty, $u_{input}^2 + u_D^2$, are estimated using Equation (8).

Evaluation of validation uncertainty requires, in addition to input and experimental uncertainties, an estimation of numerical uncertainty. In [7] numerical uncertainty is determined via the GCI approach. GCI might be a correct approach for unit problems as in the example presented by [7]. As discussed earlier, mesh size is set here to a minimum possible size in industrial crash simulation models due to considerations regarding computational time. Hence, the uncertainty due to the assumption of the mesh size can be categorized as modeling uncer-



(a) S and D histogram.



(b) E_i histogram.

Figure 4: Histogram of D_i, S_i, and E_i.

tainty rather than numerical uncertainty. Solver parameters and parallelization effects are other sources that can affect numerical uncertainty in explicit solutions of crash simulations. For all the simulations in this paper, we keep the number of CPUs constant in order to avoid numerical uncertainty due to parallelization. To examine the effect of solver parameters, we perform multiple recalculations of the approximate simulation model with varying minimum time-step size in the range of 10 percent. The standard deviation of the simulation results is considered as numerical uncertainty.

Table 2 summarizes the results of the validation. Numerical uncertainty is small in comparison to the joint input and experimental uncertainty and hence it is neglected.

Table 2: Validation results for joint evaluation of u_D and u_{input}.

		$F_{mean}(N)$
Standard uncertainties	$\sqrt{u_D^2 + u_{input}^2}$	2172.6
	u_{num}	9.1
	u_{val}	2172.6
Comparison error	E	3346.4

3.2 Separate evaluation of the standard uncertainties

A second approach, proposed by [8], is to evaluate each standard uncertainty in Equation (6) separately and calculate validation uncertainty as the root sum square of the standard uncertainties. The difference with the last approach is that we do not consider experimental results for sampling, and hence it is not necessary to estimate the probability distribution of the experimental results. Both approaches assume that standard uncertainties are independent. The V&V 20 standard accepts this as a reasonable assumption for validation problems in which the experimental value is measured directly. A discussion about the assumption follows in the next sections.

For the evaluation of input uncertainties, we draw 60 samples of the input uncertain parameters using the LHS method. Input uncertainty and experimental uncertainty are standard deviations of simulation and experimental results, and validation comparison is the difference between the mean of experimental and simulation results. We neglect u_{num} as results in the last section suggest that its value is insignificant in comparison to the other standard uncertainties.

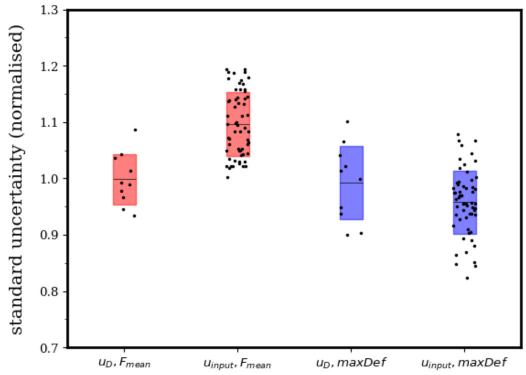
Table 3: Validation results for separate evaluation of u_D and u_{input}.

		$F_{mean}(N)$	$maxDef(mm)$	$F_{maxDef}(N)$
Standard uncertainties	u_D	1508.6	8.5	12838.7
	u_{input}	1927.4	7.4	6554.8
	u_{val}	2447.8	11.2	14415.9
Comparison error	E	3320.6	- 4.6	1190.6

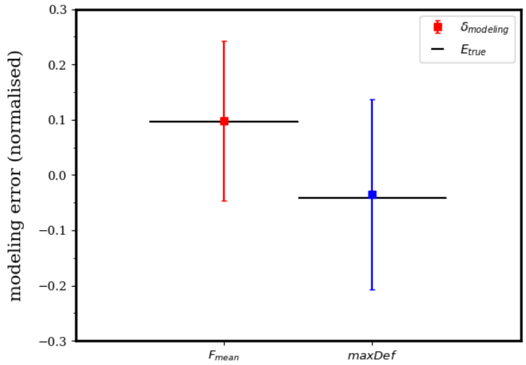
Table 3 provides the validation results from the second approach. In addition to the crash box mean force F_{mean}, we considered two further system responses for validation: maximum deformation, $maxDef$, and force corresponding to the maximum deformation, F_{maxDef}. Comparing validation results for F_{mean} in Tables 2 and 3, validation uncertainty varies by 10 percent, and the difference in comparison errors is negligible. We can conclude that validation results from both methods are comparable. Separate evaluation of standard uncertainties requires a lower number of evaluations of the simulation model because it does not consider the experimental value as a parameter for sampling. This is especially advantageous if several response quantities are desired for validation and evaluations of the simulation model are computationally expensive.

3.3 Evaluation of modeling error

The main focus of the V&V 20 validation approach is the quantification of modeling error. So far, we evaluated comparison error and validation uncertainty. Note, the validation uncertainty is not a measure of model quality but rather a measure of the validation quality. For interpretation of the validation results, we should compare validation uncertainty with comparison error. If the magnitude of u_{val} is much smaller than the magnitude of E, then the uncertainty in validation can be neglected and the modeling error has the same magnitude as the comparison error. On the other hand, if the validation uncertainty is much higher than the comparison error, the uncertainty dominates the validation practice and no conclusions can be made about the magnitude of the modeling error. The third case is when the validation uncertainty is neither negligible nor dominates the validation practice; the validation method predicts an interval for



(a) Input and experimental uncertainties, each point in scatter plot presents an experimental or simulation result.



(b) Estimated interval of modeling error with 95 percent confidence and true modeling error.

Figure 5: Validation of crash box, standard uncertainties and estimated modeling error.

the modeling error with a center at E and a radius of u_{val} for such cases. In order to avoid ambiguity, Roache [10] proposes a factor of 7 as decision criteria:

$$\frac{1}{7} \leq \frac{u_{val}}{E} \leq 7 \Rightarrow E - u_{val} \leq \delta_{model} \leq E + u_{val}. \tag{9}$$

Validation results in Table 3 for mean force and maximum deformation fulfill the criteria above. However, uncertainties dominate the validation results for force corresponding to the maximum deformation. Accordingly, we can estimate an interval for modeling error only for mean force and maximum deformation. The coverage factor, k, expands the interval of modeling error to incorporate a confidence level in the estimation. To define the value of k, an assumption is necessary for the probability distribution of the combined error term in Equation (4). k is typically in the range of 2 to 3 for uniform, triangular, and normal distributions and a 95 percent confidence interval:

$$E - U_{val,95\%} \leq \delta_{model} \leq E + U_{val,95\%}; \qquad U_{val,95\%} = k u_{val}. \tag{10}$$

We assume a coverage factor of 2 for the estimation of the modeling error. Fig. 3.3 illustrates the estimated intervals of modeling error with 95 percent confidence, as well as input and experimental uncertainties.

3.4 Assessment of the validation results

We estimated an interval for modeling error based on comparison error and standard uncertainties that affect experimental and simulation results. Some assumptions accompany the validation approach. More specifically, we assumed that: 1) the three error sources (experimental, input, and numerical) are independent, 2) experimental results have a normal probability distribution; the assumption is made because of the insufficient number of experimental results, and 3) the parent probability distribution of the three error sources has to be triangular, uniform, or normal. The assumptions affect the validation results. In this section, we aim to critically assess the validation results and question the assumptions behind the approach.

We quantify true modeling error for assessing the validation results. 60 more experimental data are generated using the high-fidelity model. LHS sampling is used in order to cover the range of uncertain parameters, in contrast to the 10 experimental data for which random sampling is employed. We define the true modeling error as the difference between the mean of simulation results and mean of the 60 experimental data. Figure 5(b) illustrates the true modeling errors via solid black lines. The estimated intervals of modeling error contain the true modeling error for both F_{mean} and $maxDef$. However, if the estimations are too conservative, then there is a higher chance that they contain the true modeling error. Thus we consider the conservativeness of the estimations as a second criterion in the assessment of validation results. We describe conservativeness as the ratio between 95 percent validation uncertainty divided by the mean value of simulation results, S:

$$conservativeness = \frac{U_{val,95\%}}{S}. \tag{11}$$

Table 4 reports the conservativeness values for F_{mean} and $maxDef$. The validation results are more conservative for $maxDef$ than for F_{mean}. Conservativeness is directly proportional to the value of the coverage factor, k. Another assumption in validation, which contributes to the conservativeness of the results, is the independence of input and experimental uncertainties. Although the experimental value is measured directly, experimental uncertainty is affected indirectly by the same parameters that we considered as input uncertain parameters. The assumption of independent u_D and u_{input} causes the effect of the common parameters to be considered twice in the evaluation of u_{val}. Neither of the standard uncertainties can be neglected in the evaluation of the validation uncertainty because of two reasons. First, in the example case presented in this paper, the same uncertainties were present in both experimental and simulation results. In contrast, experimental uncertainty in real-world results can include uncertainty sources beyond the ones known to the simulation model. Second, because experimental data are usually limited, we cannot guarantee that the experimental results contain the full range of effects by known uncertain parameters. Thus, it is still necessary to consider these parameters as uncertain in the simulation model. Therefore, we prefer results to be more conservative rather than to under-predict the interval of modeling error.

Table 4: Conservativeness of validation results.

	F_{mean}	$maxDef$
Conservativeness	0.13	0.18

4 CONCLUSION

In this paper, we considered uncertainties in the validation of component simulation models in crash. The V&V 20 validation approach is used for the validation of a crash box model. We used synthetic experimental results, following the concept of MMU, in order to evaluate the validation approach itself. The core of the validation approach is to estimate the modeling error of the simulation model with incorporated validation uncertainty. Two possible implementations of V&V 20, joint and separate evaluation of input and experimental uncertainties, resulted in comparable estimations of validation uncertainty. Additionally, we examined if the estimated interval of modeling error contains the true value of the error, and how conservative the estimations are.

The validation approach assumes that input and experimental uncertainties are independent for cases in which the validation variable is measured directly. The assumption causes the estimations to be more conservative because the effect of common uncertain parameters is considered in both input and experimental uncertainties. Additionally, the validation approach concerns only scalar SRQs at validation points. It does not address the validation of functional SRQs (e.g. curves), which are typical in crash applications.

To conclude, uncertainties can affect the crash behavior of components in a vehicle. The credibility of simulation models to predict the effect of uncertainties can be evaluated via approaches that consider uncertainties in the validation procedure. Examination of the underlying assumptions of validation approaches serves to better understand and apply the approaches to real-world applications.

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