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YIELD AND SENSITIVITY ANALYSIS OF MULTI-ELEMENT ANTENNA ARRAYS USING THE NON-LINEAR PARTIAL LEAST SQUARES POLYNOMIAL CHAOS EXPANSION TECHNIQUE

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Abstract

The Square Kilometre Array Mid-Frequency Aperture Array (SKA MFAA) uses a high number of similar elements in order to meet astronomical requirements. The Log-Periodic Dipole Antenna (LPDA) is one of the candidate elements for implementation of the MFAA. A single LPDA antenna consists of four sub-elements differentially fed, so only two sub-elements are active for a single excitation. 6.5 million LPDA elements will be required for implementation of the MFAA. Therefore, determination of yield is important for the SKA MFAA as the elements are not modified after manufacturing.

In analysis of yield, this antenna presents a high dimensional problem with 185 and 93 system parameters for the full and half models, respectively. The Non-Linear Partial Least-Squares based Polynomial Chaos Expansion (NLPLS-based PCE) was recently shown as an excellent way of calculating yield for high dimensional electromagnetic structures. Klink et al. validated the use of NLPLS-based PCE for yield analysis of a 37-variable diplexer, requiring only 30 full 3D electromagnetic analysis frequency sweeps instead of thousands required by Monte Carlo (MC) analysis [4].

In this paper, NLPLS-based PCE is implemented for estimation of yield and sensitivity of the SKA MFAA LPDA antenna. The yield specification for this problem requires that the reflection coefficient must be below -7 dB across the MFAA's band, i.e., 450 – 1450 MHz. The LPDA antenna attained 64 percent yield requiring 80 frequency sweeps while MC analysis required 300 analysis points. Furthermore, for the first time, curvature and rotation of antenna elements are considered in determination of yield.

Keywords: SKA MFAA, LPDA, NLPLS, PCE, Sensitivity, Yield.

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1 INTRODUCTION

The Square Kilometer Array Mid-Frequency Aperture Array (SKA MFAA) is a powerful radio telescope that is being designed to observe the universe at an unprecedented scale and sensitivity. Therefore, it will be one of the most challenging astronomical projects with regards to cost and power consumption. This array will compose of millions of antennas distributed within 1024 stations over a maximum baseline of 200 km [1]. Many antennas have been proposed for implementation of the MFAA. These include Crossed Octagonal Ring Antenna (C-ORA), planar Dense Dipole Array (DDA), Vivaldi arrays and the Log Periodic Dipole Antenna (LPDA). Each antenna design requires a specific number of elements for the realization of the MFAA. C-ORA MFAA realization requires 20.3 million elements for triangular grid configuration with 7% thinning, 25.3 million for Vivaldi while LPDA implementation requires 6.5 million elements [2]. Therefore, determination of yield for each MFAA candidate antenna is of importance since a high volume of elements cannot be modified after manufacturing.

Numerous methods exist for estimation of yield of an electromagnetic structure. The Monte Carlo (MC) method is the most prominent yield estimation technique. This technique involves propagating uncertainties through the model to determine the electromagnetic response. Although reliable, MC requires many repeated runs for converged estimates. Monte Carlo based yield estimation can be time consuming to implement as full physical model evaluations are required for each sample set. Moreover, this technique becomes cumbersome for high dimensional problems which leads to high computational costs and in some cases poor estimates when the number of samples available is insufficient.

Surrogate modeling techniques attempt to alleviate the drawbacks of MC-based methods by replacing the expensive to evaluate computational models with the inexpensive to evaluate surrogates. Many metamodeling techniques exist for estimation of yield. Polynomial chaos expansion (PCE) is one of the metamodeling techniques which approximate a computational model using weighted polynomials [3]. For a sufficiently accurate PCE model, statistical information, i.e., mean, variance etc., is directly extracted from the model. Yield from PCE can be approximated as the probability of the tunning variables performance resulting in a result that is better than the performance requirement. The use of PCE for estimation of yield in context of antennas has been validated in [3]. Klink *et al.* showed the use of PCE for yield analysis of a 4-parameter quad-mode antenna using the reflection coefficient as the performance metric. For a detailed explanation of this method, the reader is referred to [3-4]. Another widely used metamodeling method is referred to as Kriging (Gaussian process modelling). Kriging is a spatial interpolation technique that estimates the value at a given location while considering nearby observations. Unlike other interpolation techniques, Kriging considers the spatial correlation within the data. Kriging employs the covariance of variables to generate a model which approximates a computational model. Many variants of this method have been developed with universal Kriging being the most widely used [5]. While Kriging is relatively simple to implement, it can be computationally intensive, particularly when ordinary Kriging is used, compared with other surrogate modeling methods. In [6], Kriging was used for yield analysis of circularly polarized antennas. Furthermore, optimization of antenna yield using Kriging as the underlying surrogate modeling method in the yield optimization algorithm was shown in [6].

In order to combat limitations of some metamodeling methods and further improve performance, hybrid surrogate modelling techniques are used [7, 8]. These techniques combine two or more classical surrogate modelling methods to generate fast and robust surrogates with joint advantages of individual metamodeling techniques. Polynomial-based Kriging (PC-Kriging) is an example of a hybrid surrogate modelling method. PC-Kriging combines Kriging and PCE to create a surrogate model that is more effective compared to PCE and Kriging separately. In the model, PCE captures the global behavior of the computational model while Kriging estimates the computational models localized variations [7]. In [7], yield estimation of multiband patch antennas using polynomial-based Kriging was demonstrated. PC-Kriging required fewer samples for converged yield estimates, 93% and 80% fewer samples compared to PCE and kriging, respectively [7]. Although PC-Kriging is less computationally intensive compared to standalone PCE and Kriging techniques, large numbers of samples (model evaluations) are required as the dimensionality of the problem increases. In [4], PC-Kriging required more samples for yield convergence.

The method of NLPLS-based PCE has been demonstrated in [4, 8]. In [8], Non-Linear Partial Least Squares (NLPLS) in combination with PCE demonstrate promising results as a feasible surrogate modelling approach for high-dimensional problems. The NLPLS-based PCE is a hybrid approach that attempts to extract the relationship between observable variables and latent variables. NLPLS assumes a nonlinear relationship between the observable system parameters and latent variables making the metamodeling technique more suited for structures where the underlying process is nonlinear. PCE is integrated into NLPLS as the polynomial relating the latent variables with the response vector. The use of latent variables reduces the number of system parameters used in computing the coefficients of the NLPLS-based PCE surrogate model hence creating a less complex model. The contribution of each system parameter to the performance parameter variance (sensitivity analysis) is carried out as a post-processing step. This is achieved by transforming the NLPLS-based PCE coefficients to standard PCE coefficients from which sensitivity analysis can easily be carried out as a function of the PCE coefficients. For further explanations and mathematical validations of this technique, the reader is referred [4, 8-9]. In the context of electromagnetic structures, NLPLS-based PCE in the sensitivity analysis of an 8-variable dual patch antenna and a 37-variable diplexer, required 10 and 30 samples for converged global sensitivity, respectively [4].

In this paper, NLPLS-based PCE is used to estimate the yield of the SKA MFAA LPDA antenna. The obtained NLPLS-based PCE yield results are compared with yield estimates obtained through a Monte Carlo simulation of the LPDA structure for quantification of the effectiveness of the surrogate modelling technique. In simulation of yield, 93 system parameters were taken into consideration.

The organization of this paper is as follows. Section 2 provides a description of the LPDA's physical structure. Section 3 provides an explanation on the LPDA's manufacturing process. Here, possible uncertainties inferred on the LPDA's structure due to the fabrication process are unpacked. In section 4, the performance metric for yield analysis is introduced and NLPLS-based PCE is applied for quantification of yield. Finally, conclusions are drawn in section 5.

2 MFAA LPDA ANTENNA

The proposed LPDA antenna for MFAA implementation is shown in Figure 1. A single LPDA element is made up of 6 dipoles. The antenna comprises of four LPDA elements, two elements per polarization. The system is differentially fed, so only two sub-elements are active in the case of one excitation. A prototype of this antenna has been developed at Cambridge University [2, 11]. This antenna is now the element used in a 16-element prototype array configured in the sparse random regime [11]. Initial measurements on the 16-element array have been conducted with measurement results on scattering parameters having a good agreement with the EM simulation. For detailed description of the antenna in terms of its mechanical structure and radiation performance characteristics, the reader is referred to [2, 10-11].

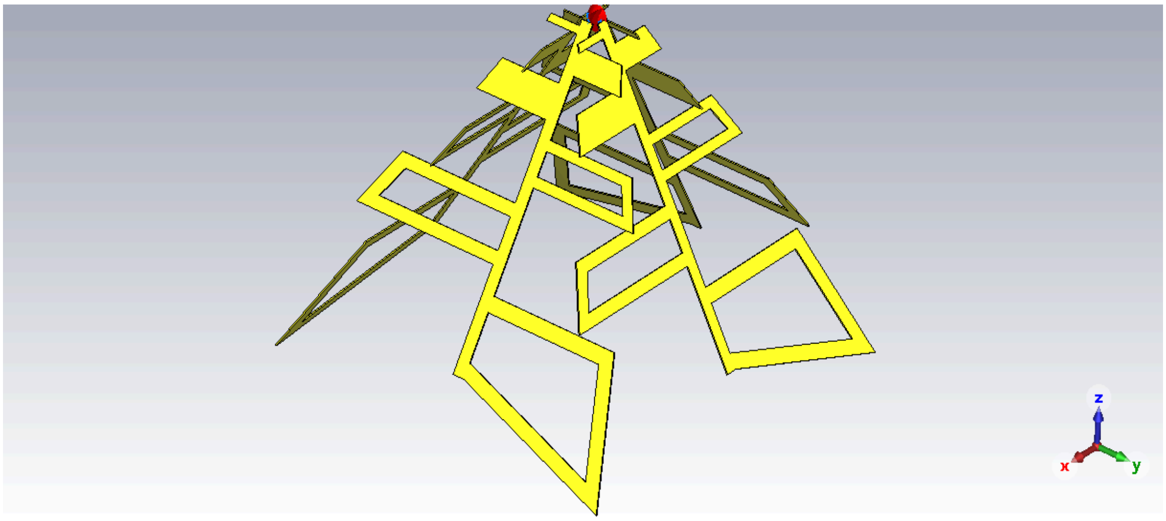

Figure 1: CST MWS model of the SKA LPDA antenna

The SKA MFAA LPDA antenna provides for a significantly high dimensional problem with 93 and 185 system parameters for the half and full models, respectively. The statistically significant parameters for the antenna half model are $x = [a_n, b_n, G, R_a, R_b]^T$, $n = 1, 2, 3, \ldots, 45$. The corresponding nominal design values are x_0 = *[1, 5, 4.8, 26.6, 5.9, 28.3, 36.4, 53.1, 26.8, 65.2, 61.6, 106.9, 45.2, 130, 81.4, 10, 99.7, 113.4, 5, 17.9, 35.2, 20.6, 43.5, 47.3, 76.5, 34.9, 93.4, 8, 70.4, 80.4, 80, 113.1, 61.4, 146, 45.8, 12, 104.1, 127.9, 17.5, 0, 0, 0, 0, 31.5, 0, , 5, 4.8, 26.6, 5.9, 28.3, 36.4, 53.1, 26.8, 65.2, 61.6, 106.9, 45.2, 130, 81.4, 10, 99.7, 113.4, 5, 17.9, 35.2, 20.6, 43.5, 47.3, 76.5, 34.9, 93.4, 8, 70.4, 80.4, 80, 113.1, 61.4, 146, 45.8, 12, 104.1, 127.9, 17.5, 0, 0, 0, 0, 31.5, 0, 33.2, 0, 0]*^{T}, with all dimensions in millimeters (mm).

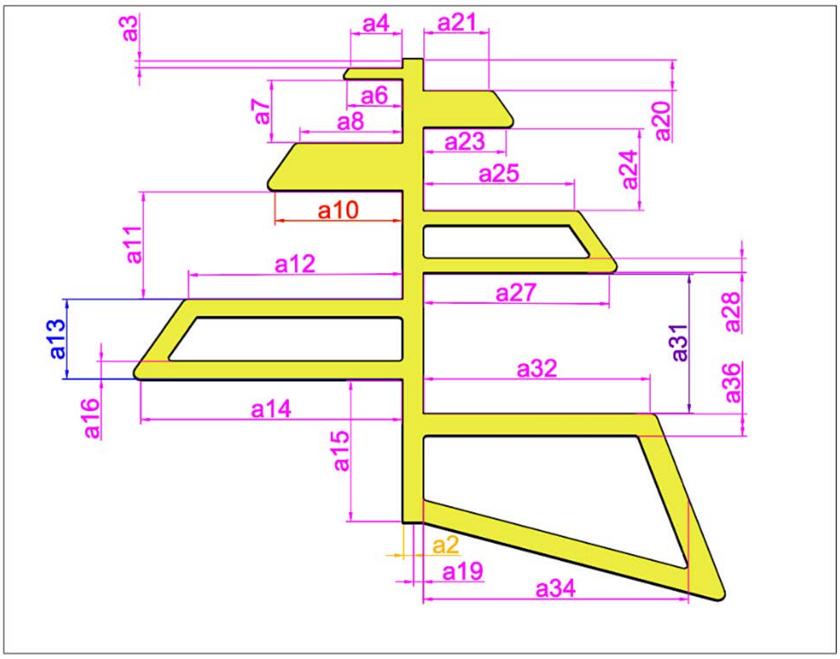

Figure 2: Parameterized single element of the SKA LPDA antenna

3 LPDA PRODUCTION PROCESS

Manufacturing of the LPDA array can be separated into three-stage; sheet metal production, shape cutting and assembly. At each production stage, uncertainties are added which ultimately contribute to the performance, i.e., reflection coefficient, radiation pattern etc., of the manufactured array deviating from the simulation. Pressure die casting is commonly used for production of high volumes of sheet metals and therefore more suitable for manufacturing of sheets for fabrication of MFAA LPDA elements. At this stage, the parameter that contributes the most to the uncertainty is the thickness of the produced sheet metals. Two primary techniques can be used for mass cutting of the LPDA elements. These are chemical etching and laser cutting. Both techniques are highly precise with low tolerances. Standard chemical etching and laser cutting can achieve tolerances of ±10% and ±5% of the metal thickness, respectively, for sheet metal thicknesses of 1-1.5 mm [12-13]. The MFAA LPDA is highly dimensional with inside and outside cuts required for realization of the structure as shown in Figures 1 and 2. During this stage, uncertainties are added to each planar dimension of the LPDA's element structure which ultimately affect the performance of the antenna. Therefore, each dimension constitutes to a probabilistic event. The final stage in the array's manufacturing process is assembly. Four LPDA elements are arranged to create two orthogonal polarizations. As the structure is assembled, variations in the mounting locations and angular positions of each element relative to the structure's reference are introduced. These deviations are shown in Figure 3 for the half model. Furthermore, the LPDA elements experience bending as they are assembled to create the antennas. The distance of each element relative to the ground plane also constitutes to an uncertain parameter.

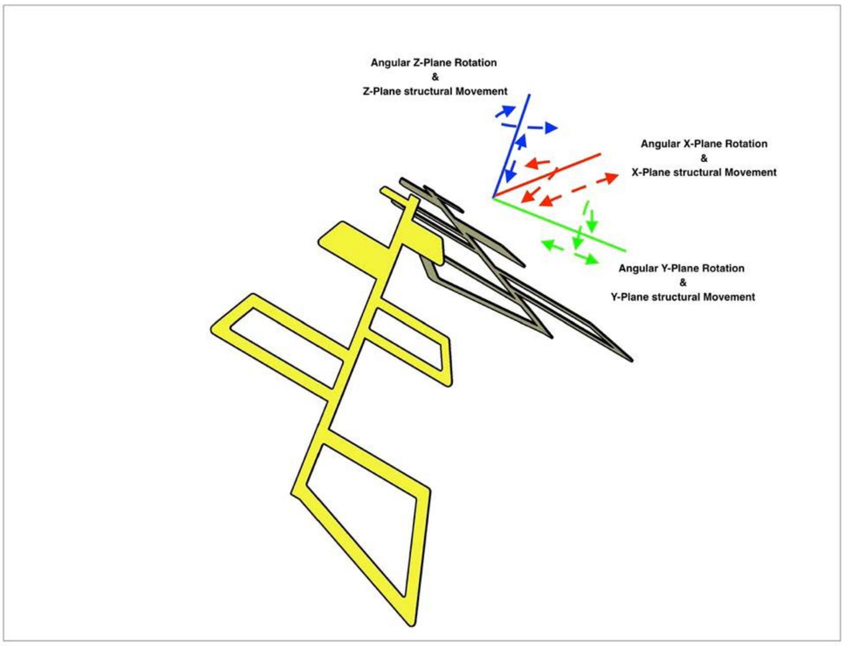

Figure 3: Possible assembly deviations of the LPDA antenna

4 LPDA YIELD AND SENSITIVITY ANALYSIS

The SKA LPDA antenna is a wideband antenna designed to operate over the MFAA's full spectrum (0.45 – 1.45 GHz) [4, 10]. For this antenna, the performance parameter used for quantification of yield is chosen as the antenna's reflection coefficient, i.e., $S_{1,1}$, as shown in Figure 4. The value of the performance parameter corresponds to the maximum reflection coefficient over the MFAA's frequency band. For the nominal design, this value is approximately -7 dB. Uncertainties due to the manufacturing process of the antenna were modeled as Gaussian with mean and variance of 0 mm and 0.05 mm, respectively (*N (0, 0.05)*). The NLPLS-based PCE yield results are shown in Figure 5. A tolerance band corresponding to ±5% of the converged MC yield estimate value is shown as an aid to determine the convergence of the NLPLS-based PCE metamodeling technique. Therefore, convergence is redefined as the minimum number of samples required to consecutively have yield estimates within the MC tolerance region. In the MC analysis, the steady state yield value is determined using the criteria whereby the change in yield between consecutive samples should be less than 1% with a 95% confidence bond, as per central limit theorem.

NLPLS-based PCE requires a minimum of 80 samples for converged yield estimates. The MC method requires a minimum of 300 samples for converged yield. Therefore, NLPLS-based PCE requires 220 fewer samples compared to MC method showcasing the robustness and applicability of the hybrid surrogate modelling technique for high dimensional problems. The attained yield for the SKA-MFAA LPDA antenna is 64%. This translates to 36% of manufactured LPDA elements not being within design specifications and therefore increased cost per unit antenna for SKA-MFAA implementation. Therefore, yield optimization of this antenna needs to be carried out for implementation of the SKA MFAA to stay within budget constraints.

Sensitivity analysis of the SKA-MFAA LPDA antenna is carried out as a post-processing step from the converged NLPLS-based PCE model. As mentioned, sensitivity analysis is done by transforming the NLPLS-based PCE coefficients to standard PCE coefficients from which the contribution of each system parameter is computed. Sensitivity analysis is important as this information is used to determine which system parameters need careful attention during manufacturing as deviations of such parameters from nominal greatly deteriorates the antennas performance. The effect of the deviation of system parameters from nominal is shown in figure 6. The antenna's radiation pattern and reflection coefficients are affected by deviations of the system parameters from their nominal design values.

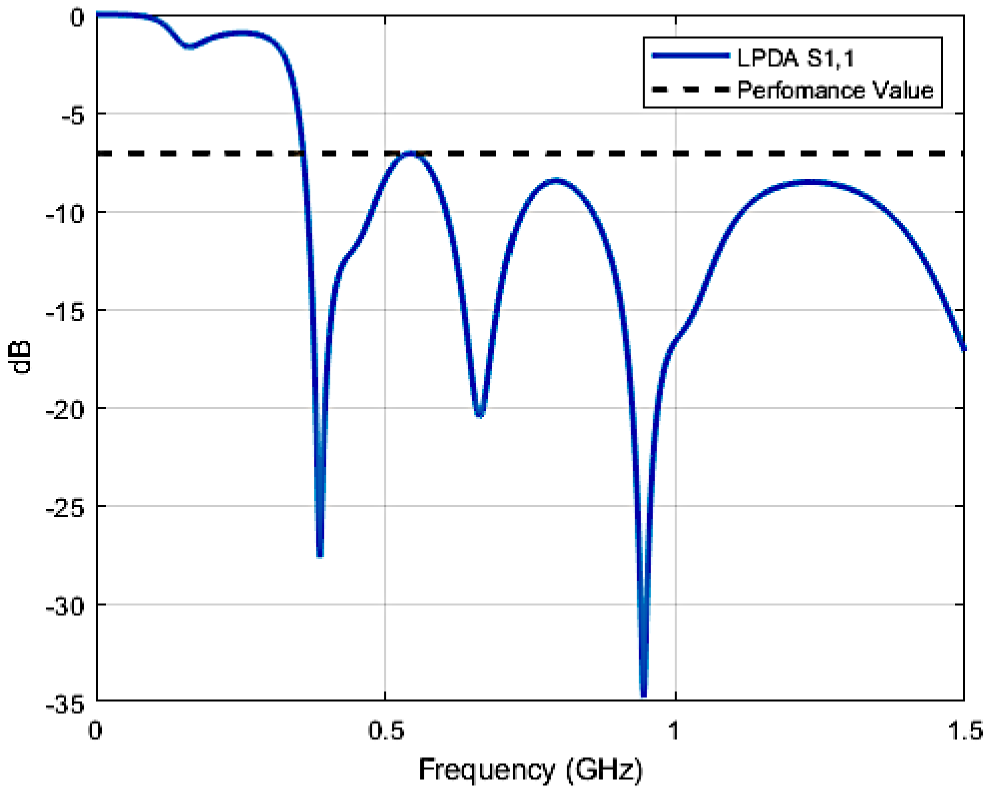

Figure 4: Simulated reflection coefficient for the SKA-MFAA LPDA antenna's nominal design

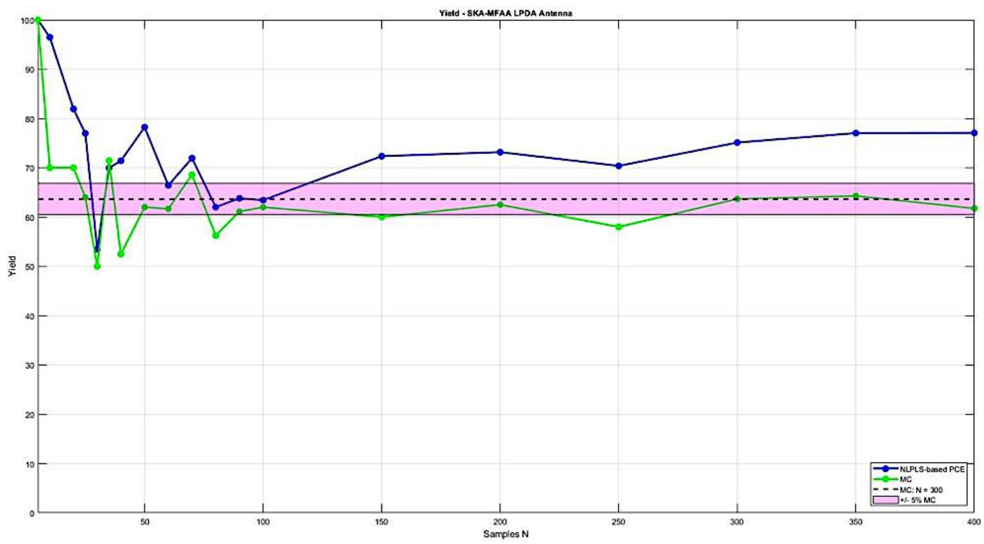

Figure 5: Yield convergence for the SKA-MFAA LPDA antenna

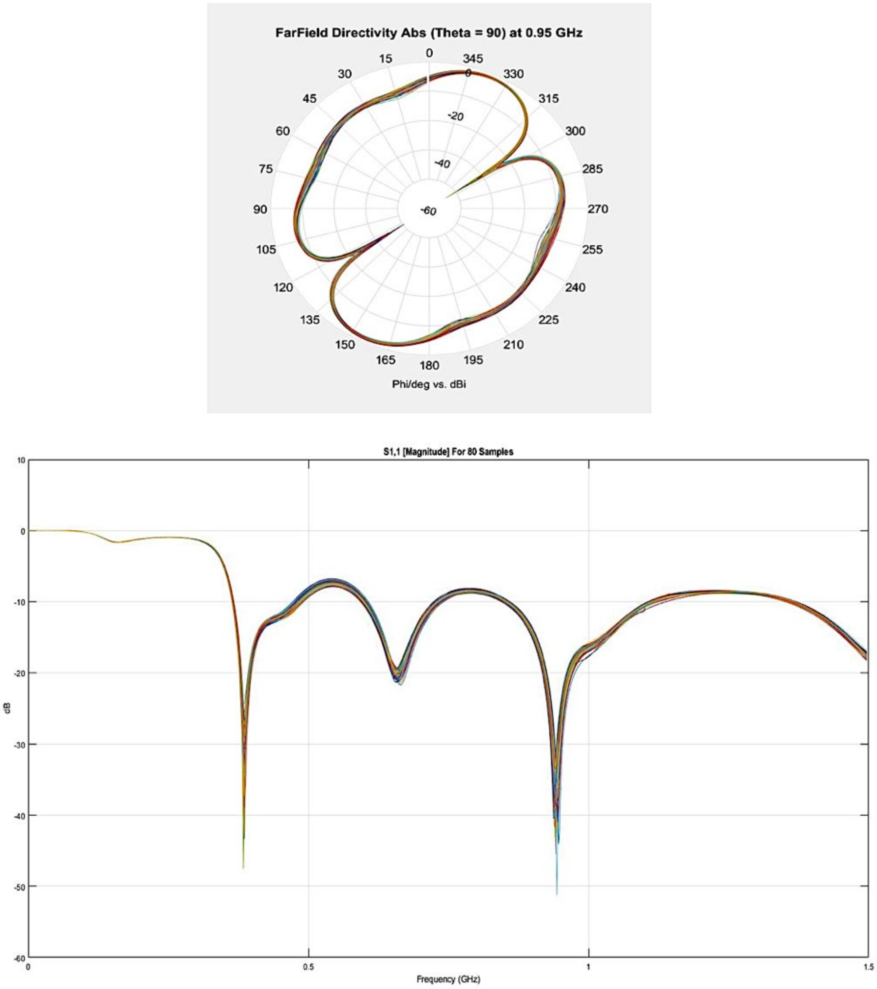

Figure 6: SKA-MFAA LPDA's radiation pattern (top) and reflection coefficient (bottom) for 80 samples

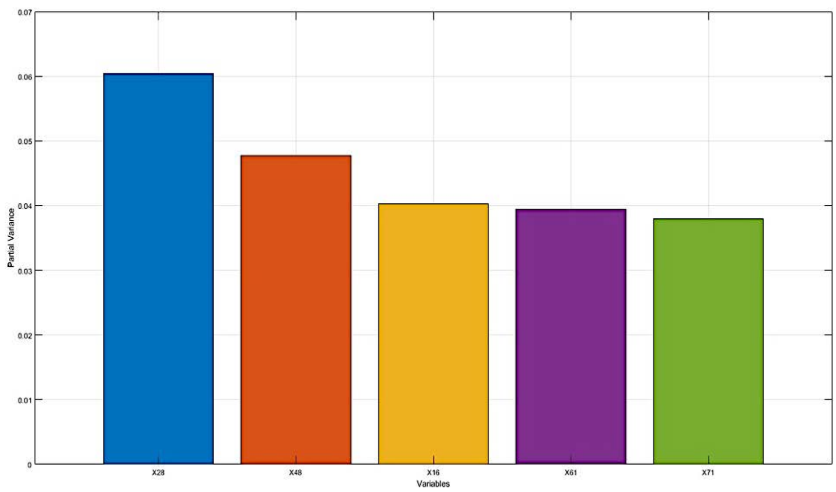

Figure 7: Most sensitive parameters

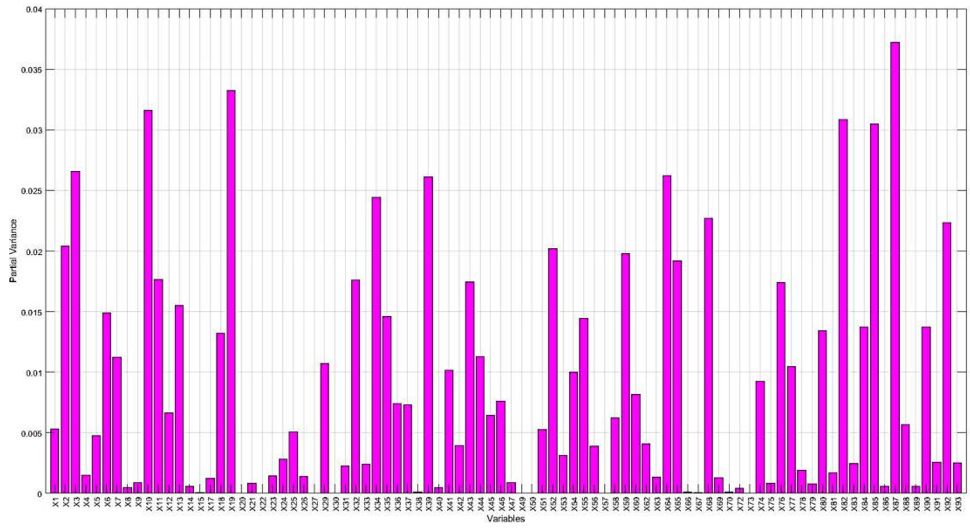

Figure 8: Least sensitive parameters

The five most sensitive system parameters are shown in figure 7 with the least sensitive parameters shown in figure 8. The most sensitive parameters are X16, X28, X48, X61 and X71. These parameters correspond to a16, a28, b3 (parameter on second LPDA element, i.e., a3 on figure 2), b16 (a16) and b26 (a26), respectively. More completely, a16, a28 and b16 correspond to the inside cut thicknesses of the second largest dipole, third largest dipole and the second largest dipole (negatively excited LPDA element), respectively. These system parameters arise from the cutting stage of the LPDA manufacturing process. The effect of these parameters on the performance of the antenna can be reduced by either changing the design of elements, i.e., exploring the LPDA design space, or exploiting more precise sheet metal cutting techniques. The latter comes with increased expenditure of which is undesirable for an already budget intensive astronomical project. Curvature, X92 (R_a) and X93 (R_b), of the antenna mainly contributed to broadening of the radiation pattern which in turn affected the antenna's gain.

5 CONCLUSION

In this paper, NLPLS-based PCE was applied to the SKA MFAA LPDA antenna for yield and sensitivity analysis. In analysis of yield, curvature over time and rotation over time of antenna elements were considered. Results showed that 80 analysis points are required to accurately estimate the yield for the 93-variable problem. For the same structure, a minimum of 300 frequency sweeps are required when analyzing yield and sensitivity of the LPDA antenna through Monte Carlo simulation. Future work will be focused on surrogate-based optimization of the LPDA antenna's yield.

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