

## **MECHANICAL PROPERTIES ESTIMATION OF STANDARD AND LIGHTWEIGHT CONCRETE THROUGH THE ELASTIC WAVES PROPAGATION**

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### **Abstract**

*Even if treated in the literature and carried out through several methods, the reliable estimation of the effective elastic properties of random media is still an open problem. Homogenization approaches, based on the study of a representative volume element, are generally used with analytical or numerical models. When considering real materials, one of the most common techniques for the estimation of effective elastic properties is based on the analysis of elastic waves propagation.*

*In this work, the first results of a comparison between the elastic properties estimation obtained by ultrasonic tests and impact tests applied to different kinds of materials are illustrated. In particular, two materials were studied: the first is a standard concrete with small-size aggregates and the latter is a lightweight concrete containing a 50% volume ratio of EPS spherical shape particles. Direct and indirect tests were carried out on both materials and, using a probabilistic approach applied to the formulation of P- and R-waves, the elastic properties of the equivalent homogenized continuum were estimated also considering confidence intervals.*

**Keywords:** Elastic waves, Concrete, homogenization, impact tests, ultrasonic tests.

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## 1 INTRODUCTION

The mechanical characterization of materials is a crucial task in civil engineering, especially when dealing with existing structures constituted by materials that can be affected by random properties due to the inner heterogeneities and/or advanced states of degradation. Homogenization techniques allow to consider such uncertainties and to obtain effective properties to be assigned to an equivalent continuum having the same behaviour of the heterogeneous one. Among these methods, dynamic homogenization is based on the analysis of the elastic waves propagation through the heterogeneous solid to estimate the effective elastic properties of the related homogenized continuum [1].

Many experimental practices are based on this concept, obtaining important information about the material, e.g. regarding non-homogeneous density distributions of the material inside a solid through tomography, or the presence of internal cracks through the study of wave reflection in the frequency domain. Within this context, the main techniques generally used in the field of Civil Engineering are the impact test [2-6] and ultrasonic test [7-9]. The first is based on the mechanical excitation of the stress waves in a solid via hammer hits, while the latter uses ultrasonic pulse-waves generated by a probe. When possible, the application of direct and indirect tests allows to estimate both the elastic constants which characterize an isotropic medium by solving a system of two equations, i.e considering volume and surface wave velocities.

In this paper, a comparison between the results obtained by the application of impact tests and ultrasonic tests on different kinds of material is presented. Two different materials are studied: the first is a standard concrete with small-size aggregates and the latter is a lightweight concrete obtained by including EPS spherical shape particles into the mixture with a volume ratio of 50%. An automated procedure for the processing of the experimental data has allowed to rapidly analyse the results of several tests with a probabilistic approach to estimate the elastic properties considering proper confidence intervals.

## 2 BASIC CONCEPT

In this Section, the basic concept on stress waves propagation in elastic solids are briefly recalled following the seminal work of Kolsky [10]. Let us consider an elastic solid media with mass density  $\rho$  in a Cartesian coordinate system  $(x, y, z)$ , a generic point with displacement vector  $(u, v, w)$ , stress tensor components  $\sigma_{ij}$  and body forces  $(X, Y, Z)$ , the equations of motion are expressed as

$$\begin{aligned}\rho \frac{d^2 u}{dt^2} &= \rho X + \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{yx}}{\partial y} + \frac{\partial \sigma_{zx}}{\partial z} \\ \rho \frac{d^2 v}{dt^2} &= \rho Y + \frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{zy}}{\partial z} \\ \rho \frac{d^2 w}{dt^2} &= \rho Z + \frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z}\end{aligned}\tag{1}$$

For an isotropic elastic solid the system of Equation (1) can be expressed in terms of displacements using Lamé constants  $\lambda$  and  $\mu$



$$\begin{aligned}
 \rho \frac{\partial^2 u}{\partial t^2} &= (\lambda + \mu) \frac{\partial \theta}{\partial x} + \mu \nabla^2 u + \rho X \\
 \rho \frac{\partial^2 v}{\partial t^2} &= (\lambda + \mu) \frac{\partial \theta}{\partial y} + \mu \nabla^2 v + \rho Y \\
 \rho \frac{\partial^2 w}{\partial t^2} &= (\lambda + \mu) \frac{\partial \theta}{\partial z} + \mu \nabla^2 w + \rho Z
 \end{aligned} \tag{2}$$

in which  $\theta = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$  and where the expression  $\frac{d^2}{dt^2}$  has been replaced with  $\frac{\partial^2}{\partial t^2}$  linearizing the differential equations.

By introducing a scalar potential  $\varphi$  and a vector potential  $\psi(\psi_1, \psi_2, \psi_3)$ , with

$$\begin{aligned}
 u &= \frac{\partial \varphi}{\partial x} + \frac{\partial \psi_3}{\partial y} - \frac{\partial \psi_2}{\partial z} \\
 v &= \frac{\partial \varphi}{\partial y} + \frac{\partial \psi_1}{\partial z} - \frac{\partial \psi_3}{\partial x} \\
 w &= \frac{\partial \varphi}{\partial z} + \frac{\partial \psi_2}{\partial x} - \frac{\partial \psi_1}{\partial y}
 \end{aligned} \tag{3}$$

and neglecting volume forces, the first equation gives

$$\begin{aligned}
 \frac{\partial}{\partial x} \left( \rho \frac{\partial^2 \varphi}{\partial t^2} \right) + \frac{\partial}{\partial y} \left( \rho \frac{\partial^2 \psi_3}{\partial t^2} \right) - \frac{\partial}{\partial z} \left( \rho \frac{\partial^2 \psi_2}{\partial t^2} \right) \\
 = (\lambda + \mu) \frac{\partial}{\partial x} \nabla^2 \varphi + \mu \frac{\partial}{\partial x} \nabla^2 \varphi + \mu \frac{\partial}{\partial y} \nabla^2 \psi_3 - \mu \frac{\partial}{\partial z} \nabla^2 \psi_2.
 \end{aligned} \tag{4}$$

The previous equations are satisfied if the functions  $\varphi$  and  $\psi_i$  are solutions of the equations

$$\nabla^2 \varphi = \frac{1}{\alpha^2} \frac{\partial^2 \varphi}{\partial t^2} \quad \nabla^2 \psi_i = \frac{1}{\beta^2} \frac{\partial^2 \psi_i}{\partial t^2} \quad i = 1, 2, 3 \tag{5}$$

in which

$$\alpha = \sqrt{\frac{\lambda + 2\mu}{\rho}} \quad \beta = \sqrt{\frac{\mu}{\rho}}. \tag{6}$$

Equations (6) describe the velocities  $\alpha$  and  $\beta$  of two types of disturbances propagated through an elastic, infinitely extended isotropic solid, which are known as volume waves. The first equation is the velocity of the expansion waves, also called longitudinal waves, associated with displacements in a direction that is equal to the direction of propagation, consisting of compressions and rarefactions that only lead to volume changes. The second equation is the velocity of the distortion waves, also called transverse waves, associated with shifts perpendicular to the direction of propagation, consisting of distortions that involve only changes in shape. Equations (6) can be also expressed through the engineering constants  $E$  and  $\nu$  in the following form



$$\alpha = V_P = \sqrt{\frac{E(1-\nu)}{\rho(1+\nu)(1-2\nu)}} \quad (7)$$

$$\beta = V_S = \sqrt{\frac{E}{2\rho(1+\nu)}} \quad (8)$$

In presence of a boundary surface, it is well known that the propagation of a third type of waves is observed: the elastic surface waves, which effect decreases rapidly with depth, also called Rayleigh waves. By solving the equation of motion at the boundary it is possible to achieve an expression for the velocity of Rayleigh waves propagation as a ratio of the transverse waves' velocity

$$V_R = \frac{0.87 + 1.12\nu}{1 + \nu} \sqrt{\frac{E}{2\rho(1 + \nu)}} = \frac{0.87 + 1.12\nu}{1 + \nu} V_S \quad (9)$$

Considering a linear elastic, homogeneous, and isotropic medium and being known the mass density, it is possible to derive the elastic constants from the equation of P-wave (7) and R-wave (9) velocities.

### 3 DESCRIPTION OF THE SPECIMENS

The experimental tests were carried out on two types of concrete material (Fig. 1): the first is a standard concrete (Conc1) characterized by small-size aggregates to achieve a more homogeneous mixture and with a nominal  $R_{ck}$  equal to 35 MPa; the latter is a lightweight concrete (Conc2) obtained by including into the mixture EPS spherical shape particles with a variable diameter in the range of 3-5 mm and with an overall volume ratio of 50%. Two cubic blocks (Block1 and Block2) were built having 1 m length per side (Fig. 2). Block1 was totally constituted by Conc1 material, while Block2 is a bi-phase material composed of Conc1 at the bottom, and by Conc2 at the top. The peculiar construction of Block2 is due to the possibility of studying the reflection and refraction of the waves passing through an interface between different materials, but this aspect has not been investigated in this study.

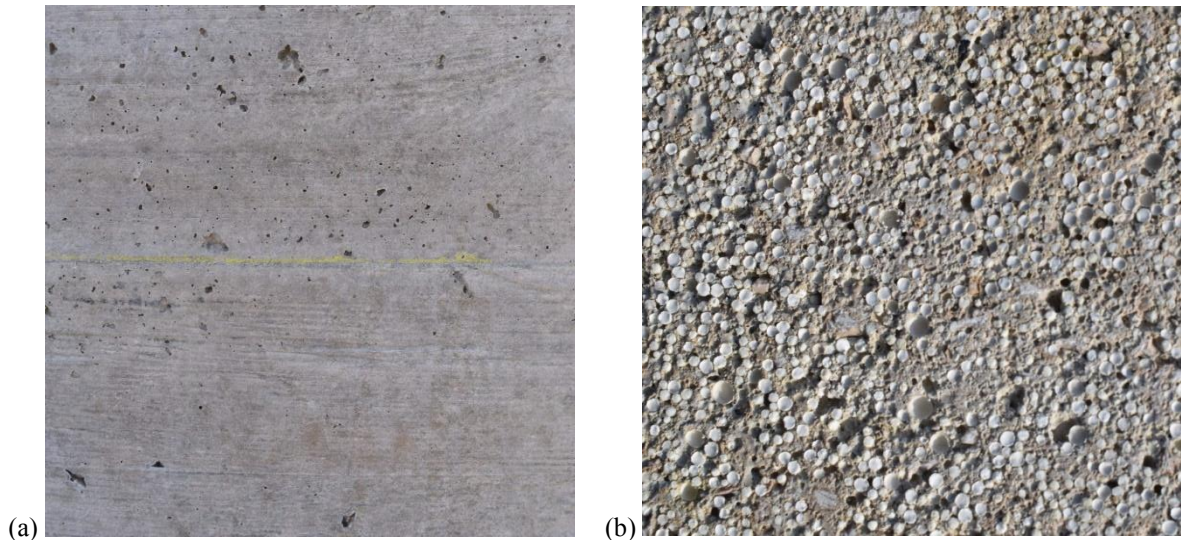


Figure 1: Surface images of Conc1 (a) and Conc2 material (b).





(a)



(b)

Figure 2: Cubic specimens used in the experimental tests: Block1 made of standard concrete (a) and Block2 characterized by a bi-phase material (b).



Figure 3: Photographic image of a generic indirect impact test carried out on Block2.

## 4 EXPERIMENTAL TESTS

### 4.1 Impact tests

The purpose of the impact tests was to calculate the propagation velocity of the elastic waves generated by an impulsive force through the analysed materials to estimate their elastic properties.

Two different types of impact tests were carried out: direct tests to study the propagation of P-waves (volume waves) and indirect tests to study the propagation of R-waves (surface waves). The direct tests consist in the application of an impulsive force on the surface of the solid and recording the wave propagated in the material on the opposite side of the solid. The velocity propagation is calculated as the ratio between the two points' distance (point of application of the pulse and point of the wave recording) and the time interval taken by the wave from the impact to the receiver. The indirect tests were performed by applying the impulsive force on the surface of the solid and recording the wave propagated on the same surface. For simplicity, two receivers on two different points aligned with the point of the pulse application were used. The propagation velocity is calculated as the ratio between the distance of the two receivers and the time interval taken by the wave to cover the distance between them (Fig. 3).



The tests were carried out using an instrumented impact hammer (model PCB 086C03) for the application of impulsive force on the sample surface. The vibrations induced by the wave passage were recorded by uniaxial piezoelectric accelerometers (model PCB 352C33 100mV/g). The data were recorded through an acquisition system (model IMC C-SERIES) with a high sample rate of 100 kHz, to catch the high velocity of the elastic waves.

The impact tests were applied on a grid of 5 points previously prepared on the lateral sample surfaces. The grid was used for the execution of the direct tests and the definition of four paths of the indirect tests on each side. To achieve a better estimation of the mechanical properties as a result of statistical analysis, 10 hits were applied in each position. Working with heterogeneous materials such as Conc2, the procedure has been used to evaluate the elastic properties of the homogenized continuum paying specific attention to the microstructure dimensions and to the distance on which the wave velocity is calculated.

## 4.2 Ultrasonic tests

Direct ultrasonic tests have been carried out on both samples by using a single position for each side. Six ultrasonic pulses have been recorded for each couple of opposite sides, exchanging the position of the 55 kHz transmitting and receiving probes. The waveforms have been recorded with a sampling frequency of 400 kHz. A total of 12 UPVs have been measured for each material, ordinary (Conc1) and lightweight (Conc2) concrete. Typical records of the ultrasonic waves are shown in Figure 4. From the comparison of the two images, it can be inferred that the attenuation in the lightweight concrete is much higher than that in the ordinary concrete.

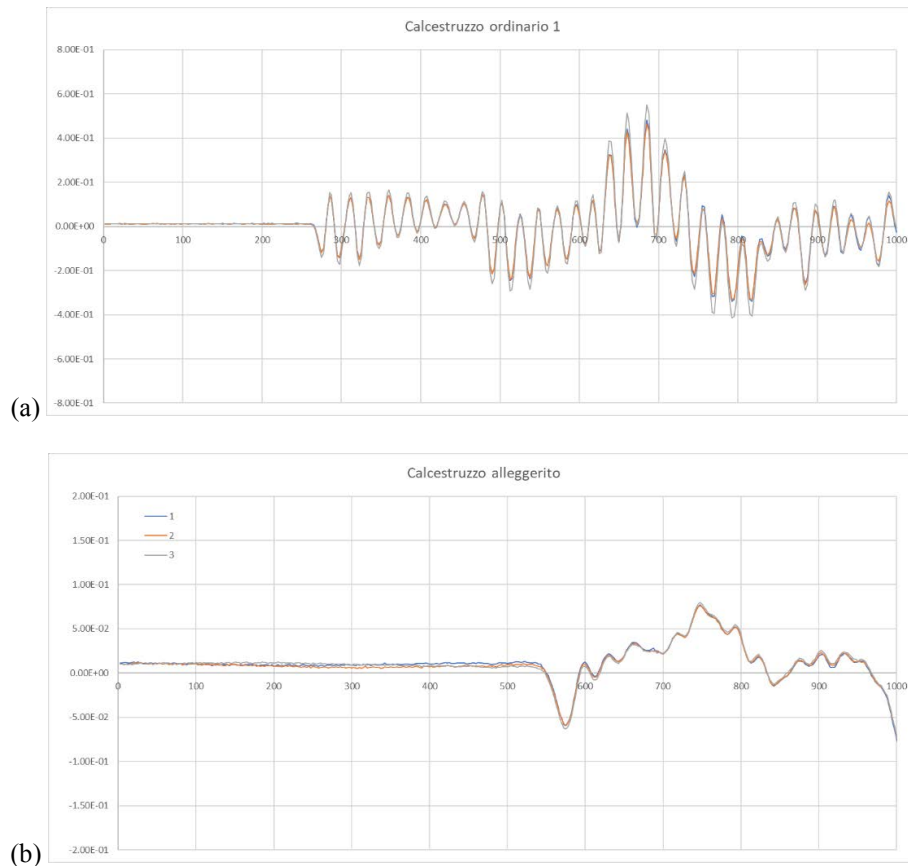


Figure 4: Typical signals acquired by the receiver probe during the ultrasonic tests carried out on the standard (a) and the lightweight concrete material (b).



## 5 ANALYSIS OF THE RESULTS

### 5.1 Results on standard concrete material

In this section, the results for the standard concrete material (Conc1) obtained by both impact and ultrasonic tests are presented. The tests were performed on the four lateral sides of the cubic samples, called A, B, C, and D, so that A is opposite to C and B is opposite to D. As described before, the impact tests were applied on five different positions for each side, while the ultrasonic tests on only one position. Table 1 and Table 2 summarize the results obtained on the P-waves from impact and ultrasonic tests, respectively. The statistical values highlight a quite good agreement of the mean velocities, even if slightly higher values are observed in the results derived from the impact tests, both global means and in the single measurements. From the standard deviations and the coefficients of variations can be asserted that the impact tests are affected by more dispersive results.

In Table 3 the results derived by indirect tests for the estimation of R-wave velocities are summarized. This kind of investigation has been carried out only through impact tests due to the impossibility to obtain consistent results by using ultrasonic sensors.

Table 1: Results on P-waves velocity estimation for Conc1 material through the impact tests [m/sec]

Path	Pos 1	Pos 2	Pos 3	Pos 4	Pos 5	Mean
From A to C	3983,34	3993,85	3914,44	3891,43	3825,82	3921,776
From B to D	3783,02	3914,53	3910,76	3726,43	3641,29	3795,206
From C to A	3937,98	3932,05	3980,93	3819,04	4035,64	3941,128
From D to B	3996,57	3946,33	4050,19	3954,22	3989,54	3987,37
Global mean (m/sec)						3911.37
Standard deviation (m/sec)						104.87
Coefficient of variation (%)						2.68

Table 2: Results on P-waves velocity estimation for Conc1 material through the ultrasonic tests [m/sec]

Path	Test 1	Test 2	Test 3	Mean
From A to C	3789.93	3794.88	3789.91	3791.58
From B to D	3802.46	3810.90	3800.45	3804.60
From C to A	3797.73	3798.46	3793.65	3796.61
From D to B	3808.63	3802.30	3805.13	3805.35
Global mean (m/sec)				3799.53
Standard deviation (m/sec)				6.62
Coefficient of variation (%)				0.17

Table 3: Results on R-waves velocity estimation for Conc1 material through the impact tests [m/sec]

	Path 1	Path 2	Path 3	Path 4	Mean
Side A	2014.53	2251.31	1863.35	2404.49	2133.42
Side B	2026.58	2063.15	2189.11	2128.84	2101.92
Side C	2105.26	2196.42	2087.71	2001.11	2097.63
Side D	2248.36	2134.96	2100.25	2108.60	2148.04
Global mean (m/sec)					2120.25
Standard deviation (m/sec)					123.94
Coefficient of variation (%)					5.85



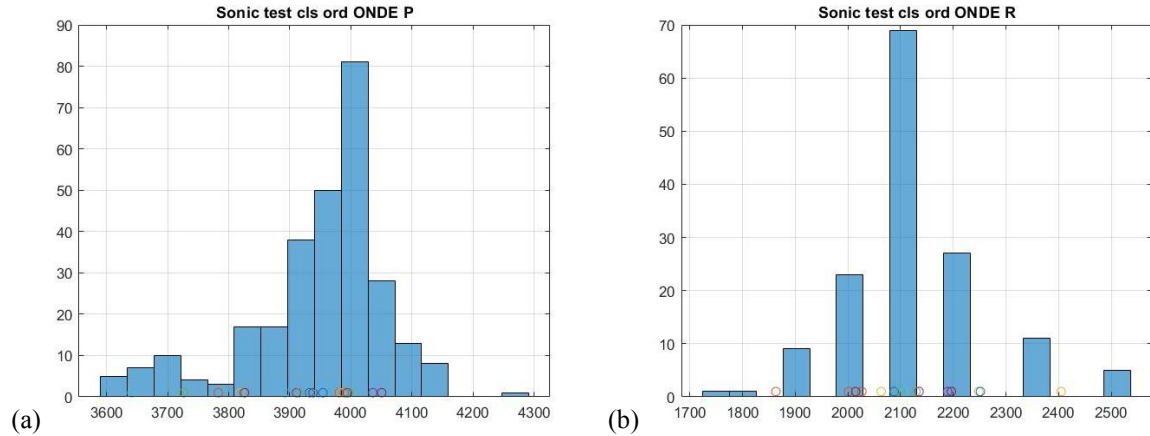


Figure 5: Frequency distribution histograms of the P-waves (a) and R-waves (b) obtained by the impact tests on the standard concrete material (Conc1).

Figure 5 illustrates the frequency distribution histograms of the results obtained from the impact tests, both for the P-wave (Fig. 5a) and R-wave velocities (Fig. 5b), respectively, which show the level of dispersions of such data. To take into account the uncertainties effects related to the process of measurement and process of experimental data, a confidence interval around the mean value can be calculated. Considering a confidence level of 95%, the ranges for the wave velocities reported in Table 4 are obtained. By solving the system of equations (7) and (8), the range of Young modulus  $E^s$  and Poisson ratio  $\nu^s$  are obtained through the impact test data and, finally, a range of Young modulus  $E^{us}$  is estimated using  $\nu^s$  in the equation (7). The material density  $\rho_1$  was directly calculated from a material sample:  $\rho_1 = 2218 \text{ kg/m}^3$ .

The results highlight that the estimation of the elastic properties obtained by ultrasonic tests is slightly lower than those obtained by impact tests. In terms of P-wave velocity, the difference is about 3% while the difference in the estimation of the elastic modulus is about 6%.

Table 4: Estimation of the mechanical parameters of Conc1 through impact tests and ultrasonic tests. (Young modulus  $E$  [MPa] and Poisson ratio  $\nu$  [-])

		Mean	St. Dev.	Conf. Interval (95%)		
Impact test	$V_{P1}^s$	3911.37	104.87	3896.83	3925.91	$\bar{E}_1^s = 29206$ [28746 ÷ 29667]
	$V_{R1}^s$	2120.25	123.94	2101.05	2139.46	$\bar{\nu}_1^s = 0.23$ [0.22 ÷ 0.24]
Ultrasonic test	$V_{P1}^{us}$	3799.53	6.62	3786.89	3802.18	$\bar{E}_1^{us} = 27559$ [27376 ÷ 27598]

## 5.2 Results on lightweight concrete material

The same methodology described in the previous section for the data analysis of the standard concrete was applied to the data obtained on the lightweight concrete material (Conc2). Owing to the reduced dimensions of the material, being Block2 constituted by both Conc1 and Conc2, the number of paths used on the lateral sides for the estimation of the R-waves velocities is smaller than in the previous case. For this reason, further tests were carried out on the upper face of the block.



Table 5: Results on P-waves velocity estimation for Conc2 material through the impact tests [m/sec]

Path	Pos 1	Pos 2	Pos 3	Pos 4	Pos 5	Mean
From A to C	2097.51	2176.12	2163.55	2475.86	2181.65	2218.938
From B to D	2231.33	2205.25	2383.74	2584.64	2528.39	2386.67
From C to A	2303.72	2049.06	2289.33	2492.47	2351.4	2297.196
From D to B	1983.26	2203.5	2195.53	2289.28	2524.02	2239.118
Global mean (m/sec)						2285.48
Standard deviation (m/sec)						169.08
Coefficient of variation (%)						7.40

Table 6: Results on P-waves velocity estimation for Conc2 material through the ultrasonic tests [m/sec]

Path	Test 1	Test 2	Test 3	Mean
From A to C	1843.22	1852.79	1858.07	1851.36
From B to D	1880.31	1883.44	1860.70	1874.82
From C to A	1850.30	1863.79	1877.44	1863.84
From D to B	1881.37	1875.24	-	1878.30
Global mean (m/sec)				1866.06
Standard deviation (m/sec)				14.13
Coefficient of variation (%)				0.76

Table 7: Results on R-waves velocity estimation for Conc2 material through the impact tests [m/sec]

	Path 1	Path 2	Path 3	Path 4	Mean
Side A	1149.2	1054.13	-	-	1101.67
Side B	961.85	992.77	-	-	977.31
Side C	945.35	1082.75	-	-	1014.05
Side D	1120.68	1037.39	-	-	1079.03
Upper side	983.46	982.75	976.07	936.61	969.72
Global mean (m/sec)					1028.36
Standard deviation (m/sec)					69.75
Coefficient of variation (%)					6.78

In Tables 5-7 the results obtained by impact tests and ultrasonic tests on the Conc2 material are summarized. As expected, the values of P- and R-wave velocities are substantially lower than those of Conc1 due to the material properties. It should be noted as, differently from the case of Conc1, the gap between the values of P-waves given by the impact tests and the ultrasonic tests increased up to about 20%. This fact should be related to the inner heterogeneity of the material, which could affect the measurements obtained by ultrasonic tests less reliable, as also asserted by [11].

As for the previous case, Figure 6 illustrates the frequency distribution histograms of the results obtained from the impact tests, both for the P-wave (Fig. 6a) and R-wave velocities (Fig. 6b), respectively, showing the dispersion level of the data, and Table 8 reports the mechanical properties of homogenized continuum associated to the heterogeneous medium Conc2. The material density  $\rho_2$  was directly calculated from a material sample:  $\rho_2 = 1237 \text{ kg/m}^3$ .

The results highlight a significant difference in the estimation of the Young modulus using the two methods, with a reduction of almost 35% obtained with ultrasonic tests with respect to the impact test method. This outcome is in agreement with the observations described in several papers, in which a lower reliability of the experimental data is obtained in the case of heterogeneous media [11].



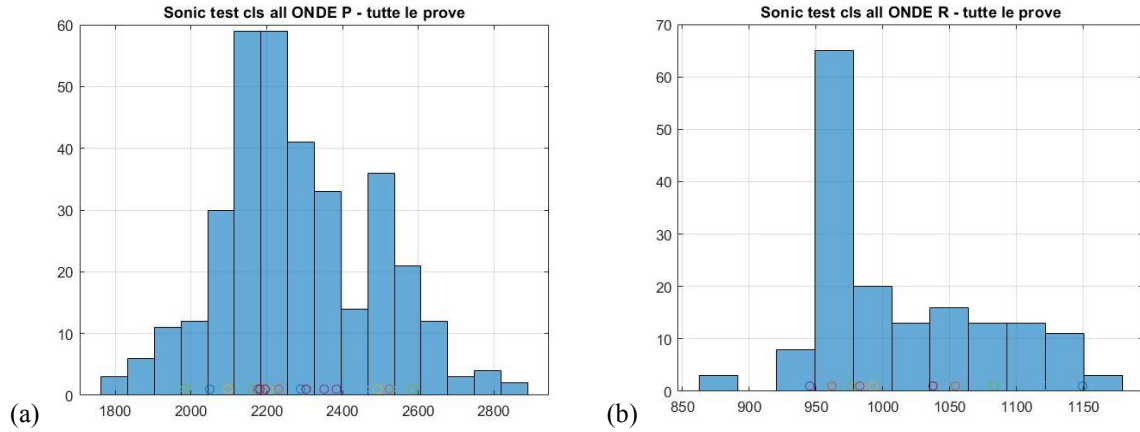


Figure 6: Frequency distribution histograms of the P-waves (a) and R-waves (b) obtained by the impact tests on the lightweight concrete material (Conc2).

Table 8: Estimation of the mechanical parameters of Conc2 through impact tests and ultrasonic tests. (Young modulus  $E$  [MPa] and Poisson ratio  $\nu$  [-])

		Mean	St. Dev.	Conf. Interval (95%)		
Impact test	$V_{P2}^S$	2285.48	169.08	2262.05	2308.92	$\bar{E}_2^S = 4039$ [3943 ÷ 4136]
	$V_{R2}^S$	1028.36	69.75	1015.88	1040.84	$\bar{\nu}_2^S = 0.35$ [0.34 ÷ 0.36]
Ultrasonic test	$V_{P2}^{us}$	1866.06	14.13	1861.18	1872.99	$\bar{E}_2^{us} = 2693$ [2679 ÷ 2713]

## 6 CONCLUSIONS

In this paper, the first results of a comparison between the elastic properties of two different materials estimated through ultrasonic tests and impact tests have been illustrated. In particular, the materials were characterized by standard concrete with small-size aggregates and lightweight concrete containing a 50% volume ratio of EPS spherical shape particles. Direct and indirect tests have been carried out on both materials. Using a statistical approach applied to the values of P- and R-wave velocities, the elastic properties of the equivalent homogenized continuum have been estimated. The experimental results have shown a good agreement of the results obtained by the two methods in the case of a material almost homogeneous, as for the case of the standard concrete, while for the case of heterogeneous material the two methods have provided significant differences in terms of wave velocities and related mechanical parameters estimation. Even if several papers in the literature point out a lower reliability of ultrasonic tests in the cases of heterogeneous media, a deeper study is necessary to achieve a consistent understanding of the observed phenomena.

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