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RELATIVE ENTROPY-BASED RELIABILITY ASSESSMENT OF HYBRID TELECOMMUNICATION SKELETAL TOWERS

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Abstract

This paper concerns an alternative methodology of reliability assessment in civil engineering structures based on the relative entropy idea. This relative entropy measures a distance between two random distributions describing an extreme structural effort and its resistance and may be used in time-dependent reliability problems, which is illustrated by dynamical excitation of the skeletal metal tower. The tower under consideration has hybrid character, which enables flexible mass minimization and it consists of upper aluminum part and lower segments made of steel. This structure has been modeled deterministically in the FEM system ABAQUS, while stochastic analysis based upon the generalized stochastic perturbation technique, has been programmed in the computer algebra package MAPLE. The relative entropy-based reliability indices have been compared with these obtained from the Eurocode FORM methodology while consecutively randomizing various parameters in this case study.

Keywords: Relative entropy, reliability assessment, hybrid telecommunication towers, stochastic perturbation method.

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1 INTRODUCTION

There is no doubt that numerical modeling and full-scale experiments with skeletal tower structures are still very challenging [1] in engineering practice. It is also well-known that one of the most dangerous loadings is in this particular case a dynamic wind pressure [2,3]. Usually, such towers are designed using different structural steels, but this is not always an optimal solution, because higher segments have their structural efforts rather distant from an optimum. A hybrid steel-aluminum telecommunications tower subjected to a dynamic wind pressure distribution having uncertain characteristic velocity is the subject of this study [4]. It is designed with lower segments manufactured with stainless steel and upper ones – with the use of some aluminum alloys, according to the patent of the first Author. Such a structural solution allows to reduce remarkably the overall weight of the tower in comparison to analogous, original steel tower of the same height and signal transmission functionality [5]. The basic Finite Element Method (FEM) model has been completed in commercial system ABAQUS® using displacement-based formulation, its 3D beam and truss linear elements, and also elastic supports. Dynamic response of the tower in the context of displacement and stresses histories has been determined numerically using the Hilber-Hughes-Taylor (HHT) [6] and contrasted with the results obtained from the classical Newmark scheme. Probabilistic analysis, which can be conducted in many ways [7], has been completed here using the iterative generalized stochastic higher-order perturbation technique, where dynamic structural polynomial response functions have been found by an application of the Least Squares Method (LSM) and special series of the FEM experiments. The LSM approximation procedures as well as all non-deterministic procedures have been implemented all in the environment of computer algebra system MAPLE 2020®. The expected values, coefficients of variation, skewness as well as kurtosis of extreme tower displacements and reduced stresses, where the first two of these parameters were used in the final reliability assessment. The following Gaussian random input parameters have been considered in this study: antenna masses, soil compliance, environmental temperature, structural round pipes thicknesses as well as wind characteristic velocity.

The basic engineering reliability analysis has been provided here with the use of the First Order Reliability Method (FORM) recommended by Eurocode 0 [8], and this approach has been contrasted with the probability distance model provided by Bhattacharyya [9]. Such a distance seems to be more appropriate measure to study the differences in-between admissible and extreme functions inherent in the serviceability and ultimate limit states, especially when the structural response exhibits non-Gaussian distribution. Nevertheless, some analytical expression similar to the FORM analysis has been provided. Considering the fact that such a probabilistic distance exhibits the same sensitivity with respect to the input uncertainty level as the FORM index but it shows totally different numerical values range, a special upscaling procedure has been successfully proposed. It has been demonstrated that the hybrid aluminum-steel towers are interesting alternative for traditional steel structures, and also that the probability distance approach proposed may be used instead of the FORM approach in many engineering reliability studies.

2 GOVERNING EQUATIONS

The reliability index $\beta=\beta(t)$ is investigated in this work for the solid mechanics problem governed by the following dynamic equilibrium equations system relevant to a domain Ω within the given time interval $[t_0,t_k]$:

- equation of motion

$$\rho \ddot{u}_i = \sigma_{ij,j} + \rho f_i, \quad \mathbf{x} \in \Omega, \quad t \in [t_0, t_k], \tag{1}$$

- constitutive equations

$$\sigma_{ij} = C_{ijkl} \varepsilon_{kl}, \quad \mathbf{x} \in \Omega, \quad t \in [t_0, t_k], \tag{2}$$

- geometrical equations

$$\varepsilon_{kl} = \frac{1}{2}\left(u_{i,j} + u_{j,i} + u_{k,i} u_{k,j}\right), \quad \mathbf{x} \in \Omega, \quad t \in [t_0, t_k], \tag{3}$$

- boundary conditions

$$\sigma_{ij} n_j = \hat{t}_i, \quad \mathbf{x} \in \partial\Omega_\sigma, \quad t \in [t_0, t_k], \tag{4}$$

$$u_i = \hat{u}_i, \quad \mathbf{x} \in \partial\Omega_u, \quad t \in [t_0, t_k], \tag{5}$$

- and also initial conditions

$$u_i = u_i^0, \quad \mathbf{x} \in \Omega, \quad t = t_0, \tag{6}$$

$$\dot{u}_i = \dot{u}_i^0, \quad \mathbf{x} \in \Omega, \quad t = t_0. \tag{7}$$

A notation within these equations is classical, so that displacements u_i and stresses σ_{ij} as well as their time-dependent behavior $u_i(t)$, $\sigma_{ij}(t)$ are determined.

The uncertainty sources consecutively considered in this study include a subsoil stiffness, elasticity modulus as well as environmental actions including external temperature constant for the entire structure as well as elevation-dependent dynamic wind pressure. They reflect a natural subsoil inhomogeneities, structural material imperfections and ageing, structural corrosion as well as unpredictable character of atmospheric conditions and/or wind short-period fluctuations. All structural elements are modelled according to the Euler-Bernoulli beam theory considering the fact that the tower is designed with the use of tubular structural elements only; so that further simplifications within the initial equations system (1-6) are applicable.

The reliability index analyzed here is some function of the first two probabilistic moments of the structural response, and the analysis is restricted to two cases – the ultimate limit state (ULS), and also serviceability limit state (SLS). These indices must fulfill the following inequality throughout the entire time domain:

$$\beta_{SLS} \geq \hat{\beta}_{SLS} \;\wedge\; \beta_{ULS} \geq \hat{\beta}_{ULS}, \quad \mathbf{x} \in \Omega, \quad t \in [t_0, t_k], \tag{8}$$

where $\hat{\beta}_{SLS}, \hat{\beta}_{ULS}$ stand for the admissible values in these two states recommended by the designing code Eurocode 0. The most popular approach is the First Order Reliability Method (FORM) and due to its theoretical basis it can be interpreted in this dynamic problem as

$$\beta(t)=\frac{E\left[R-E(t)\right]}{\sqrt{Var\left(R-E(t)\right)}},\quad \mathbf{x}\in\Omega,\quad t\in\left[t_0,t_k\right], \tag{9}$$

where E[.] and Var(.) denote the expected values and variances of the given response function, whereas R and $E(t)$ correspond to the structural resistance and extreme dynamical effort. This equation is most frequently represented in practical case studies assuming a lack of knowledge concerning any correlation functions in the following shorter form:

$$\beta(t)=\frac{E\left[R\right]-E\left[E(t)\right]}{\sqrt{Var(R)+Var\left(E(t)\right)}},\quad \mathbf{x}\in\Omega,\quad t\in\left[t_0,t_k\right]. \tag{11}$$

Some interesting alternative is the Bhattacharyya relative entropy measuring a probabilistic distance in-between two distribution functions (PDFs) of R and $E(t)$ in each discrete time moment. It can be represented under the same assumptions by the following relation:

$$\begin{aligned}H_B(t)=&\frac{1}{4}\frac{\left(E\left[R\right]-E\left[E(t)\right]\right)^2}{Var(R)+Var\left(E(t)\right)}\\&+\frac{1}{2}\ln\left(\frac{Var(R)+Var\left(E(t)\right)}{2\sqrt{Var(R)Var\left(E(t)\right)}}\right),\quad \mathbf{x}\in\Omega,\quad t\in\left[t_0,t_k\right]\end{aligned}. \tag{12}$$

This relative entropy (probabilistic divergence) may be relatively easily rescaled to the variability interval of the FORM index using a similarity in-between its first component and the FORM formula to be applicable in practical reliability assessment. The following rescaling procedure is proposed:

$$\beta'(t)\equiv\frac{\sqrt{H_B(t)}}{2},\quad \mathbf{x}\in\Omega,\quad t\in\left[t_0,t_k\right]. \tag{13}$$

The aforementioned equations system (1-6) have been discretized using classical procedure inherent in the Finite Element Method and the FEM system ABAQUS has been employed with its non-linear dynamic analysis, where integration of the equations of motion has been carried out by the Hilber-Hughes-Taylor (HHT) algorithm. Following solutions have been fount with full Newton method. Due to significant slenderness of the structure, and imposed mechanical nonlinearities (i.e. spring supports) some improvement of solution accuracy have been introduced by self-updating of stiffness matrix at each iteration. The HHT algorithm is based on time discretization shown in (2) and the governing equation of the method is shown in (1).

$$\mathbf{M}\ddot{x}_{n+1}+(1+\alpha)\mathbf{C}\dot{x}_{n+1}-\alpha\mathbf{C}\dot{x}_n+(1+\alpha)\mathbf{K}x_{n+1}-\alpha\mathbf{K}x_n=F(t_n+\alpha\Delta t), \tag{14}$$

$$\begin{cases}x_{i+1}=x_i+\Delta t\cdot\dot{x}_i+\left(1/2-\beta\right)\cdot\left(\Delta t\right)^2\cdot\ddot{x}_i\\ \dot{x}_{i+1}=\dot{x}_i+\left(1-\gamma\right)\cdot\Delta t\cdot\ddot{x}_i\end{cases} \tag{15}$$

The coefficients β and γ in (15) are directly associated with numerical damping of the HHT method. These parameters are related to the parameter α directly representing damping coefficient in the method, which can be taken from the interval [-0.3,0.0]. Lower bound on the parameter α corresponds to a greater numerical damping, which may prevent accuracy loss even with interferences of vibrations at higher frequencies. Their interrelations are proposed by the following formulas:

$$\beta = \frac{(1-\alpha)^2}{4}, \quad \gamma = \frac{1-2\alpha}{2} \tag{16}$$

Maximum time step of solution finding has been set hare as equal to 0.5 second (usually 1.0 sec is adopted) and the system ABAQUS automatically divides this maximum time step accordingly to iteration steps providing desired accuracy of the method. Also desired structural response values have been set to be recovered at strictly chosen time points of analysis which were every 0.5 s meaning finally for each desired structural response type there have been 1200 discrete values to be recovered.

3 NUMERICAL SIMULATION

Numerical models for both investigated types of skeletal towers have been all created in the system ABAQUS (see Fig. 1). The main legs and bracing elements have been modelled as linear beam elements in space named B31 finite elements. Model consists of 882 nodes and 996 finite elements resulting in total number of algebraic equations exceeding 5200. At the bottom of the skeletal tower spring pin supports have been prescribed involving some additional inter-relation of actions between foundation of skeletal tower and soil. A dynamic wind pressure has been adopted according to Fig. 2, while a detailed listing of all structural members have been contained in Table 1. Numerical solution of the reliability index FORM histories determined for various uncertainty sources has been contained in Fig. 3. Figs. 4 & 5 include a comparison of the FORM and entropy-based indices calculated while randomizing antenna masses in the ULS (extreme reduced von Mises stresses) and SLS (extreme horizontal displacement at the tower's top), respectively.

Within the scope of the simulations, the structure undergo dynamic wind excitation as stated before. Wind direction acting along the Z axis has been considered as it applies maximum compressive stress to only one leg of the tower whereas to other legs undergo tensile stress. This wind load has been applied as line load assigned to the main legs of the structure with its value gradually increasing along with the altitude of leg members of the tower [11,12]. Additionally, some point loads have been applied to the top nodes of the tower simulating the aerodynamic resistance of the telecommunication equipment (antennas). This antennas have been placed at the very top of the tower (all 3 nodes – 2 panel/sector antennas at each node) and also one level below (all 3 nodes – 1 parabolic antenna at each node). Naturally, dead load in form of self-weight of the structure have been considered in this work accordingly [8].

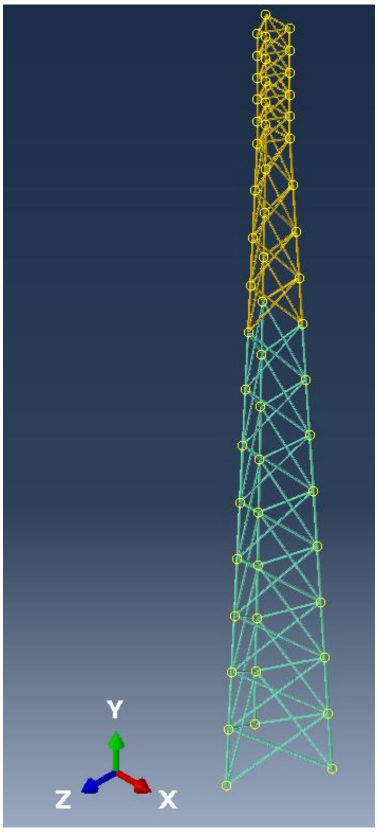

Fig. 1. Numerical model of the proposed hybrid steel-aluminum structure

Segment no.	Segment height	Legs section	Legs material	Bracing section	Bracing material
1	3.00 m	CHS 139.7x6.3	S355	L 120x80x8	S355
2	3.00 m	CHS 139.7x6.3	S355	L100x75x8	S355
3	3.00 m	CHS 127x6.3	S355	L100x75x8	S355
4	3.00 m	CHS 127x6.3	S355	L100x75x8	S355
5	3.00 m	CHS 127x5.6	S355	L90x60x8	S355
6	3.00 m	CHS 127x5.6	S355	L90x60x8	S355
7	3.00 m	CHS 108x5	S355	L60x60x5	S355
8	3.00 m	CHS 108x5	S355	L60x60x5	S355
9	2.50 m	CHS 110x6	AW-6061 T6	L80x80x6	AW-6061 T6
10	2.50 m	CHS 110x6	AW-6061 T6	L80x80x6	AW-6061 T6
11	2.50 m	CHS 110x4	AW-6061 T6	L80x80x6	AW-6061 T6
12	2.50 m	CHS 110x4	AW-6061 T6	L80x80x6	AW-6061 T6
13	1.20 m	CHS 90x5	AW-6061 T6	L80x80x6	AW-6061 T6
14	1.20 m	CHS 90x5	AW-6061 T6	L80x80x6	AW-6061 T6
15	1.20 m	CHS 90x5	AW-6061 T6	L80x80x6	AW-6061 T6
16	1.20 m	CHS 90x5	AW-6061 T6	L80x80x6	AW-6061 T6
17	1.20 m	CHS 90x5	AW-6061 T6	L80x80x6	AW-6061 T6
Weight of steel part (height 24.0 m)				4542 kg	
Weight of aluminum part (height 16.0 m)				622 kg	
Total weight of numerical model				5164 kg	

Table 1: Cross sections and materials of the aluminum skeletal tower

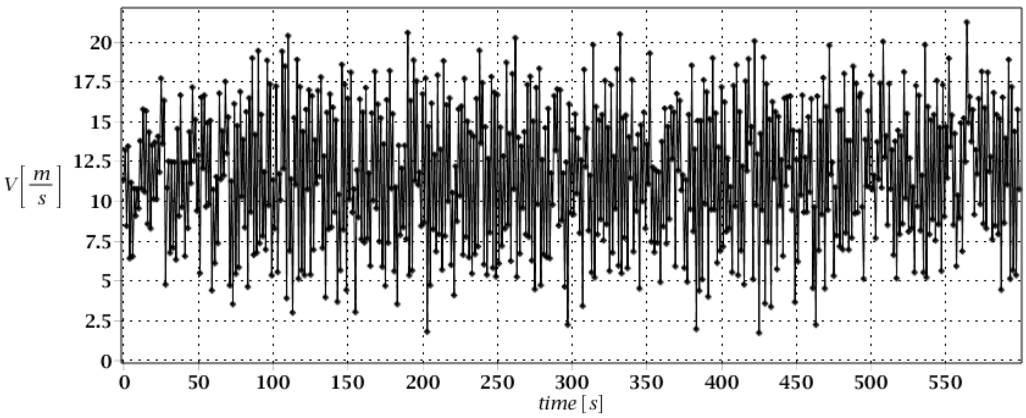

Fig. 2. Dynamic wind spectrum applied to the tower structure

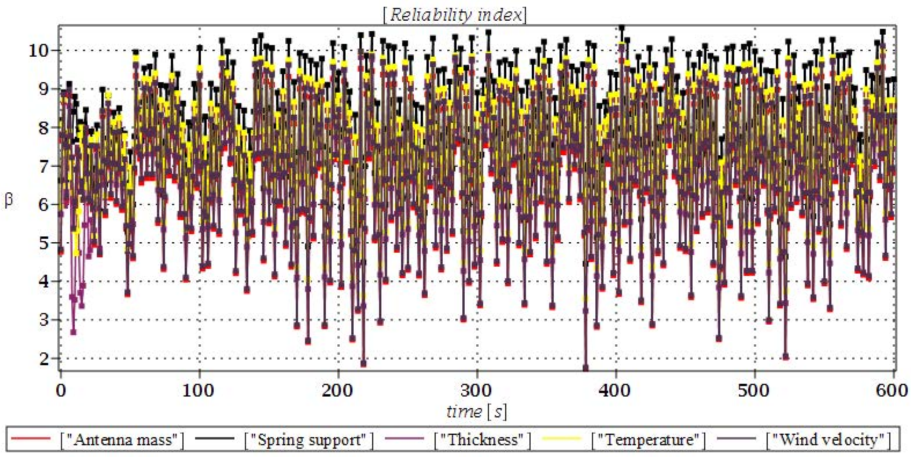

Fig. 3. Reliability indices of the hybrid tower due to various uncertainty sources

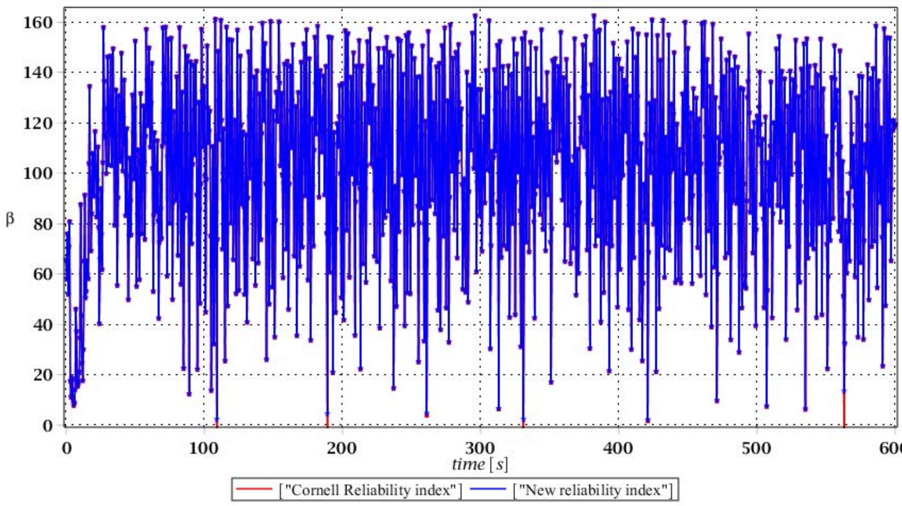

Fig. 4. A comparison of the FORM and relative entropy-based reliability indices for the SLS while randomizing antenna masses

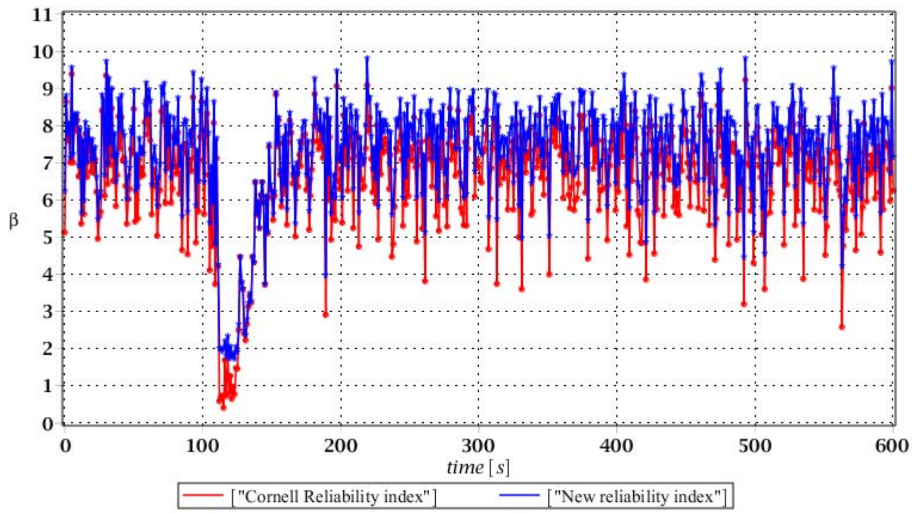

Fig. 5. A comparison of the FORM and relative entropy-based reliability indices for the ULS while randomizing antenna masses

Fig. 3 documents quite similar sensitivity of the structure towards proposed input uncertainty sources as dynamic reliability index spectrums corelates to one another greatly. It should be also noticed that reliability indices drop below theoretical admissible limit (equal to 3.8) which is stated for static ULS according to [8]. This strongly underlines that studies related to dynamic analysis of structural reliability should be forwarded as there is no such limit value stated explicitly for failure probability for dynamic ULS.

Figs. 4-5 visualize that proposed approach based on probabilistic entropy concept might be treated as successfully concurrent to the FORM indicated in [8] as dynamic reliability indices spectra follow the same distribution in time history and they both follow the characteristic temporary peak values of wind velocity. More importantly proposed probabilistic entropy-based approach slightly reduces the influence of the underestimation of reliability index which occurred in this study around 120 second of dynamic wind excitation. This phenomenon of local reliability index underestimation have been induced by the overestimation of output COV, which comes from the non-precise WLSM approximation of SRF estimation caused by partial ringing of the structure (numerical issues) which could not have been erased even by the maximum numerical damping of HHT method. It only has to be stated that such rough dynamic wind excitation as proposed in this work is rather extreme and wind velocities in time history naturally tend to vary within smaller amplitudes [13,14]. Nevertheless this only underlines that for any real-life scenario proposed approach based on probabilistic entropy concept might be implemented into real-life monitoring of the structures.

4 CONCLUDING REMARKS

Computer simulation presented in this study documents that mean wind velocity and principal structural element thickness have both paramount importance while analyzing reliability of such a structure, while antenna masses, external temperature as well as support stiffness are less influential here. It is also documented that rescaling procedure proposed for the Bhattacharyya entropy to the variability interval of the Cornell index is quite efficient, although predictions made on the basis of the traditional reliability assessment are less optimistic. From the engineering point of view it is proven that the hybrid tower structure is more optimal telecommunication solution as having more than 10% smaller mass and no necessity for corrosion protection of the upper part during its exploitation.

Further computational reliability study should be based upon an application of the shell finite elements discretization of the structural elements [15], and also usage of the 3D solid elements to model the bolts and the welds in-between the rebars and legs of the tower.

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REFERENCES

[1] J. Szafran, K. Juszczyk, M. Kamiński, Experiment-based reliability analysis of structural joints in a steel lattice tower. *Journal of Constructional Steel Research*, **154**, 278-292, 2019.

[2] G. Ballio, F. Maberini, G. Solari, A 60 year old, 100 m high steel tower: limit states under wind action. *Journal of Wind Engineering & Industrial Aerodynamics*, **43**(1-3), 2089-2100, 1992.

[3] W. Zhang, C. Li, G. Liu, Y. Xiao, Reconstruction of dynamic wind forces on a transmission steel lattice tower using aeroelastic wind tunnel test data. *Engineering Structures*, **275** A, 115167, 2023.

[4] M. Kamiński, Hybrid telecommunication tower. Patent PL no. 239992, Polish Patent Office, Warsaw, 2017.

[5] M. Kamiński, M. Solecka, Optimization of the truss-type structures using the generalized perturbation-based stochastic finite element method. *Finite Elements in Analysis and Design*, **63**, 69-79, 2013.

[6] H.M. Hilber, T.J.R. Hughes, R.L. Taylor, Improved numerical dissipation for time integration algorithms in structural dynamics. *Earthquake Engineering & Structural Dynamics*, **5**, 282-292, 1977.

[7] G. Stefanou, The stochastic finite element method: past, present and future. *Computer Methods in Applied Mechanics & Engineering*, **198**, 1031-1051, 2009.

[8] EN 1990:2002+A1, Basis of Structural Design. European Committee for Standardization, Brussels, 2002.

[9] R. Bredow, M. Kamiński, Structural safety of the steel hall under dynamic excitation using the relative probabilistic entropy concept. *Materials*, **15**(10), 3587, 2022.

[10] M. Kleiber, A. Siemaszko, R. Stocki, Interactive stability-oriented reliability-based design optimization. *Comput. Meth. Appl. Mech. & Engrg*. **168**, 243-253, 1999.

[11] Eurocode 1: Actions on structures – Part 1-4: General actions – Wind actions – European Committee for Standardization, Brussels, 2010

[12] Eurocode 3: Design of steel structures – Part 3-1: Towers, masts and chimneys – Towers and masts – European committee for Standardization, Brussels, 2006.

[13] W.H. Hu et al., Vibration-based structural health monitoring of a wind turbine system. Part I: Resonance phenomenon, *Engineering Structures*, **89**, 260-272, 2015.

[14] J.Y. Kim et. al., Long-term monitoring of wind-induced responses of a large-span roof structure, *Journal of Wind Engineering and Industrial Aerodynamics*, **99**, 955-963, 2011.

[15] J. Argyris, M. Papadrakakis, G. Stefanou, Stochastic finite element analysis of shells. *Computer Methods in Applied Mechanics & Engineering*, **191**, 4781-4804, 2002.