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BRIDGE MODEL UPDATING USING MONITORED SEISMIC RESPONSES: INSIGHTS FROM A NUMERICAL PARAMETRIC STUDY

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Abstract. *Model updating is a critical process in various branches of engineering, particularly in civil engineering, and plays a key role in areas such as structural health monitoring (SHM), risk assessment, decision-making, and vulnerability analysis. By integrating observed data into numerical models, engineers can refine key damage-sensitive model parameters, thereby improving the assessment of the reliability and safety of existing structures. However, many model updating methodologies, whether probabilistic or deterministic, face significant challenges. One of the main difficulties is the lack of uniqueness in the inverse calibration problem (or system identification), especially when dealing with large-scale models with various fitting parameters and limited experimental observations. Factors such as the selection of appropriate objective functions, constraints, and penalty functions in deterministic methods, and the choice of likelihoods and priors in stochastic (Bayesian) approaches, as well as decisions between using different modeling approaches such as finite element, fiber, or discrete element models, can all affect the accuracy and efficiency of the model updating process. In this research, a specific case of model updating is addressed: bridge model updating using monitored seismic responses. The core idea is that earthquake-induced responses provide a natural avenue for refining estimates of structural parameters as more seismic events are recorded. By formulating the cost function as the error between monitored and simulated responses in the time domain, the updating process can capture not only linear but also nonlinear behavior, even at intensity levels below ultimate limit states. To explore this approach further, a comparative study is conducted to evaluate the strengths and limitations of various numerical techniques. A case study involving a reinforced concrete (RC) cantilever bridge pier is presented, in which an accelerometer is deployed to update specific model parameters based on seismic response records.*

Keywords: Model updating, Bayesian model updating, Stochastic methods, Metaheuristic algorithms, Markov Chain Monte Carlo, System identification

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1 INTRODUCTION

Finite Element Model Updating (FEMU) has become a foundational tool in civil engineering to enhance the reliability of numerical simulations of structural behavior by aligning model predictions with observed responses. Applications of FEMU range from structural health monitoring (SHM) and damage detection to seismic vulnerability assessment and lifecycle reliability analysis. By calibrating uncertain parameters—such as stiffness, mass density, damping, and material properties—FEMU helps reduce parameter uncertainty and modeling errors, improve decision-making, and increase confidenc in structural evaluations under both service and extreme conditions [1].

A critical challenge in FEMU is the *non-uniqueness of the inverse problem*, which refers to the fact that multiple combinations of parameter values may produce similar structural responses. This ill-posed nature is exacerbated when updating large-scale or nonlinear models with limited or noisy measurement data [1]. Ill-conditioning and identifiabilit issues can lead to unrealistic parameter estimates, undermining the reliability of the updated model. Several studies have attempted to address this problem by introducing regularization methods [3], model reduction techniques [2], and sensitivity-based parameter subset selection [1, 5].

According to the classificatio presented by Ereiz et al. [1], FEMU approaches can be broadly divided into *deterministic* and *stochastic (probabilistic)* methods. Deterministic methods treat model updating as a single-objective or multi-objective optimization problem, where the cost function—typically define in terms of discrepancies between observed and simulated data—is minimized using optimization techniques such as gradient-based methods or metaheuristics like Genetic Algorithms (GAs) [6]. These methods are often computationally efficien but provide no insights into the uncertainty of the estimated parameters. In contrast, *stochastic methods*, rooted in Bayesian inference, aim to estimate the full posterior probability distribution of the model parameters by incorporating prior knowledge and measured data. Techniques such as Markov Chain Monte Carlo (MCMC) sampling offer rigorous uncertainty quantifi ation but are often computationally intensive [3, 4].

Previous works in the literature have investigated ways to *improve accuracy, reduce computational cost, and quantify uncertainty* of both deterministic and probabilistic approaches. For deterministic approaches, researchers have developed hybrid optimization methods, regularization strategies, and multi-objective formulations to enhance convergence and robustness [7, 6]. Probabilistic studies, on the other hand, have employed advanced Bayesian techniques such as Transitional MCMC and approximate Bayesian comutation methods to reduce computational demand while maintaining rigorous uncertainty estimation [8, 9]. These studies also highlight how factors like data quality, parameter sensitivity, and prior distribution assumptions can affect the outcome of the updating process.

In this paper, we build upon these foundations by conducting a *comparative study of deterministic and probabilistic model updating strategies* applied to a *reinforced concrete (RC) cantilever bridge pier* subjected to seismic excitation and considering the nonlinear structural response as model evidence. Using acceleration time-history data collected at the pier's free end, we update three key structural parameters influencin the nonlinear behavior. The deterministic FEMU is performed using a *GA*, while the probabilistic update is implemented via *MCMC*, both coupled with *OpenSees* for nonlinear time history analysis (NLTHA). The general framework for model updating is implemented within the MATLAB R2024 programming environment, enabling data handling, optimization, and numerical simulation workfl ws. Our goal is to assess and compare the *accuracy, efficiency* of both approaches in a practical, earthquakeexcited structural scenario.

The remainder of this paper is structured as follows: Section 2 presents the theoretical formulation and general framework of the utilized methods; Section 3 describes the RC bridge pier case study and the FEMU workfl w and discusses the results and key finding from both deterministic and stochastic approaches; and Section 4 closes the study with concluding remarks and future research directions.

2 Structural Dynamic Model Updating using vibration measurements

This study aims to compare the accuracy and computational efficien y of multi-variable FEMU strategies for RC structures exhibiting nonlinear behavior under seismic excitation, employing both probabilistic (stochastic) and deterministic approaches. The general objective is to update certain selected model parameters to minimize the discrepancy between the measured vibration response and the numerically simulated response of the structure under dynamic loading.

The nonlinear finit element (FE) model of the structure under investigation is governed by a collection of uncertain parameters. These include, but are not limited to, mass and inertia properties, applied gravitational loads, geometric dimensions, boundary and constraint conditions, damping characteristics, and parameters definin the nonlinear material behavior. For brevity, these will collectively be referred to as model parameters. It is assumed that selected parameters for updating are time-invariant throughout the duration of the seismic event. Two distinct model updating strategies are explored: a deterministic approach using a GA and a stochastic approach using Bayesian inference via four different algorithms.

2.1 Nonlinear FEMU with GA (deterministic approach)

In the deterministic model updating framework, the problem is formulated as an optimization task in which the optimal set of model parameters $\boldsymbol{\theta}^*$ minimizes an objective function $J(\boldsymbol{\theta}, \mathbf{x})$, define as the mean squared error between the measured and simulated structural responses:

$$\boldsymbol{\theta}^* = \arg\min_{\boldsymbol{\theta}} J(\boldsymbol{\theta}, \mathbf{x}) = \arg\min_{\boldsymbol{\theta}} \mathbb{E}[\|\mathbf{y}_{\text{meas}} - \mathbf{y}(\boldsymbol{\theta}, \mathbf{x})\|^2] \tag{1}$$

where $\mathbf{x}$ denotes the non-updated parameters influencin the nonlinear structural response, such as the ground acceleration time history, initial displacements, and velocities. $\boldsymbol{\theta} \in \mathbb{R}^n$ represents the vector of model parameters to be identifie (e.g., material properties such as concrete elasticity modulus, steel yield strength, or damping ratios), $\mathbf{y}_{\text{meas}} \in \mathbb{R}^m$ denotes the vector of experimentally measured structural responses (e.g., accelerations), and $\mathbf{y}(\boldsymbol{\theta}, \mathbf{x}) \in \mathbb{R}^m$ is the corresponding vector of simulated responses obtained from the FE model given the parameter set $\boldsymbol{\theta}$. The objective function $J(\boldsymbol{\theta}, \mathbf{x})$ corresponds to the expected mismatch between the observed and simulated data, with $\mathbb{E}$ in Eq. (1) representing the expectation operator.

To solve this optimization problem, a GA is employed. GA is a population-based, evolutionary metaheuristic inspired by the principles of natural selection. It does not rely on gradient information, which makes it particularly suitable for problems characterized by nonlinearity, nonconvexity, or noisy measurements, as commonly encountered in structural dynamic analyses under seismic excitation. It iteratively evolves a population of candidate solutions across successive generations to minimize a predefine fitnes (objective) function. The workfl w based on the GA, is illustrated schematically in the fl wchart presented in Fig. 1. The deterministic FEMU procedure using GA proceeds as follows:

1. Initialize a population of N_p individuals (parameter vectors) uniformly distributed within predefine lower and upper bounds for each component of $\boldsymbol{\theta}$.

2. For each individual:

   - Conduct a nonlinear time-history dynamic analysis using the OpenSees platform.
   - Extract the acceleration response at the monitored degree of freedom(s).
   - Evaluate the objective function value $J(\boldsymbol{\theta}, \mathrm{x})$ in Eq. (1) based on the discrepancy with the corresponding measured data.

3. Apply selection, crossover, and mutation operators to generate a new population, and guide the evolutionary process toward convergence. Selection promotes the survival of fitte individuals, while crossover and mutation introduce diversity and exploration within the parameter space.

4. Repeat the evaluation and evolution steps until a convergence criterion is met (e.g., maximum number of generations or fitnes stagnation threshold).

Due to the high computational demand of nonlinear dynamic simulations, particularly when evaluating large populations over many generations, parallel computing techniques are employed to accelerate the optimization process. Specificall , MATLAB' s `parfor` loop structure is utilized to evaluate the fitnes of individuals concurrently. Each GA population member is simulated independently via OpenSees on separate CPU cores, enabling substantial reductions in total computational time.

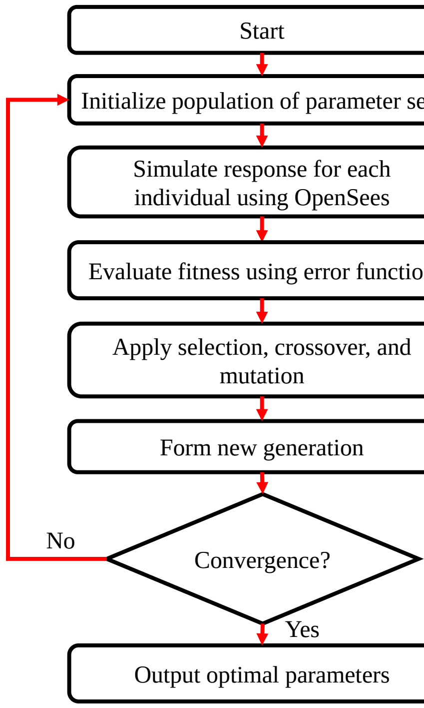

Figure 1: Deteministic FEMU with GA and OpenSees

2.2 Nonlinear FEMU with MCMC (probabilistic approach)

MCMC methods are widely used in Bayesian inference to sample from complex posterior distributions where analytical solutions are intractable. In the context of *FEMU* based on vibration measurements, MCMC provides a powerful tool for estimating the posterior distribution of uncertain model parameters $\boldsymbol{\theta}$ given observed dynamic response data $\mathbf{y}_{\text{meas}}$.

The Bayesian formulation treats the model parameters as random variables and updates their distribution based on measured data. Bayes' theorem define the posterior distribution as:

$$p(\boldsymbol{\theta}|\mathbf{y}_{\text{meas}}) = \frac{p(\mathbf{y}_{\text{meas}}|\boldsymbol{\theta}) \cdot p(\boldsymbol{\theta})}{p(\mathbf{y}_{\text{meas}})} \tag{2}$$

where:

- $p(\boldsymbol{\theta})$ is the prior distribution reflectin prior knowledge or assumptions about the model parameters,
- $p(\mathbf{y}_{\text{meas}}|\boldsymbol{\theta})$ is the likelihood function, which measures how well the parameters $\boldsymbol{\theta}$ explain the observed vibration data,
- $p(\mathbf{y}_{\text{meas}})$ is the marginal likelihood or evidence, typically unknown and not needed in MCMC.

The posterior distribution $p(\boldsymbol{\theta}|\mathbf{y}_{\text{meas}})$ encapsulates the updated knowledge about the parameters after incorporating the measurements. MCMC methods allow for sampling from this posterior without explicitly computing the evidence. Assuming normally distributed errors in the model prediction, the likelihood is expressed as:

$$p(\mathbf{y}_{\text{meas}}|\boldsymbol{\theta}) \propto \exp\left(-\frac{1}{2}(\mathbf{y}_{\text{meas}} - \mathbf{y}(\boldsymbol{\theta}, \mathbf{x}))^\top \boldsymbol{\Sigma}^{-1}(\mathbf{y}_{\text{meas}} - \mathbf{y}(\boldsymbol{\theta}, \mathbf{x}))\right), \tag{3}$$

where $\mathbf{y}(\boldsymbol{\theta}, \mathbf{x})$ denotes the FE model response, and $\boldsymbol{\Sigma}$ is the covariance matrix representing the measurement noise and modeling uncertainty. The value of Σ plays a critical role in shaping the likelihood function, effectively controlling how strongly the data influenc the posterior distribution. The sampling of the posterior distribution using a MCMC method enables not only parameter estimation but also the quantificatio of uncertainty associated with the identifie parameters.

2.3 MCMC Techniques for Bayesian FEMU

This section presents the theoretical formulation and salient features of four different MCMC techniques which will be comparatively evaluated in the subsequent sections. These techniques include the classical Metropolis-Hastings (MH) algorithm [13, 14], the Adaptive Metropolis (AM) algorithm [11], the Delayed Rejection (DR) method [12], and the Delayed Rejection Adaptive Metropolis (DRAM) algorithm [15], which combines the features of DR and AM.

2.3.1 Metropolis-Hastings (MH) Algorithm

The Metropolis-Hastings (MH) algorithm (based on the work of Metropolis et al. [13] and later Hastings [14]) is the cornerstone of MCMC sampling. It generates a Markov chain whose stationary distribution converges to the target posterior distribution $p(\boldsymbol{\theta}|\mathbf{y}_{\text{meas}})$, where $\boldsymbol{\theta}$ denotes the model parameters and $\mathbf{y}_{\text{meas}}$ the observed data. Given a current state $\boldsymbol{\theta}_k$, a new candidate $\boldsymbol{\theta}^*$ is drawn from a proposal distribution $q(\boldsymbol{\theta}^*|\boldsymbol{\theta}_k)$. The target distribution, commonly denoted as $\pi(\boldsymbol{\theta})$ in the MCMC literature, typically represents the posterior distribution of the model parameters given the observed data, i.e., $\pi(\boldsymbol{\theta}) = p(\boldsymbol{\theta} \mid \mathbf{y}_{\text{meas}})$ [18]. The acceptance probability α is computed as:

$$\alpha(\boldsymbol{\theta}_k, \boldsymbol{\theta}^*) = \min\left(1, \frac{\pi(\boldsymbol{\theta}^*)q(\boldsymbol{\theta}_k|\boldsymbol{\theta}^*)}{\pi(\boldsymbol{\theta}_k)q(\boldsymbol{\theta}^*|\boldsymbol{\theta}_k)}\right) \tag{4}$$

The candidate $\boldsymbol{\theta}^*$ is accepted with probability α, otherwise the chain remains at $\boldsymbol{\theta}_k$. The algorithm proceeds as follows:

Initialization $(i = 0)$

- Choose an initial guess $\boldsymbol{\theta}^{(0)}$
- Set Σ
- Select a proposal distribution q
- Set Priors

Iterative Sampling

- **Proposal:** Generate a candidate sample $\boldsymbol{\theta}^*$ from a Gaussian proposal distribution $q(\boldsymbol{\theta}^*|\boldsymbol{\theta}_k)$
- **Acceptance Ratio:** Compute the Metropolis-Hastings acceptance ratio in Eq. (4)
- **Acceptance Step:** Accept the new sample with probability α, i.e.,

$$\boldsymbol{\theta}^{(i+1)} = \begin{cases} \boldsymbol{\theta}^* & \text{with probability } \alpha \\ \boldsymbol{\theta}_k & \text{otherwise} \end{cases}$$

- Increment $k \leftarrow k + 1$ and repeat.

The likelihood function $p(\mathbf{y}_{\text{meas}}|\boldsymbol{\theta})$ is constructed based on the discrepancy between simulated and observed structural response (e.g., acceleration), which can be modeled using the Gaussian error model in Eq. (3).

After sufficien iterations, the Markov chain generated by the MH algorithm converges to the posterior distribution. The samples can then be used to estimate posterior statistics such as the mean, variance, credible intervals, and to perform sensitivity or uncertainty analysis. Diagnostic tools such as trace plots, acceptance rates, and autocorrelation functions are commonly used to assess convergence and mixing quality of the chain. A fl wchart of MH is represented in Fig. 2

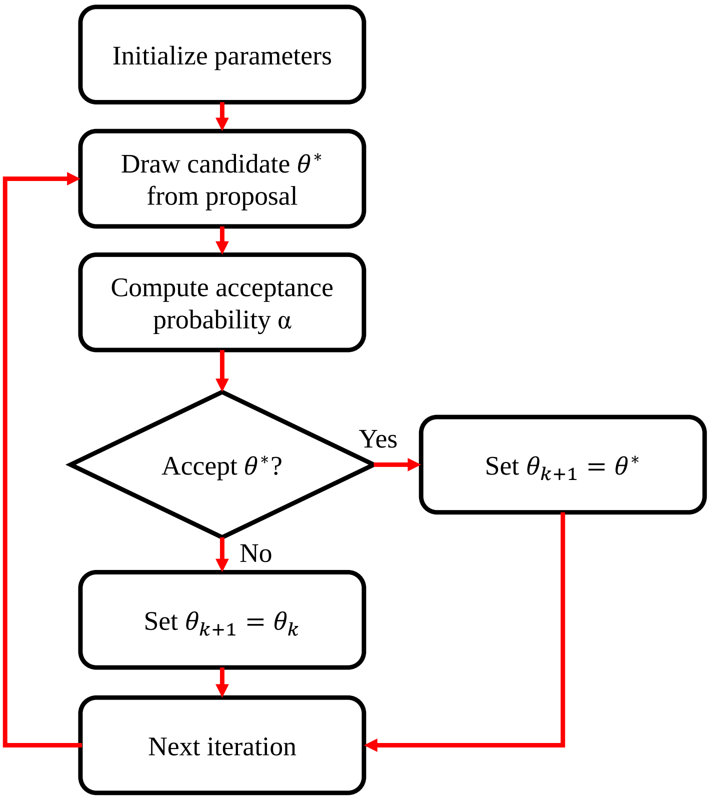

Figure 2: Metropolis-Hastings (MH) Algorithm

2.3.2 Adaptive Metropolis (AM)

Although MH is simple and robust, its performance can degrade in high-dimensional or highly correlated parameter spaces due to ineffic ent exploration. To alleviate some of these limitations, an Adaptive Metropolis (AM) algorithm [11] enhances the classical MH by allowing the proposal distribution to adapt during the sampling process. The proposal covariance matrix is updated online using the empirical covariance of the samples in the Markov chain. This adaptation aims to improve the efficien y and convergence of the chain by tuning the proposal to the shape of the target distribution. At iteration k, the candidate θ^* is drawn from a multivariate normal proposal distribution:

$$\boldsymbol{\theta}^* \sim \mathcal{N}(\boldsymbol{\theta}_k, s_d\Sigma_k), \tag{5}$$

where Σ_k is the empirical covariance matrix computed from $\{\theta_0, \ldots, \theta_{k-1}\}$, and s_d is a scaling parameter typically set as $s_d = 2.4^2/d$, with d being the dimension of the parameter space [17]. The adaptation is performed only after a burn-in period to ensure convergence properties.

2.3.3 Delayed Rejection (DR)

The Delayed Rejection (DR) technique [12] is an extension of MH that aims to improve acceptance rates by allowing a second (or more) proposal if the firs one is rejected. Instead of remaining at $\boldsymbol{\theta}_k$ when $\boldsymbol{\theta}^*$ is rejected, a new candidate $\boldsymbol{\theta}^{**}$ is proposed using a different, usually narrower, proposal distribution. The first-stag acceptance probability is the presented in Eq. 4.

If $\boldsymbol{\theta}^*$ is rejected, the second-stage candidate $\boldsymbol{\theta}^{**}$ is sampled from $q_2(\boldsymbol{\theta}^{**}|\boldsymbol{\theta}_k, \boldsymbol{\theta}^*)$ with acceptance probability:

$$\alpha_2(\boldsymbol{\theta}_k, \boldsymbol{\theta}^*, \boldsymbol{\theta}^{**}) = \min\left(1, \frac{\pi(\boldsymbol{\theta}^{**})q_1(\boldsymbol{\theta}^*|\boldsymbol{\theta}^{**})q_2(\boldsymbol{\theta}_k|\boldsymbol{\theta}^{**}, \boldsymbol{\theta}^*)\left[1 - \alpha_1(\boldsymbol{\theta}^{**}, \boldsymbol{\theta}^*)\right]}{\pi(\boldsymbol{\theta}_k)q_1(\boldsymbol{\theta}^*|\boldsymbol{\theta}_k)q_2(\boldsymbol{\theta}^{**}|\boldsymbol{\theta}_k, \boldsymbol{\theta}^*)\left[1 - \alpha_1(\boldsymbol{\theta}_k, \boldsymbol{\theta}^*)\right]}\right) \tag{6}$$

This approach can significantl increase the efficien y of the sampling process, especially in complex target distributions with narrow high-probability regions.

2.3.4 Delayed Rejection Adaptive Metropolis (DRAM)

The DRAM algorithm [15] combines the adaptivity of AM with the multiple proposal strategy of DR, aiming to achieve both fast convergence and high acceptance rates. The adaptation of the proposal distribution is performed in conjunction with the delayed rejection steps, allowing the algorithm to efficientl explore the parameter space. Key features of DRAM include:

- Online adaptation of the covariance matrix to fi the posterior distribution.
- Multiple proposal stages to avoid stagnation and improve acceptance.
- Capability to efficientl handle correlated or multimodal posteriors.

The DRAM algorithm has been shown to perform better than MH, AM, or DR individually, particularly in Bayesian FEMU problems involving high-dimensional and poorly scaled parameter spaces. A fl wchart illustrating the workfl w of DRAM is illustrated in Fig. 3

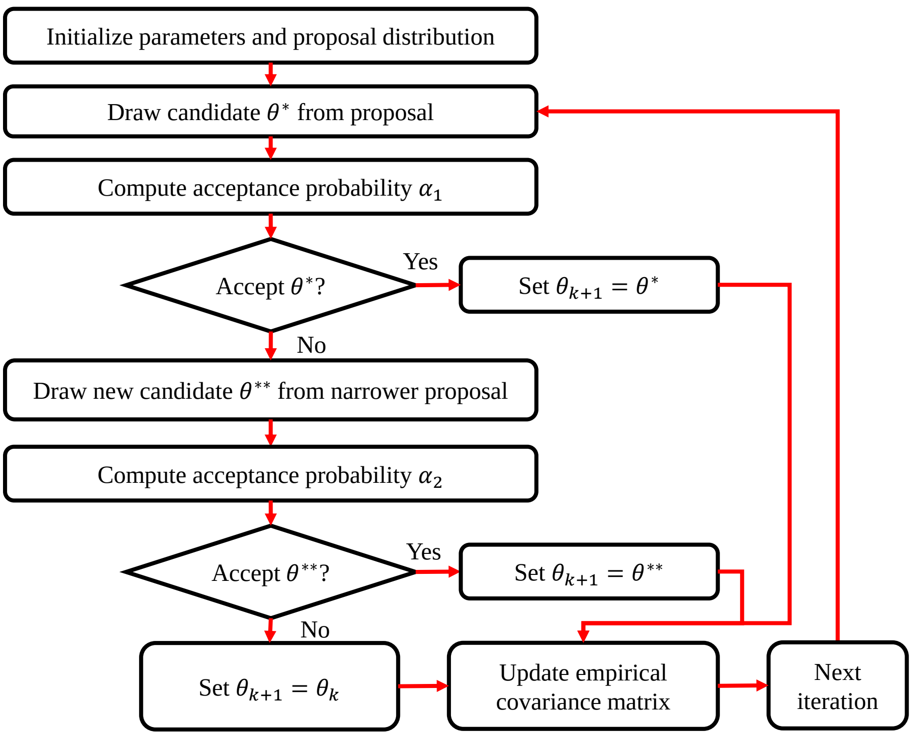

Figure 3: Delayed rejection adaptive metropolis (DRAM) algorithm.

3 Numerical example and discussion

To compare the effectiveness of the model updating approaches presented in the previous sections, a circular RC bridge pier is selected as a case study. This choice is motivated by the widespread use of such piers in bridge structures located in seismically active regions worldwide [16]. The geometric layout and material properties of the pier are illustrated in Fig. 4.

The mass contributing to the computation of inertial forces is define by accounting for both the dead load of the superstructure (including the deck and cap beam), modeled as a lumped mass of 113,456 kg, and the self-weight of the pier, modeled as a distributed mass of 1,672 kg/m. The pier is fi ed at the base, representing a cantilever configuration and the P–Δ effect is included in the analysis to account for geometric nonlinearity.

For the dynamic excitation, an ground acceleration from the 1994 Northridge earthquake (station: Castaic - Old Ridge Route/360°, 6.4 Ml, 4:30:55 AM PST, 17 Jan 1994) [10] is applied. This ground motion is selected to ensure that the pier, when modeled with its initial (true) parameters, exhibits nonlinear plastic behavior. A NLTHA is then performed, and the resulting acceleration response at the top of the pier is considered the reference output in this analysis. To simulate realistic measurement conditions, artificia noise is added to the acceleration response, with a standard deviation corresponding to 5% of gravity. The generated time series data is considered henceforth as synthetic experimental data.

In the model updating process, three parameters are selected for calibration: the elastic modulus of concrete E_c, the yield strength of reinforcing steel f_y, and the damping ratio ζ for the firs elastic mode (T1: 0.5733 sec). The assumed true values for E_c, f_y, and ζ are 2.5×10^{10} Pa, 4.6×10^{8} Pa, and 5%, respectively. For both deterministic FEMU, where the bounds act as constraints, and probabilistic FEMU, where they defin the limits of uniform prior distributions, the following parameter ranges are adopted: E_c varies between 2.13×10^{10} and 2.88×10^{10} Pa; f_y ranges from 4.23×10^{8} to 4.97×10^{8} Pa; and ζ is considered within the interval 3.60%, and 6.40%.

The hysteretic behavior of the pier, accelerogram, acceleration and displacement response (at free end) are shown in Fig. 5, and as it can be noted in this figur that the pier experienced nonlinearity during this ground motion.

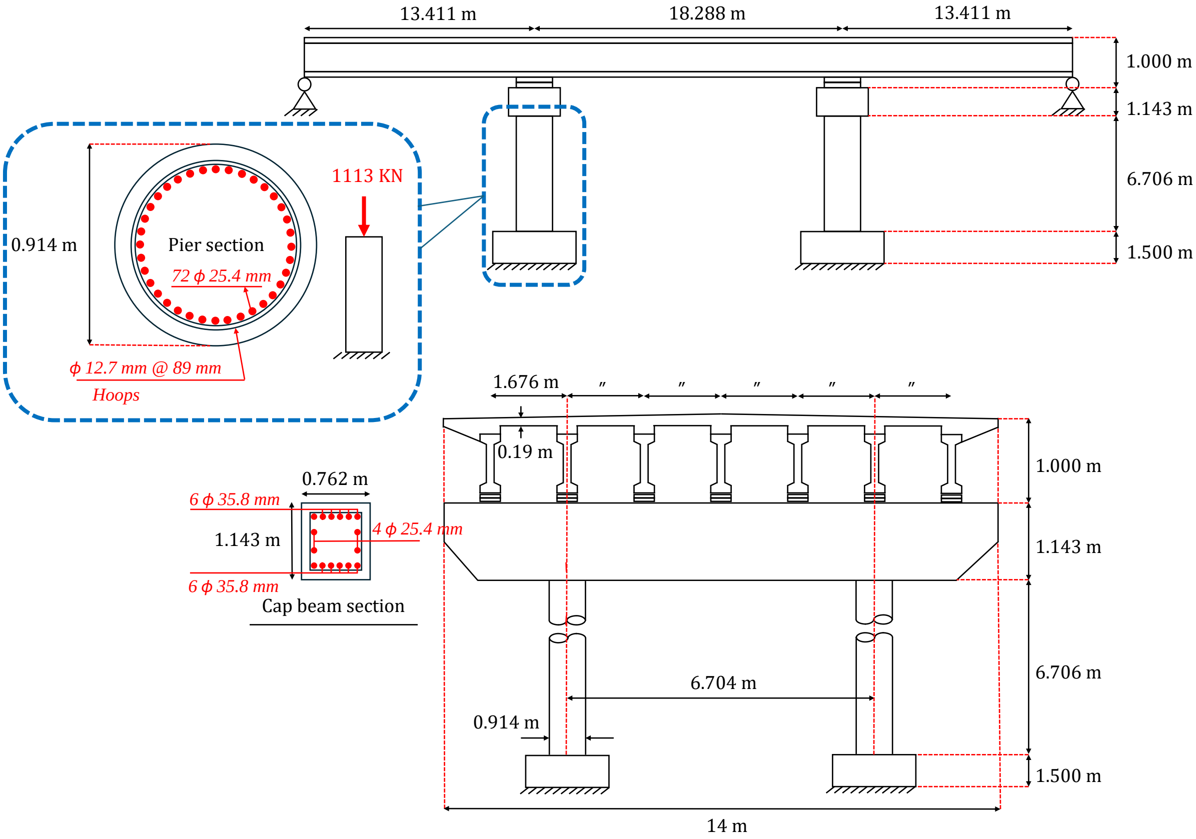

Figure 4: Considered layout of multispan continuous concrete I-girder bridge

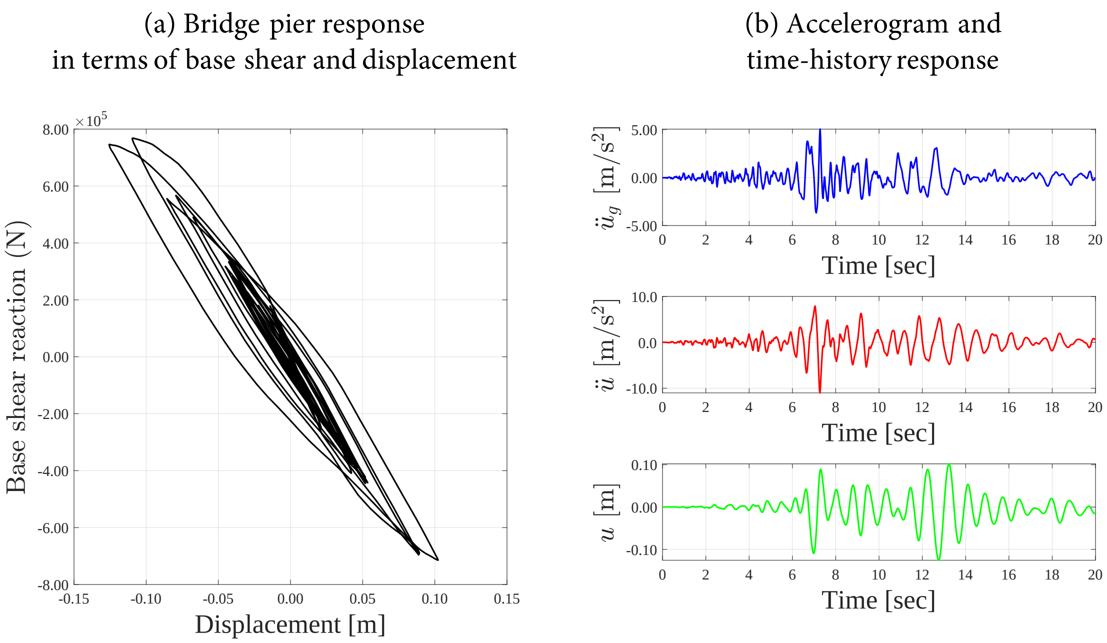

Figure 5: Bridge pier response to 1994 Northridge earthquake

3.1 Deterministic FEMU results

The results in Table 1 summarize the performance of a deterministic FEMU approach using a GA for two different population sizes, namely a population of 20 (P20) and 50 (P50). The optimal solutions obtained with the larger population size (P50) show significantl better accuracy compared to those with P20, as evidenced by the lower estimation errors across all parameters. In particular, the estimation error for ζ drops from 6.4% to just 0.4%, highlighting the sensitivity of this parameter to the population size in the optimization process. The estimation error for each model parameter is computed as the absolute difference between the estimated value and the true value, normalized by the true value and expressed as a percentage.

The objective function values in the fina generation further confir the improvement in solution quality with a larger population. Both the mean and best values of the objective function are lower for P50, indicating better convergence behavior. This suggests that increasing the population size enhances the global search ability of the GA, reducing the risk of premature convergence and allowing more accurate identificatio of the true model parameters.

However, this improved accuracy comes at the cost of higher computational time. The stopping criteria were set to 100 generations for P20 and 40 generations for P50. The total runtime increased from 6,974 seconds for P20 to 8,504 seconds for P50. This increase is attributed to the added computational complexity and overhead associated with evaluating a larger population in each generation of the GA. The implementation of parallel computing significantl reduces the computational time, achieving a speedup of more than 14 times for both the P20 and P50 cases as shown in Table 1. Fig. 6 presents the convergence behavior of both the mean and best values of the objective function for the deterministic approach. The results clearly indicate that the P50 case yields lower objective function values in both metrics compared to the P20 case. Furthermore, Fig. 7 illustrates the convergence of the estimated model parameters. A comparison with the true values demonstrates that, across all three parameters, the P50 case provides more accurate estimations than P20.

Table 1: Results of nonlinear FEMU with GA

	Population	
	P20	P50
Optimal solution		
E_c [Pa]	2.4451×10^{10}	2.5272×10^{10}
f_y [Pa]	4.6474×10^{8}	4.5905×10^{8}
ζ [%]	5.32	5.02
Estimation error		
E_c [%]	2.2000	1.0900
f_y [%]	1.0300	0.2070
ζ [%]	6.4000	0.4000
Objective function in last generation		
Mean value $(\mathrm{m}^2/\mathrm{s}^4)$	3.5864×10^{3}	2.8278×10^{3}
Best value $(\mathrm{m}^2/\mathrm{s}^4)$	3.3472×10^{3}	2.7668×10^{3}
Computational cost		
Time (sec)	6974	8504
Time (with parallel computation, 7 cores) (sec)	484	590

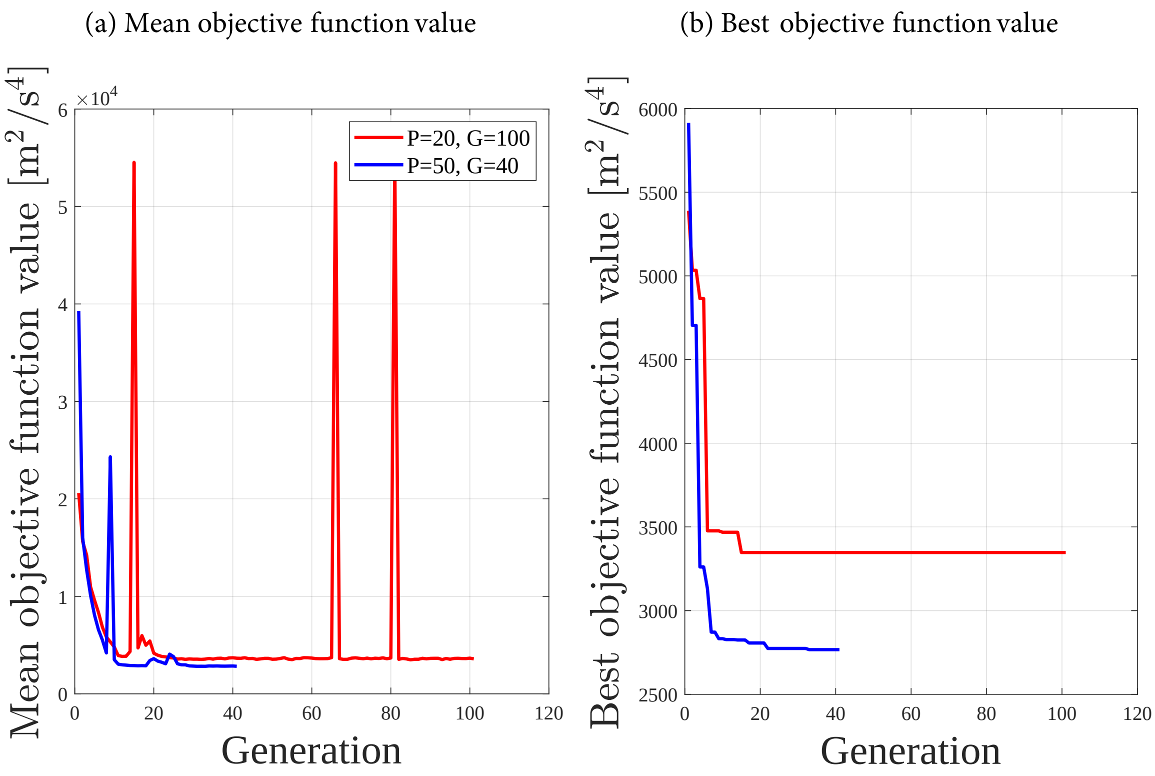

Figure 6: Objective function convergence in deterministic approach

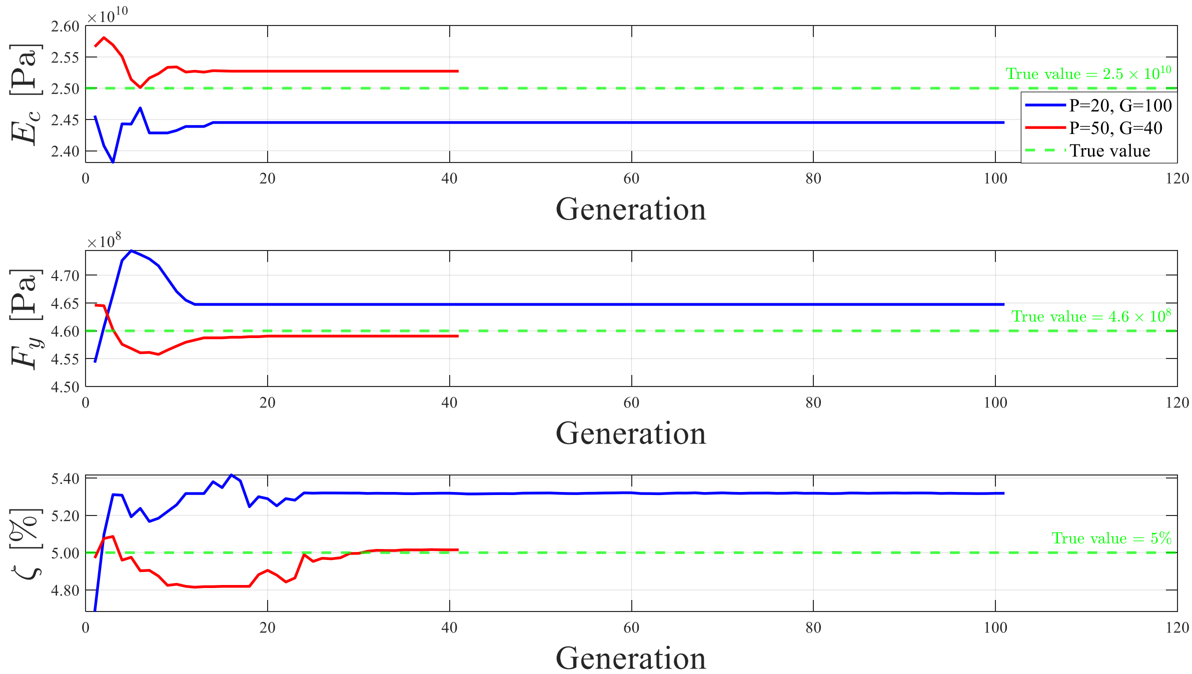

Figure 7: Convergence of model parameters in deteministic approach

3.2 Probabilistic FEMU results

The probabilistic FEMU results using MCMC algorithms (Table 2) demonstrate that all four methods – MH, DR, AM, and DRAM – produce nearly identical mean estimates for the key parameters: Young's modulus ($E_c \approx 2.49$–2.50×10^{10} Pa), steel yield stress ($f_y \approx 4.58$–4.60×10^{8} Pa), and damping ratio ($\zeta \approx 0.0502$–0.0506). This consistency across algorithms indicates robust convergence in the solution space for this class of inverse problems.

However, significan differences emerge in computational efficien y, with AM being the fastest (23,460 s), MH moderately efficien (27,247 s), and both DR and DRAM requiring approximately twice the computational time of AM (46,000 s) despite employing the same number of samples (8000). These observations suggest that the adaptive mechanism in AM provides substantial computational advantages without compromising solution accuracy, while the delayed rejection strategy in DR and DRAM incurs significan additional cost.

Although in this case study the number of model parameters is limited to three—resulting in comparable accuracy among the algorithms—their performance may diverge with an increased number of parameters. In particular, algorithms specificall designed for high-dimensional or highly correlated parameter spaces are expected to yield more accurate and efficien estimations under such conditions.

Analysis of the coefficien of variation (COV) reveals consistent patterns in parameter uncertainty across all methods. Among model parameters (ζ) shows substantially greater COV (9.63–9.83%) compare to the others and yield stress exhibits the lowest COV (3.53–3.72%). The consistently higher variability in ζ estimates suggests either greater sensitivity to algorithmic differences or inherent challenges in precisely constraining this parameter through the available measurements. This pattern holds across all methods, indicating it may reflec fundamental properties of the inverse problem rather than algorithm-specifi behaviors.

The results demonstrate that while different MCMC variants achieve equivalent mean solutions for deterministic FEMU problems, their computational performance and stability characteristics differ significantl . The AM algorithm emerges as particularly advantageous, combining computational efficien y with reliable convergence. The MH method offers slightly better stability for f_y estimation at moderate additional computational cost, while DR and DRAM provide comparable accuracy but require substantially greater resources. The consistently higher uncertainty in ζ estimates warrants particular attention in practical applications, suggesting that either additional measurement information or alternative parameterization approaches may be needed to better constrain this parameter. These finding provide valuable guidance for algorithm selection in FEMU applications, where the choice between methods may depend on the relative importance of computational efficien y versus parameter stability for specifi engineering problems.

Table 2: Results of nonlinear FEMU with MCMC

	MCMC algorithm			
	MH	DR	AM	DRAM
Optimal solution (mean value)				
E_c [Pa]	2.49×10^{10}	2.50×10^{10}	2.50×10^{10}	2.49×10^{10}
f_y [Pa]	4.58×10^{8}	4.59×10^{8}	4.60×10^{8}	4.59×10^{8}
ζ [%]	5.02	5.06	5.06	5.04
Optimal solution (COV)				
E_c [%]	4.88	4.84	4.87	4.9
f_y [%]	3.53	3.630	3.72	3.64
ζ [%]	9.68	9.63	9.83	9.82
Computational cost				
Number of samples	8000	8000	8000	8000
Time (sec)	27247	46044	23460	45899

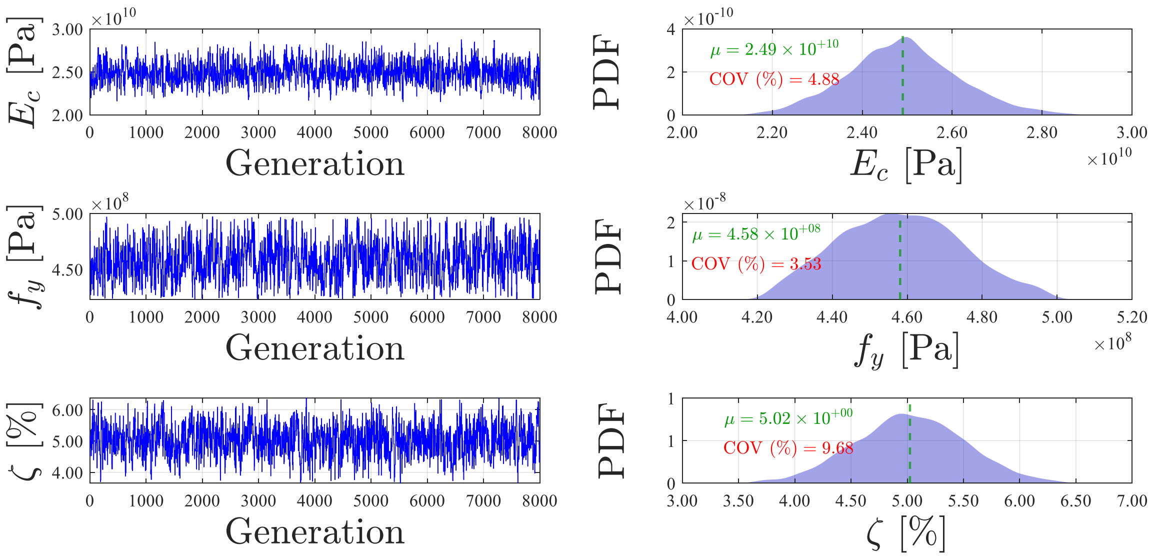

Figure 8: MCMC trace and posterioir PDF using the MH algoritm

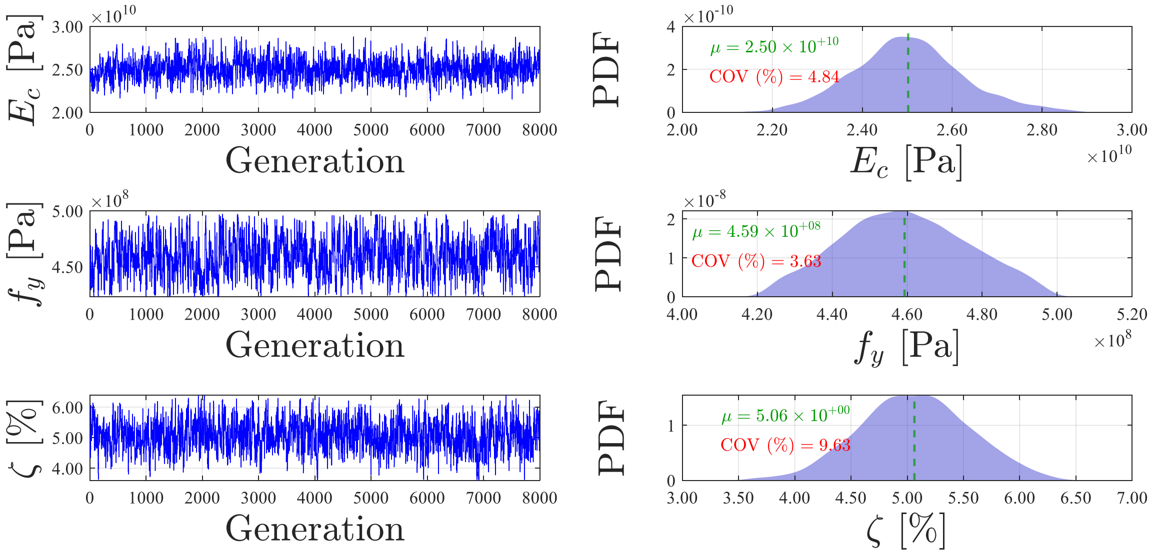

Figure 9: MCMC trace and posterioir PDF using the DR algoritm

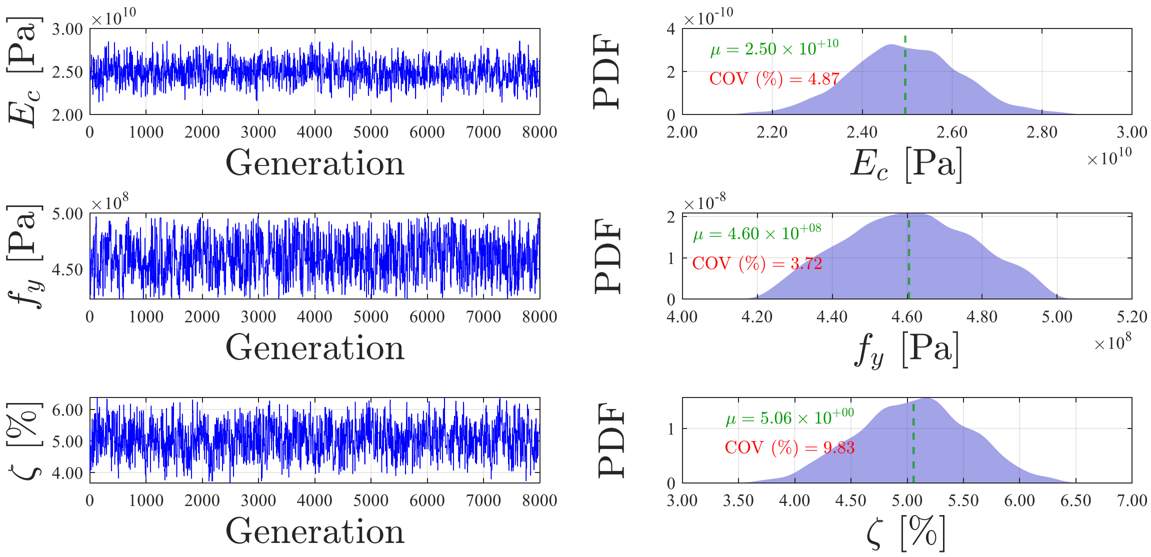

Figure 10: MCMC trace and posterioir PDF using the AM algoritm

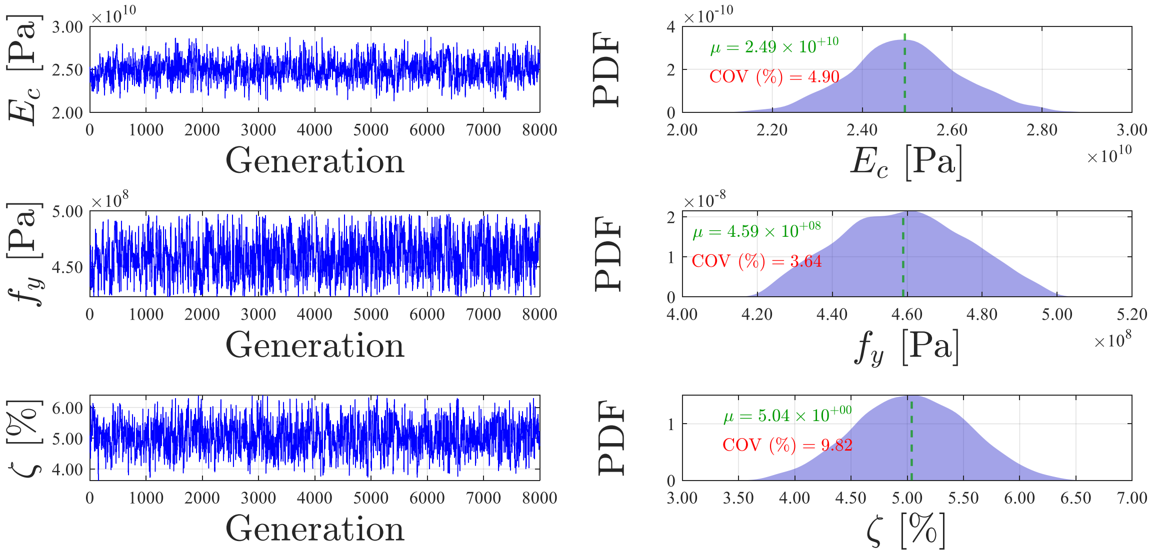

Figure 11: MCMC trace and posterioir PDF using the DRAM algoritm

4 Conclusions

In this study, two distinct approaches—deterministic and probabilistic—were investigated for nonlinear finit element model updating (FEMU) of a reinforced concrete bridge pier subjected to seismic loading. As bridge piers are critical components in the seismic response of highway bridges, their accurate modeling is essential for post-earthquake assessment.

The results have demonstrated that both deterministic and probabilistic approaches are capable of accurately estimating the true values of the selected model parameters in this case study. While the probabilistic approach offers the additional advantage of uncertainty quantification by updating the full probability distributions of the parameters, it comes with significantl higher computational costs and increased complexity—particularly in tuning hyperparameters to ensure chain stability. The comparison between different sampling algorithms within the probabilistic framework revealed similar accuracy levels due to the limited number of parameters (three in this case). However, this similarity may not hold in higher-dimensional problems, where algorithmic efficien y and accuracy could diverge.

The use of OpenSees proved highly effective due to its computational efficien y in nonlinear dynamic analyses. Parallel computing was successfully employed within the genetic algorithm (GA) used for the deterministic approach, leading to substantial reductions in computational time. In contrast, traditional MCMC algorithms do not inherently support parallelization, making them less efficien for large-scale applications. To address this limitation, more advanced sampling methods such as Transitional Markov Chain Monte Carlo (TMCMC) are recommended for future probabilistic FEMU studies. In conclusion, the integration of dynamic measurements, numerical modeling, and optimization algorithms offers a powerful toolset for enhancing the predictive accuracy of structural models. This approach holds promise for application in structural health monitoring, seismic vulnerability assessment, and performance-based engineering of reinforced concrete structures.

Several directions are proposed for further research to enhance FEMU methodologies:

- Inclusion of a larger set of model parameters, particularly those influencin nonlinear behavior.

- Accounting for correlations between parameters and updating them jointly.
- Development of a general framework to automatically select or optimize hyperparameters based on the structural model and parameter set.

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