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CAPACITY BOUNDS OF UNREINFORCED MASONRY WALLS WITH INTERVAL MECHANICAL PROPERTIES

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Abstract

The analysis of unreinforced masonry walls with uncertain mechanical properties is addressed. The interval model of uncertainty is adopted to deal with the limited availability of experimental data on masonry. Accordingly, the uncertain properties are modelled as interval variables with assigned lower bound and upper bound. Focusing on the in-plane behaviour of unreinforced masonry walls, a finite element model based on a simplified micro-modelling approach is developed in ABAQUS. The masonry wall shear capacity bounds are efficiently predicted by applying a ratio-of-polynomials response surface. A benchmark wall under in-plane monotonic loading is analysed to assess the accuracy and efficiency of the proposed approach.

Keywords: Unreinforced Masonry Walls, In-Plane Resistance, Simplified-Micro Modelling, ABAQUS, Interval Analysis, Response Surface Method.

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1 INTRODUCTION

Extensive experimental, analytical, and numerical studies have been devoted to the prediction of the load-bearing capacity of unreinforced masonry (URM) walls (see e.g., [1],[2]). Finite Element (FE) approaches for analysing masonry structures are commonly classified into three main modelling strategies based on the treatment of units and mortar joints [3]-[6]: detailed micro-modelling, macro-modelling, and simplified micro-modelling. Detailed micro-modelling offers high accuracy but is computationally demanding and often impractical for engineering applications. In contrast, macro-modelling reduces computational effort but fails to accurately predict cracking patterns and failure modes. Simplified micro-modelling provides a good balance between computational efficiency and accuracy, making it a commonly used approach.

Reliable predictions from numerical simulations require an accurate definition of the mechanical properties of masonry. Such properties can vary in a wide range due to several sources of uncertainty relating to workmanship techniques, block shape and texture, properties of single components (mortar and bricks/stones), environmental conditions, past damage, deterioration, etc. The aim of this paper is to study the capacity of URM walls with uncertain mechanical properties under in-plane loading.

The traditional probabilistic model has been extensively adopted in the literature to predict the influence of uncertainties on the behaviour of masonry structures (see e.g., [7]-[11]). A major drawback is the limited availability of experimental data for the accurate probabilistic characterization of the uncertain mechanical properties of masonry, especially for historical buildings. To cope with this issue, the present study represents uncertainties as interval variables [12] with assigned Lower Bound (LB) and Upper Bound (UB), relying on the *Improved Interval Analysis* (*IIA*) via *Extra Unitary Interval* (*EUI*) [13]. The interval model requires only range information on the uncertain parameters, in line with the prescriptions provided by Building Codes and Guidelines (see e.g., [14]) as well as visual approaches for the mechanical characterization of masonry, such as the Masonry Quality Index (MQI) method [15].

Once the interval mechanical properties are incorporated into a FE model, the in-plane resistance of the masonry wall also exhibits an interval nature. The evaluation of the capacity bounds is a challenging task due to the nonlinear behaviour of the masonry wall and the high computational cost of deterministic FE analyses. Surrogate models (see e.g., [7],[11]) have been applied in the literature to efficiently analyse the ultimate capacity of masonry structures with random mechanical properties. The proposed approach for the propagation of interval uncertainties relies on the use of a ratio-of-polynomials Response Surface (RS), which requires only a few deterministic analyses at selected sampling points. The assumed RS was originally developed within the probabilistic framework [16] and later extended to linear problems with interval parameters [17],[18]. This study investigates the extension of the ratio-of-polynomials RS to the propagation of interval uncertainties in nonlinear problems, along with the influence of higher-order terms.

Focusing on URM walls under in-plane loading, a 3D FE model is developed in ABAQUS [19] based on a simplified micro-modelling approach and the Extended Finite Element Method (XFEM). A benchmark URM wall subjected to monotonic in-plane loading [20] is selected as a case study to assess the accuracy and efficiency of the proposed approach in predicting the bounds of the in-plane resistance and the associated failure modes in the presence of interval mechanical properties.

2 MASONRY WALL 3D FINITE ELEMENT MODEL

The 3D behaviour of unreinforced masonry (URM) walls is simulated using a simplified micro-modelling approach [4] implemented in the commercial Finite Element (FE) software ABAQUS [19]. This approach employs the Surface-Based Cohesive Behaviour (SBCB) and the Drucker-Prager (DP) plasticity models. Crack propagation within masonry units is simulated using the Extended Finite Element Method (XFEM), without the need to predefine crack locations.

The SBCB model captures failure mechanisms, such as tensile cracking and shear sliding of joints, based on surface interactions. The initial joint behaviour is described by a linear traction-separation relationship, characterized by stiffness constants depending on the elastic properties of mortar and brick units. Tensile cracking and shear sliding of joints are governed by their tensile and shear strengths, respectively. The calculation of joint shear strength relies on the Mohr-Coulomb failure criterion, which involves the cohesion, c, and coefficient of friction, μ. Damage initiation is assumed to occur when contact stresses satisfy the quadratic stress failure criterion [21]. After damage initiation, the reduction of the interface stiffness is assumed to follow a linear softening law.

The nonlinear compressive response of masonry units is simulated using the DP model. Specifically, a linear yield criterion is adopted, which requires the definition of the friction angle and flow stress ratio as well as the uniaxial compressive stress–strain curve (see, e.g., [22]) of masonry units.

For further details on the implementation and parameters of the models described above, readers are referred to the relevant literature (see, e.g., [3], [4], [19]).

3 UNCERTAIN MASONRY MECHANICAL PROPERTIES

A reliable FE analysis of masonry walls requires the accurate selection of the key mechanical properties, considering the various sources of uncertainty. Since only a limited amount of experimental data is typically available for existing masonry buildings, uncertainties can be appropriately described using the interval model [12], which relies solely on range information. The uncertain parameters are described as interval variables with assigned Lower Bound (LB) and Upper Bound (UB). To limit conservatism, typically affecting the *Classical Interval Analysis* (*CIA*) [12], the so-called *Improved Interval Analysis* via *Extra Unitary Interval* (*IIA* via *EUI*) [13] is adopted. Based on this approach, the $i-$th interval mechanical property is expressed in the following affine form:

$$d_i^I = \left[\underline{d}_i, \overline{d}_i\right] = d_{0,i}\left(1+\alpha_i^I\right) = d_{0,i}\left(1+\Delta\alpha_i \hat{e}_i^I\right) \tag{1}$$

where the superscript I indicates interval variables; $\underline{d}_i$ and $\overline{d}_i$ denote the LB and UB of the interval variable d_i^I, respectively; $d_{0,i}$ represents the nominal value; $\alpha_i^I = \left[\underline{\alpha}_i, \overline{\alpha}_i\right] \in \mathbb{IR}$ is the fluctuation around the nominal value, described by a dimensionless symmetric interval variable ($\overline{\alpha}_i = -\underline{\alpha}_i$) with deviation amplitude $\Delta\alpha_i$; $\mathbb{IR}$ is the set of all real interval numbers; $\hat{e}_i^I = \left[-1,1\right]$ is the *EUI* associated with the $i-$th interval variable. Based on Eq. (1), the interval variable d_i^I can be defined by its LB and UB or by the midpoint and normalized deviation amplitude:

$$d_{0,i} = \mathrm{mid}\left\{d_i^I\right\} = \frac{\underline{d}_i + \overline{d}_i}{2}; \qquad \Delta\alpha_i = \frac{\Delta d_i}{d_{0,i}} = \frac{\overline{d}_i - \underline{d}_i}{2d_{0,i}} \tag{2a,b}$$

where $\mathrm{mid}\{\bullet\}$ denotes the midpoint operator; Δd_i is the deviation amplitude of d_i^I, and $\Delta\alpha_i < 1$ to guarantee positive values of the uncertain mechanical properties.

The definition of these quantities is quite challenging as the values of masonry mechanical properties may vary significantly due to various factors, such as masonry typology, workmanship techniques, environmental conditions, etc. The analyst can rely on ranges of the mechanical properties recommended by Building Codes and Guidelines (see, e.g., [14]), visual approaches such as the Masonry Quality Index (MQI) method [15], or experimental data, if available.

4 CAPACITY BOUNDS VIA RESPONSE SURFACE APPROACH

Let us assume that N mechanical properties of the FE modelled URM wall are described as interval variables d_i^I, $i = 1,2,\ldots,N$, whose fluctuations are collected into the interval vector $\boldsymbol{\alpha}^I = [\underline{\boldsymbol{\alpha}}, \overline{\boldsymbol{\alpha}}] \in \mathbb{IR}^N$. All possible responses obtained as the uncertain parameters vary within their interval define a solution set, which is typically approximated by the smallest conservative hypercube. Focusing on the generic response quantity, $y^I = f(\boldsymbol{\alpha}^I)$, with $f : \mathbb{R}^N \to \mathbb{R}$ denoting a deterministic mapping, the LB and UB are formally defined as:

$$\underline{y} = \min_{\boldsymbol{\alpha}\in\boldsymbol{\alpha}^I}\{f(\boldsymbol{\alpha})\}; \qquad \overline{y} = \max_{\boldsymbol{\alpha}\in\boldsymbol{\alpha}^I}\{f(\boldsymbol{\alpha})\}. \tag{3a,b}$$

Such bounds can be evaluated by solving a global optimization problem which requires repeated deterministic FE analyses of the masonry wall, resulting in a high computational burden. Surrogate models are often used in conjunction with global optimization to reduce the computational cost associated with the evaluation of the interval response bounds.

The uncertainty propagation strategy proposed in this study relies on the use of a ratio-of-polynomials RS, which can be defined by performing a certain number of deterministic FE analyses.

Since the deterministic nonlinear static analysis of the URM wall is performed using an incremental procedure associated with the Newton-Raphson method, the RS is determined at each load increment. As a first step, the following expression for the generic interval response quantity at each load increment, $y^I(\boldsymbol{\alpha})$, is assumed:

$$y^I \equiv y(\boldsymbol{\alpha}^I) = y_0 + \sum_{i=1}^{N} y_i(\alpha_i^I) \tag{4}$$

where y_0 denotes the nominal value, i.e., the response quantity estimated for $\boldsymbol{\alpha} = \mathbf{0}$ and $y_i(\alpha_i^I)$ is the deviation due to the $i-$th uncertain parameter, evaluated setting $\alpha_j^I = 0$ for $j \neq i$. To simplify notation, the load step is omitted. By interval extension of the RS proposed in Ref. [16], the deviation $y_i(\alpha_i^I)$ associated with the $i-$th uncertain parameter is approximated as a rational function of the fluctuation α_i^I, so that Eq. (4) reads:

$$y^I(\boldsymbol{\alpha}) = y_0 + \sum_{i=1}^{N}\sum_{k=1}^{s}\frac{\alpha_i^I}{A_i^{(k)} + B_i^{(k)}\alpha_i^I} = y_0 + \sum_{i=1}^{N}\sum_{k=1}^{s}\frac{\Delta\alpha_i\hat{e}_i^I}{A_i^{(k)} + B_i^{(k)}\Delta\alpha_i\hat{e}_i^I} \tag{5}$$

where s is the number of natural deformation modes of the relevant FE affected by the $i-$th fluctuation α_i^I; $A_i^{(k)}$ and $B_i^{(k)}$ are $2N \times s$ unknown coefficients, which can be evaluated by fitting the approximate solution to the actual response at selected sampling points. By applying a *saturated design*, $1 + 2s \times N$ deterministic analyses are needed, one to compute the nominal solution y_0 and $2s \times N$ to evaluate the "exact" implicit response, $y(\boldsymbol{\alpha}^I)$ for $2N \times s$ arbitrary realizations of the interval parameters $\alpha_i^{(j)} \in \alpha_i^I$ ($i = 1, 2, \ldots, N$; $j = 1, 2, \ldots, 2s$). It is worth mentioning that the coefficients of the ratio-of-polynomials RS are almost insensitive to the selection of the sampling points [16]. Once the coefficients $A_i^{(k)}$ and $B_i^{(k)}$ are evaluated, Eq. (5) can replace the time-consuming numerical model of the masonry wall, allowing the efficient determination of the LB and UB of the response quantity of interest (Eqs. (3a,b)).

5 NUMERICAL APPLICATION

A well-known experimental benchmark masonry wall [20] is selected as a case study to validate the proposed procedure (see Fig. 1). The URM wall comprised 18 courses of solid clay bricks bonded with a 10 mm-thick mortar. The upper and lower courses were secured to steel beams using high-performance mortar. The effective dimensions of the wall were 990 mm in length, 1000 mm in height, and 100 mm in thickness. Two identical specimens, labeled J4D and J5D, were tested under a uniform vertical pressure of 0.3 MPa. Before applying a 4 mm horizontal displacement, the vertical movement and rotation of the upper steel beam were constrained. To simulate the behaviour of the benchmark masonry wall, a FE model is built in ABAQUS using the simplified micro-modelling approach described in Section 2. 3D hexahedral elements with eight nodes, reduced integration, and hourglass control (C3D8R) [19] are used to model the expanded units. A mesh, consisting of 7×3×2 elements per masonry unit, resulting in 4520 elements, is adopted. The mechanical properties used to perform numerical simulations are taken from the experimental findings [20], as well as the data available in the literature [4]. Nonlinear static analysis is carried out by applying displacement control incrementally, using the Newton-Raphson algorithm. Automatic load stepping with specified increments up to a maximum of 10^5 is employed. A viscosity parameter of 0.002 is implemented for regularization to ensure convergence during stiffness degradation in the masonry joints.

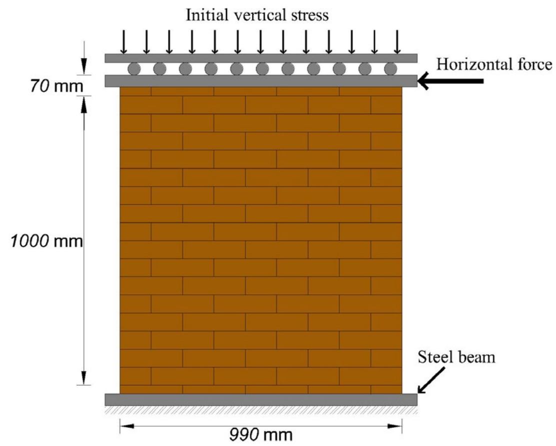


Figure 1: Test setup for the URM wall under confined shear loading (based on [20]).

As a first step, the FE model built in ABAQUS is validated by comparison with experimental results and numerical simulations available in the literature [4]. The load-displacement curves plotted in Fig. 2 demonstrate the ability of the implemented FE model to accurately capture the in-plane behaviour of the masonry wall.

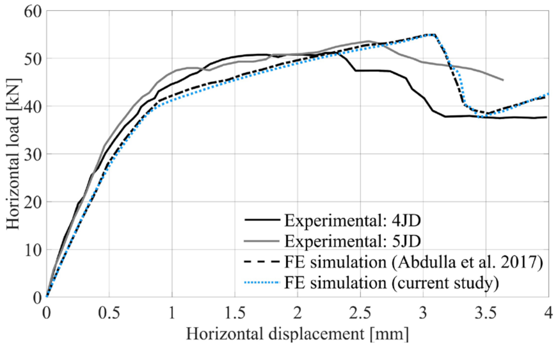
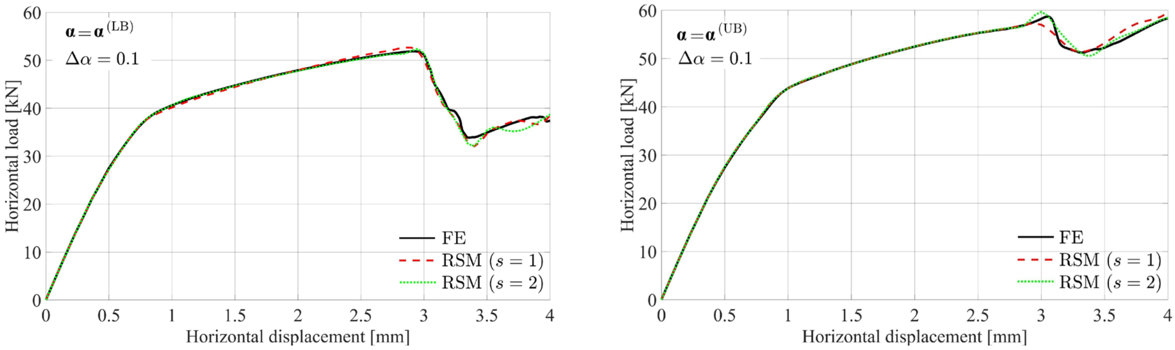


Figure 2: Load-displacement curves from FE simulations and experimental results for the benchmark URM wall under in-plane loading.

Given their significant impact on the in-plane behaviour of URM walls [5], the cohesion, c, and the coefficient of friction, μ, of the masonry joint interfaces are assumed uncertain. Both parameters are allowed to vary independently across two identical horizontal portions of the wall, each having a height of 500 mm. As a result, four interval parameters are involved, c_b^I, μ_b^I, c_t^I, and μ_t^I, representing the cohesion and coefficient of friction in the bottom (b) and top portions (t) of the URM wall. The nominal values of the uncertain parameters are assumed to be the same for the two portions of the wall and equal to those of the benchmark wall. The dimensionless fluctuations of the uncertain parameters around their nominal value, α_i^I, $i = c_b, \mu_b, c_t, \mu_t$, characterized by the same deviation amplitude $\Delta\alpha$, are collected into the interval vector $\boldsymbol{\alpha}^I = \left[\alpha_{c_b}^I \quad \alpha_{\mu_b}^I \quad \alpha_{c_t}^I \quad \alpha_{\mu_t}^I\right]^{\mathrm{T}}$.

Approximate estimates of the LB and UB of the masonry wall in-plane resistance are determined by applying the ratio-of-polynomials RS in conjunction with a combinatorial procedure. As a first step, the problem is discretized by subdividing the range of the interval parameters into smaller subintervals. Then, deterministic analyses are performed for all possible combinations of the associated endpoints, and the minimum and maximum among the obtained capacity samples are identified. For instance, subdividing the range of each interval parameter into ten subintervals and applying the ratio-of-polynomials RS (Eq. (5)) with $s = 2$, the bounds of the in-plane resistance of the masonry wall with $\Delta\alpha = 0.1$ are achieved for the following combinations of the uncertain parameters: $\boldsymbol{\alpha}^{(\mathrm{LB})} = \left[-0.1 \quad -0.1 \quad 0.1 \quad -0.1\right]^{\mathrm{T}}$ and $\boldsymbol{\alpha}^{(\mathrm{UB})} = \left[0.1 \quad 0.1 \quad 0.1 \quad 0.1\right]^{\mathrm{T}}$.

For validation purposes, Fig. 3 shows the comparison between the load-displacement curves obtained by FE analysis and the proposed Response Surface Method (RSM) with $s = 1$ and $s = 2$ for the realizations of the interval parameters $\boldsymbol{\alpha}^{(\mathrm{LB})}$ and $\boldsymbol{\alpha}^{(\mathrm{UB})}$. A very good agreement can be observed, even when $s = 1$.



(a) (b)

Figure 3: Load-displacement curves provided by FE analysis and RSM for the realizations of the uncertain parameters yielding the (a) LB and (b) UB of the shear capacity ($\Delta\alpha = 0.1$).

Table 1 lists the LB and UB of the masonry wall in-plane resistance predicted by FE analysis and RSM (with $s = 1$ and $s = 2$), R_{FE} and R_{RSM} , for two different values of the normalized deviation amplitude $\Delta\alpha$, along with the associated percentage error $\varepsilon(\%) = ((R_{\mathrm{RSM}} - R_{\mathrm{FE}}) / R_{\mathrm{FE}}) \times 100$. It can be observed that the RS yields very accurate estimates of the bounds of the capacity, even when $s = 1$ and larger degrees of uncertainty ($\Delta\alpha = 0.2$) are considered. The percentage errors for $\Delta\alpha = 0.2$ are less than 3.5% and 2.5% for $s = 1$ and $s = 2$, respectively.

	LB					UB				
$\Delta\alpha$	R_{FE} [kN]	R_{RSM} [kN] s=1	$\varepsilon(\%)$ s=1	R_{RSM} [kN] s=2	$\varepsilon(\%)$ s=2	R_{FE} [kN]	R_{RSM} [kN] s=1	$\varepsilon(\%)$ s=1	R_{RSM} [kN] s=2	$\varepsilon(\%)$ s=2
0.10	51.88	52.58	1.35	52.38	0.96	58.71	57.19	-2.59	59.70	1.69
0.20	48.62	49.54	1.90	49.23	1.27	62.61	60.49	-3.38	61.18	-2.27

Table 1: LB and UB of the masonry wall in-plane resistance provided by FE analysis and RSM, and associated percentage errors.

6 CONCLUSIONS

An efficient approach for analysing the in-plane behaviour of unreinforced masonry walls with uncertain mechanical properties has been presented. Given the limited availability of experimental data on masonry, uncertainties have been described as interval variables, fully characterized by their lower bound and upper bound. The masonry wall behaviour has been simulated by a 3D Finite Element (FE) model built in ABAQUS, based on a simplified micro-modelling approach. Uncertainty propagation has been efficiently performed by applying a ratio-of-polynomials Response Surface (RS), which requires only a few deterministic FE analyses at selected sampling points. Different orders of the RS and degrees of uncertainty have been investigated. Numerical results on a benchmark masonry wall under monotonic in-plane loading have demonstrated the accuracy of the proposed approach in predicting the bounds of the interval capacity.

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PROPOSAL: Tech4You - Technologies for climate change adaptation and quality of life improvement.
NRRP THEMATIC AREA: CLIMATE, ENERGY, SUSTAINABLE MOBILITY TERRITORY OF REFERENCE: Calabrian and Lucanian territorial areas.
Spoke 4 - Technologies for resilient and accessible cultural and natural heritage: Unibas Affiliates: Unical, Unirc, CNR - Goal 4.7: Protection and enhancement of natural and cultural heritage and identity of the inner territories.
Pilot Project 4.7.1 - Open platform "phigital space" (physical and digital) of the type "user profiling" for the advanced and dynamic codesign of interventions on the built and ex novo.

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