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ACOUSTIC WAVE TRANSMISSION OF MICROPOLAR PLATES

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Abstract. *This study investigates the interaction of sound waves with expanded (EPS) polystyrene plates, which possess a microstructure composed of interacting grains. The Cosserat or micropolar theory of elasticity is recognized for its ability to account for the internal length scales inherent to materials with microstructure. This theory introduces additional independent rotational degrees of freedom, along with associated curvature strain and couple stress descriptors, enabling the continuum to support a diverse range of bulk waves. Therefore, we apply the Cosserat theory to model through-thickness vibrations of linear elastic EPS plates, aiming to reproduce the experimental response. Experiments were carried out to measure the transmission coefficient of such low density plates. With a comprehensive experimental setup including a loudspeaker, a microphone, and a data acquisition system, through-thickness frequencies were observed within the frequency range between 2 kHz and 70 kHz. The comparison between theoretical and experimental results allows for the identification and calibration of material parameters.*

Keywords: Transmission Coefficients Micropolar Elasticity, Cosserat Continuum, Through-Thickness Frequency.

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1 INTRODUCTION

Following Voigt's foundational work [1], significan discrepancies emerged between experimentally observed dynamics and classical elasticity predictions, particularly in polymers, granular, and porous solids. These deviations stem from microstructural effects, absent in classical theories [2]. In granular and multimolecular materials, these effects become pronounced, giving rise to novel wave types unexplained by classical elasticity.

Expanding on the work of the Cosserat brothers [3], the theory of asymmetric elasticity introduces a framework where each material point in a continuum is associated with a rigid *triad* that can rotate independently of its translational displacement. This formulation established the balance equations for momentum in dynamic cases. A rigorous and complete linear theory of micropolar elasticity, incorporating microinertia effects, was later developed by Eringen [4, 5].

This study focuses exclusively on *micropolar linear elastic media*, hereafter referred to as the *Cosserat continuum*. This framework preserves the reversibility of deformations under mechanical stress while explicitly accounting for microstructural effects by introducing additional rotational degrees of freedom at each material point. Most research on micropolar elastodynamics has remained largely theoretical [6]. However, recent experimental studies have examined polystyrene plates, both expanded and extruded [7, 8].

Beyond fundamental insights, this study conducts a through-thickness analysis of vibrations, establishing the dependence of longitudinal resonances on the Cosserat coupling modulus κ—an effect absent in classical elasticity. Experimental validation over the 2 – 70 kHz range confirm predicted resonances on a polystyrene plate sample, enabling parameter identification Sections 2-4 shortly present the theoretical model and the analysis carried out, leading to numerical values of through-thickness frequencies. Section 5 details the experimental setup and results, while Section 6 concludes with key findings

2 DESCRIPTION OF THE PROBLEM

A sketch of the experiment is shown in Figure 1a. Sound waves generated by a loudspeaker propagate through the air and impinge on a plate. Within the plate, wave propagation occurs in the form of elastic waves, which reflec and transmit through its thickness. The oscillation of the plate's surface generates transmitted sound waves in the surrounding medium, which are recorded by a microphone placed beyond the obstacle. In our model, we focus solely on the solid plate and assume that it is immersed in vacuum, neglecting any fluid-structur interaction. Our objective is to investigate whether and how the microstructure of the plate, described by a micropolar model, influence wave propagation within the solid.

3 EQUATIONS OF MOTION

The model of the micropolar continuum is based on Eringen's theory of micropolar elasticity [5]. The set of coupled differential equations that govern the fiel motion of a micropolar continuum are the following:

$$
\begin{aligned}
(\mu+\alpha)\Delta\mathbf{u} + (\lambda+\mu)\nabla(\mathrm{div}\,\mathbf{u}) + \kappa\,\mathrm{curl}\,\boldsymbol{\theta} &= \rho\ddot{\mathbf{u}} \\
\gamma\Delta\boldsymbol{\theta} + (\alpha+\beta)\nabla(\mathrm{div}\,\boldsymbol{\theta}) + \kappa\,\mathrm{curl}\,\mathbf{u} - 2\kappa\boldsymbol{\theta} &= \rho J\ddot{\boldsymbol{\theta}}
\end{aligned}
\tag{1}
$$

where $\mathbf{u}$ and $\boldsymbol{\theta}$ are, respectively, the unknown displacement and microrotation fied that are a function of time t and of the position vector $\mathbf{x} = \{x_1, x_2, x_3\}^T$ (Figure 1 b). The double dots indicate the second derivative with respect to time t. Moreover, $\lambda, \mu, \alpha, \beta, \gamma$ and κ are material moduli, ρ is the density of the material, and J is the microinertia. The differential operators, when applied to a vector fiel $\mathbf{v} = \{v_1, v_2, v_3\}^T$ provide the following results: $\Delta \mathbf{v}_i = v_{i,i} + v_{i,j} + v_{i,k}$, $\operatorname{div} \mathbf{v} = v_{1,1} + v_{2,2} + v_{3,3}$, $\operatorname{curl} \mathbf{v}_i = (-1)^{(i-1)}(v_{k,j} - v_{j,k})$ where the comma subscript indicates the derivation with respect to the spatial coordinates and $i = 1, 2, 3$, $j = 1, 2, 3$, $k = 1, 2, 3$. The gradient ∇ applied to a scalar fiel ϕ gives the vector $\nabla\phi = \{\phi_{,1}\ \phi_{,2}\ \phi_{,3}\}^T$.

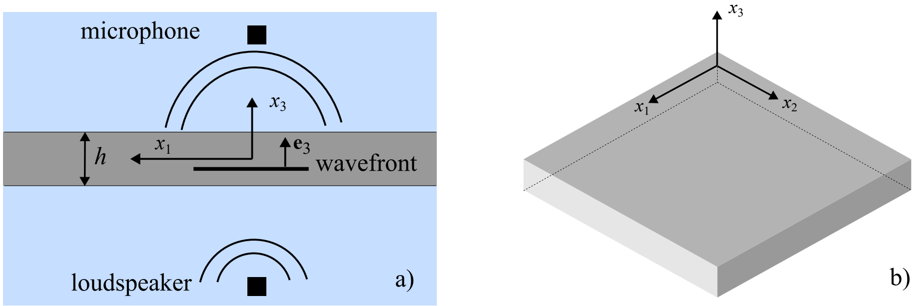

Figure 1: Sketch of the experiment and cross-section of the plate (a) and system of coordinates (b).

4 THROUGH-THICKNESS FREQUENCIES OF A MICROPOLAR PLATE

A cross section of the plate under study is reported in Figure 1a. We want to describe the vibrations of the plate when a plane wave with wavefront parallel to the upper and lower free-stress surfaces propagates within the plate thickness. Such kind of harmonic progressive waves with frequency ω (in rad/s) advancing in the positive direction of the vector $\mathbf{e}_3$, which is normal to the wavefront, are expressed as:

$$\mathbf{u}(x_3, t) = \mathbf{a}e^{I(k\,x_3 - \omega t)} \quad , \quad \boldsymbol{\theta}(x_3, t) = \mathbf{b}e^{I(k\,x_3 - \omega t)} \tag{2}$$

where k is the wavenumber, $\mathbf{a}$ and $\mathbf{b}$ are the amplitude vectors, and I is the imaginary unit.

Substitution of the tentative solution (2) into the fiel equations (1), leads to a firs decoupled eigenvalue problem where in-plane and out-of-plane vibrations behave independently. In this eigenvalue problem, which has quadratic structure, the eigenvalue is the wavenumber k and the eigenvectors are the amplitude vectors, which are both functions of ω. In-plane and out-of-plane vibrations are associated with displacement components $\{u_1, u_3, \theta_2\}$ and $\{u_2, \theta_1, \theta_3\}$, respectively. We restrict our interest to in-plane vibrations. The six related eigenvalues k_i with $i = 1..6$ are associated with eigenvectors involving displacements along u_3 only, which are longitudinal waveforms, or displacements along u_1 and microrotations θ_2, which will be called shear/micropolar waveforms. The in-plane longitudinal and shear/micropolar waves are also decoupled. Therefore, the displacement field will be expressed as the following modal superposition:

$$u_3 = a_1 e^{I(k_1\,x_3 - \omega t)} + a_2 e^{I(k_2\,x_3 - \omega t)} \tag{3}$$

for longitudinal waves, and

$$u_1 = a_3 e^{I(k_3 x_3 - \omega t)} + a_4 e^{I(k_4 x_3 - \omega t)} + a_5 e^{I(k_5 x_3 - \omega t)} + a_6 e^{I(k_6 x_6 - \omega t)}$$
$$\theta_2 = R(k_3) a_3 e^{I(k_3 x_3 - \omega t)} + R(k_4) a_4 e^{I(k_4 x_3 - \omega t)} + R(k_5) a_5 e^{I(k_5 x_3 - \omega t)} + R(k_6) a_6 e^{I(k_6 x_3 - \omega t)} \tag{4}$$

for shear micropolar waves, where a_i are the modal amplitudes and $R(k_i)$ are the ratios between the modal amplitudes b_i/a_i, derived from the solution of the firs eigenvalue problem. It should be noted that the in-plane eigenvalue problem, due to the constitutive relationships ([5]) does not depend on the constitutive constants α and β.

Among the solutions (3) and (4), we want to select those that satisfy the free stress conditions on the two upper and lower surfaces of the plate, that is:

$$\sigma_{33} = 0 \quad \sigma_{13} = 0 \quad m_{23} = 0 \quad \text{on} \quad x_3 = \pm h/2 \tag{5}$$

where σ_{33}, σ_{13} are, respectively, the normal and tangent stress components acting on the surface of the plate, and m_{23} is the microcouple inducing rotation about the axis x_2, normal to the plane of the plate; h is the height of the plate. By substituting equations (3) and (4) into equations (5), a second problem of eigenvalues is obtained, whose eigenvalues are the square of the frequencies ω^2 at which oscillations such as those expressed by the tentative solution (2) can occur. These frequencies are called through-thickness frequencies in classical Cauchy continua, and we will extend this nomenclature to the Cosserat media.

The second eigenvalue problem associated with Equations (5) also decouples into longitudinal and shear/microrotation waves. For longitudinal waves, it provides infinit discrete analytical solutions that are:

$$\omega = \frac{\sqrt{\lambda + 2\mu + \kappa}}{h\sqrt{\rho}} n\pi \tag{6}$$

where n is an integer number representing the mode order. Unlike Cauchy continua, the through-thickness frequency of longitudinal waveforms also depends on the constitutive coupling parameter κ. The eigenvalues of the shear microrotation waves are obtained numerically. The through thickness frequencies in the range of 0-100 kHz for a sample plate are reported on the vertical axis of Figure 2 as a function of the order of the mode. The red points refer to longitudinal modes, while the blue and cyan points, respectively, refer to shear and micropolar modes. The classificatio is carried out by observing the components u_1 and θ_2. In shear modes, we have $u_1 = 1, \theta_2 = 0$, while in micropolar modes $u_1 = 0, \theta_2 = 1$. Figure 2 presents a rich spectrum, with several shear/micropolar modes in-between two subsequent longitudinal through-thickness modes.

In the experimental response, we expect that the longitudinal thickness frequencies will dominate over the shear/micropolar thickness frequencies, since the shear vibration will transmit in the air beyond the obstacle only because of viscosity. Moreover, micropolar vibration modes could possibly transmit beyond the obstacle only due to surface irregularities. Based on that, the relationship (6) can be used to determine the material parameters from the observation of the peaks in the transmitted response spectrum. In particular, when longitudinal thickness frequencies are known together with the density and thickness of the plate, by inverting (6), it is possible to determine the combination of material parameters $\lambda + 2\mu + \kappa$.

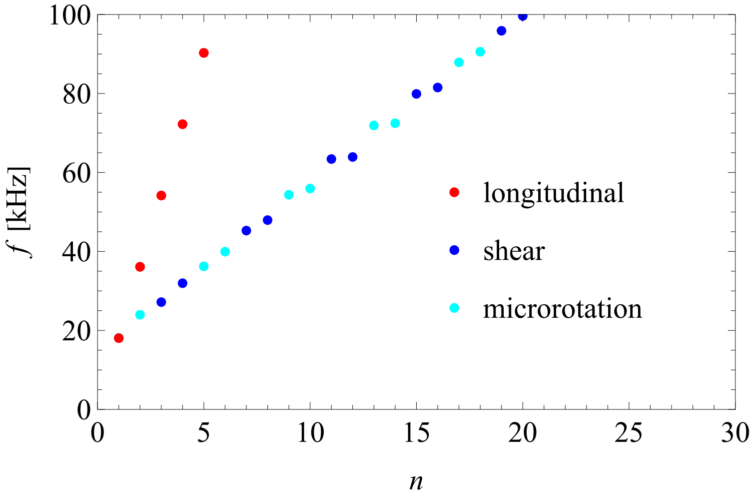

Figure 2: Through-thickness frequencies as a function of the mode order of a sample micropolar plate with the following mechanical and geometrical parameters: $h = 20$ mm, $\lambda = 1.3\,10^7$ N/m^2, $\mu = 2.7\,10^6$N/m^2, $\kappa = 3.0\,10^6$ N/m^2, $\gamma = 1.7\,10^6$N, $\rho = 32$ kg/m^3, $J = 4.0 10^{-1}$m^2.

5 EXPERIMENTAL METHOD

5.1 Sample characteristics and experimental setup

A plate-shaped expanded polystyrene (EPS) sample was used for acoustic transmission analysis (Fig. 3, left). EPS is widely used for its thermal and acoustic insulation properties in building applications. The sample specification are summarized in Table 1.

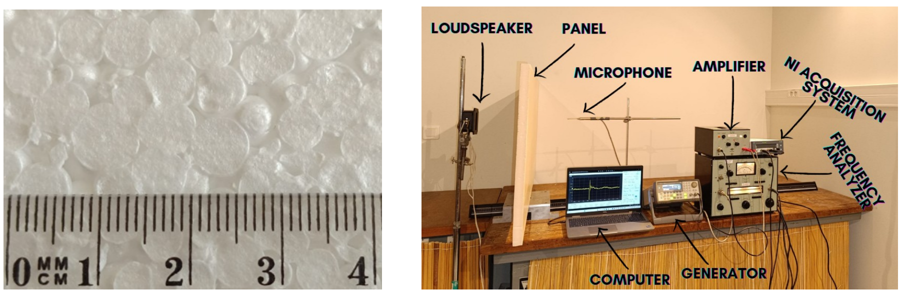

Figure 3: Expanded polystyrene sample (left) and experimental setup (right).

Our ref.	Material	Thickness	Manufacturing process
Plate G	Expanded Polystyrene (EPS)	20 mm	Thermal Expansion

Table 1: Descriptive characteristics of the sample used in this study.

The experimental configuratio (Fig. 3, right) ensures normal-incidence plane wave excitation ($\theta = 0°$). A 15 μs pulsed signal, generated by an Agilent 33220A function generator, excites a Visaton MHT 12 loudspeaker. The transmitted wave is detected by a Brüel & Kjær 4138-C-006 microphone, relayed to a National Instruments PXIe-1090 acquisition system. A

Brüel & Kjær 2706 power amplifie stabilizes excitation, while a 2121 frequency analyzer enhances signal fidelit . Temporal acquisitions undergo FFT processing for spectral characterization.

5.2 Experimental results

Fig. 4 illustrates both incident and transmitted signals in the time domain (left), along with their corresponding frequency spectra (right). Significan attenuation of the transmitted signal amplitude is observed, highlighting the sound-absorbing characteristics of the sample. Furthermore, the transmitted signal appears earlier in the time domain compared to the incident wave (as shown in the zoomed-in region of Fig. 4, left). This phenomenon can be attributed to the interaction between the airborne longitudinal wave and the elastic structure of the plate. Upon encountering the sample, the acoustic wave induces vibrations within the material, facilitating the generation and propagation of additional waves within the plate, thereby modifying the effective wave velocity. The spectral content of the incident signal reveals a broadband energy distribution ranging from 2 kHz to approximately 70 kHz, which is particularly suitable for analyzing the fundamental propagation modes.

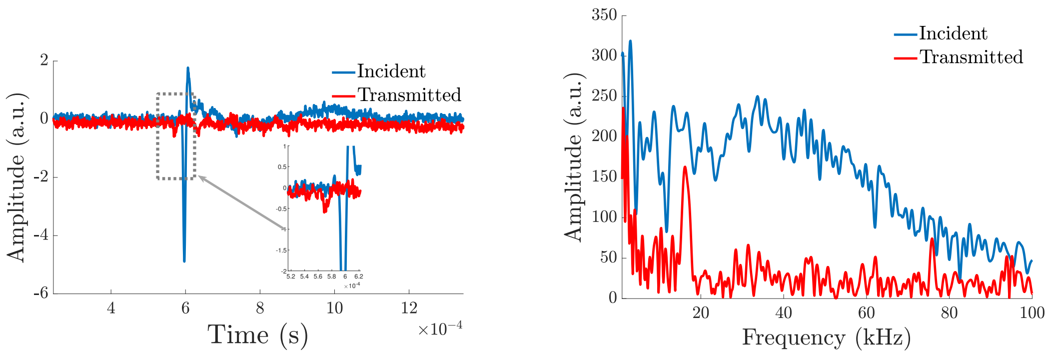

Figure 4: Time-domain representation of the incident and transmitted signals (left) and their corresponding frequency spectra (right) for the 20 mm thick EPS plate.

The magnitude of the transmission coefficien T, shown in Fig. 5, was computed using Matlab®'s `tfestimate` function, applied to the acquired incident and transmitted signals. To mitigate spectral leakage and improve the robustness of the frequency domain estimation, appropriate windowing techniques were implemented during preprocessing [9]. From Fig. 5, it is clear that several peaks that are integer multiples of the firs (16.35 kHz) arise, as predicted by Equation 6. In-between adjacent couples of these integer multiples, a rich spectrum appears, which could be attributed to other thickness resonances, as shown in Figure 3. Then, on knowing the density of the plate and its thickness, based on the firs peak of the transmission spectrum, inversion of Equation 6 provides $\lambda + 2\mu + \kappa = 1.3\,10^7$ N/m^2.

6 CONCLUSIONS

This study has explored the transmission of acoustic waves through plates made of polystyrene, employing the Cosserat theory of elasticity. By incorporating microstructural effects, the micropolar model enables the description of additional wave modes that cannot be captured by classical elasticity theories. The theoretical formulation led to a system of coupled differential equations that govern both displacement and microrotation fields Analysis of through-

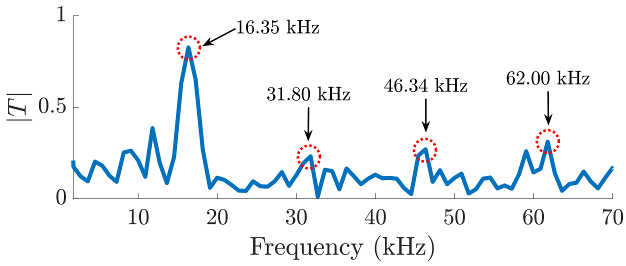

Figure 5: Experimental magnitude of the transmission coefficien for the 20 mm thick EPS plate.

thickness frequencies demonstrated that, unlike classical elasticity, longitudinal resonance frequencies depend on the micropolar coupling parameter κ. In addition, the study identifie shear and microrotation modes, highlighting the role of microinertia in wave propagation. The experimental measurements, conducted over a broad frequency range (2 kHz – 70 kHz), confirme the presence of predicted resonance peaks. The comparison between theoretical and experimental results allowed the identificatio of a combination of material parameters, specificall $\lambda + 2\mu + \kappa$, demonstrating the impact of micropolar elasticity in low-density polymeric materials.

7 ACKNOWLEDGMENTS

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