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SECTIONAL HOMOGENIZATION WITH A GENERAL NONLINEAR CONSTITUTIVE LAW FOR CORRUGATED BOARD ANALYSIS

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Abstract

Corrugated board is widely used in packaging applications due to its favorable mechanical properties and cost-effectiveness. This study proposes a two-step, multiscale, sectional homogenization approach to determine the effective elastic parameters and predict the mechanical behavior of cardboard under various loading conditions. In the first step, a representative 3D periodic model of corrugated board is developed to extract its effective elastic properties. This homogenization process reduces the complex 3D structure to a simplified 2D shell model. The second step focuses on computing deformations, strains, and stresses in cardboard structures. Instead of using a conventional plastic constitutive model, the 3D Representative Volume Element (RVE) model is employed. Strains from the 2D model are applied as boundary conditions to the 3D RVE, where corresponding stresses are determined based on pre-calibrated strain-stress relationships obtained from uniaxial tests. Stiffness degradation is captured, from a computational viewpoint, at Gauss points in the 2D model. The results demonstrate that the proposed approach accurately reproduces the real mechanical behavior of cardboard while utilizing a simplified 2D model.

Keywords: Corrugated Board, Sectional Homogenization, Nonlinear Material Modelling, Inverse Representative Volume Element Evaluation, Composite Materials and Structures.

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1 INTRODUCTION

Corrugated cardboard, invented in the late 19th century, has become one of the most widely used structural materials in the packaging industry [1, 2]. Its success lies in the unique combination of low weight, high stiffness, cost-efficiency, and recyclability. Composed of one or more corrugated core layers (fluting), sandwiched between flat linerboards, such structure provides resistance to bending, compression, and impact, while using minimal raw material. These characteristics have made corrugated board indispensable, not only in transport packaging, but also in furniture design, temporary structures, and architectural prototypes (see, e.g., [3, 4, 5]).

Despite its widespread use, an accurate modeling of the mechanical behavior of corrugated board remains a challenging task. Its geometry is highly heterogeneous and periodic, exhibiting nonlinear responses under realistic loading conditions, including localized buckling, delamination, and stiffness degradation. Standard computational methods often fail to efficiently capture these effects.

Homogenization techniques have long been used to approximate the macroscopic behavior of materials with periodic microstructures. Classic methods, such as asymptotic homogenization, periodic cell analysis, and numerical Representative Volume Element (RVE) approaches, allow the reduction of complex 3D problems to simplified 2D models [6, 7]. These methods are especially effective when analyzing large-scale structures, where solving the full geometry would be computationally infeasible.

In the case of corrugated cardboard, numerical homogenization has been successfully applied to estimate effective elastic parameters, including in-plane moduli and transverse shear stiffness [8]. Previous research by Garbowski and Gajewski [9] proposed a Finite Element-based homogenization approach suitable for extracting the transverse shear stiffness of sandwich panels with corrugated cores. However, such methods are typically limited to the elastic regime and do not account for local instability phenomena, such as localized buckling of liners in unconstrained regions [10].

In order to address these limitations, in the present research contribution a novel sectional, two-step multiscale homogenization method has been developed. This approach divides the full 3D periodic unit into distinct segments (sections) based on their mechanical role and local behavior. Each section is then independently homogenized, allowing for more accurate modeling of nonlinear effects, such as, e.g., local buckling of unsupported liner segments.

The proposed method also incorporates a general nonlinear constitutive law, calibrated from uniaxial tests, which is used in the second step of the procedure. Consistently, strain values obtained from a simplified 2D shell model are applied as boundary conditions to the local 3D RVE sections. The stress response, including stiffness degradation, is then assessed and mapped back into the global model. This workflow enables realistic simulations of mechanical behavior, even in the post-elastic range, while maintaining computational efficiency [11].

This work builds upon the broader field of multiscale modeling, where local RVEs are embedded within global continuum models, often in the form of shells or plates [12, 13]. Such methods have been proven effective in composite structures and advanced sandwich panels, and their extension to the modeling of corrugated board opens new possibilities for structural optimization, durability analysis, and virtual prototyping, among the aims of the present contribution.

Following the current introduction and brief literature survey, the article is organized in three main sections. Section 2 is devoted to the development of the two-step computational method, both regarding constitutive modelling and computational features. Consequently, Section 3 presents a numerical validation example, also providing relevant discussion about the results and the devised methodology in Section 4. Closing remarks are gathered in an additional section, to highlight the main outcome and the innovative key aspects.

2 MATERIALS AND METHOD

2.1 Sectional two-step homogenization

The proposed method is based on the concept of sectional two-step homogenization. In the first step, effective elastic properties of the corrugated board are determined using a Representative Volume Element modeled in 3D. Unlike classic approaches that consider a single periodic unit, here, the periodic unit is divided into mechanically distinct sections, reflecting different local behaviors under load. These sections may include:

- compressed linerboard segments that are either supported or unsupported by the fluting;
- regions transmitting shear between liners and the corrugated core;
- zones prone to local or global buckling.

Such segmentation allows the method to capture nonlinear effects that are typically averaged out, in traditional homogenization.

Each defined section is separately homogenized, and the resulting properties are assigned to Gauss points in the simplified 2D shell model. This 2D model reproduces the overall geometry of the real structure (e.g., a packaging wall), assuming that each point exhibits different effective stiffness properties depending on the underlying section of the corrugated geometry.

The general structure of the method is illustrated in Figure 1, which outlines the complete workflow, from the construction of the 3D geometry and sectional RVE identification, through homogenization, to the strain-to-stress mapping in the simplified model.

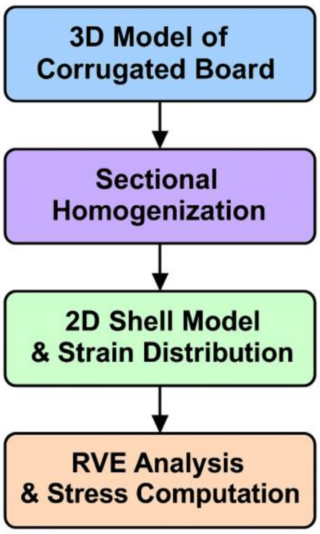


Figure 1. Flowchart illustrating the proposed two-step sectional homogenization methodology. The workflow includes 3D geometry modeling, sectional homogenization, 2D shell modeling, and reverse mapping through pre-calibrated RVE responses.

2.2 Inverse constitutive calibration

In traditional finite element simulations, especially those involving shell or plate elements, nonlinear behavior is typically introduced through built-in constitutive models, such as elastoplasticity or damage mechanics. However, these models operate at the macroscopic generalized level and require global parameters that are often difficult to calibrate for complex heterogeneous, possibly anisotropic, materials like corrugated cardboard. Moreover, when homogenization is applied in the first step, information about the material microstructure is typically lost, and, with it, its local instability mechanisms.

To overcome this limitation, the second step of the current devised approach uses a non-standard constitutive procedure based on inverse RVE evaluation. Instead of assuming a predefined constitutive model for the 2D shell elements, a reverse projection technique is employed: local strain tensors computed at Gauss integration points, in the 2D model, are imposed as boundary conditions on the corresponding 3D RVE sections.

To this aim, it is worth to note that, each RVE has been previously calibrated using either:

- laboratory uniaxial tests (e.g., compressive or tensile tests in Machine Direction/Cross-machine Direction), or
- detailed numerical simulations for each RVE section under representative loading conditions.

From these, nonlinear strain-stress curves are obtained and stored as lookup tables or interpolation functions. These curves represent how each section responds to deformation, also accounting for effects such as: (a) stiffness softening due to local damage or buckling; (b) nonlinear stress redistribution in post-elastic ranges; (c) anisotropic behavior, if present in liners or fluting.

In the simulation workflow, for each Gauss point: (i) the local strain tensor ($\boldsymbol{\varepsilon}$) is computed in the 2D shell element; (ii) the point is associated with a specific RVE section (e.g., compressed unsupported liner); (iii) the corresponding RVE model is loaded, and $\boldsymbol{\varepsilon}$ is applied as a boundary condition; (iv) the stress tensor ($\boldsymbol{\sigma}$) is computed based on pre-calibrated nonlinear response curves.

The stress is then transferred back to the 2D shell model, replacing what would otherwise be computed via a simple constitutive law. This decouples the geometry from the constitutive behavior while retaining local mechanical realism. It also allows for heterogeneous nonlinear responses within a single shell element, which is especially valuable for structures exhibiting localized phenomena such as:

- folding or crushing in unsupported liner zones,
- tension stiffening in stretched regions,
- shear-lag effects between layers.

Importantly, this procedure preserves computational efficiency: the 3D RVEs are not solved in real time, but their responses are interpolated from precomputed data, making the approach suitable for large-scale simulations. Moreover, it can be extended to time-dependent behavior (e.g., creep or cyclic fatigue) by enriching the database of RVE responses.

3 NUMERICAL EXAMPLE

3.1 Three-point bending test

To evaluate the accuracy and predictive capability of the proposed sectional homogenization method, we performed a numerical simulation of a classic three-point bending test, on a specimen made of single-wall corrugated cardboard. This test setup is commonly used in experimental studies to evaluate flexural stiffness and to observe failure mechanisms such as local buckling, delamination, or crushing. A schematic representation of the experimental configuration is presented in Figure 2.

The tested specimen had the following dimensions: length 200 mm, width 50 mm, and total thickness 4 mm (including two liners and a sinusoidal fluting). The sample was simply supported at both ends with a hinge and a roller boundary condition, respectively. A concentrated vertical force was applied at midspan using a rigid cylindrical indenter. The loading process was displacement-controlled, to facilitate detailed observation of post-buckling behavior.

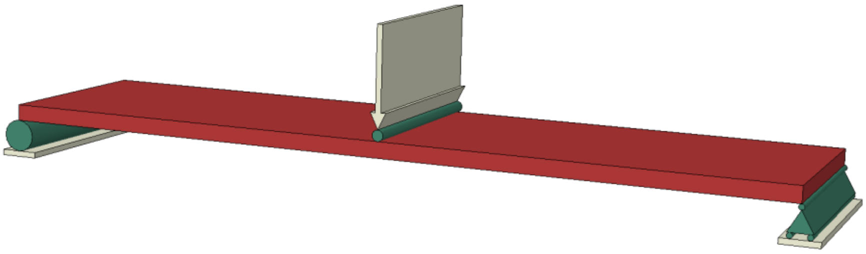
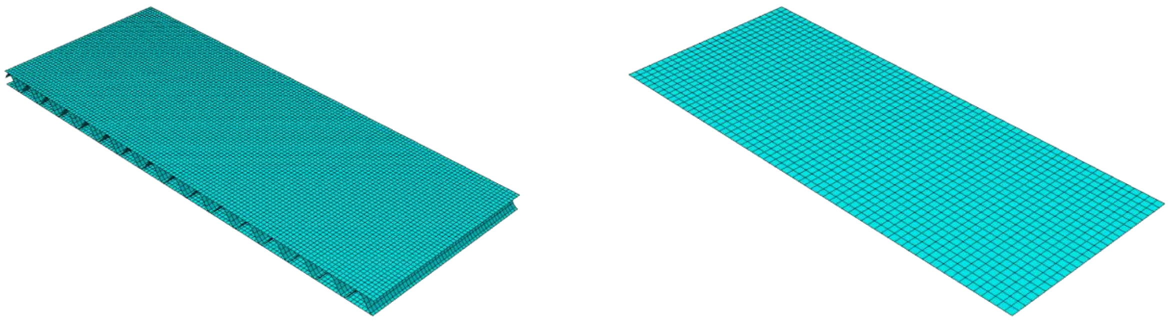


Figure 2. Experimental layout of the three-point bending test: a corrugated board sample is placed on two supports with a concentrated load applied in the center.

To investigate the efficiency and fidelity of the proposed method, two numerical models were constructed:

Model A – Full 3D geometry (Figure 3a). This model explicitly reproduces the detailed geometry of the corrugated core and linerboards. The fluting was modeled using four-node fully integrated shell elements, discretized to follow the actual sinusoidal profile. Both linerboards were represented by layered shell elements, co-meshed with the fluting at contact nodes. Material orthotropy was defined in accordance with Machine Direction (MD) and Cross-machine Direction (CD), and nonlinear behavior, including local buckling, was captured through geometric nonlinearity and material softening laws.

Model B – 2D shell with sectional RVEs (Figure 3b). In the simplified model, the entire structure was represented using a single-layer shell mesh with effective elastic properties derived from the first step of the sectional homogenization procedure. However, the key innovation lies in how material behavior is assigned: each Gauss point in the shell mesh is linked to a pre-defined sectional RVE, representing specific geometric and mechanical conditions (e.g., compressed liner above a flute, or in unsupported regions). During simulation, local strains at each Gauss point are used to retrieve nonlinear stresses via calibrated strain-stress curves specific to that RVE. This enables the model to replicate local stiffness degradation and post-buckling behavior, while maintaining a significantly reduced computational cost.



(a) (b)

Figure 3. Numerical models used in the analysis: (a) half of the full 3D model (with 22,050 shell 4-node elements) representing the actual geometry of the corrugated structure with shell elements for fluting and liners; (b) half of the simplified 2D shell model (with 1 575 shell 4-node elements) using sectional RVEs to represent local nonlinear behavior.

This comparison aimed to verify whether the simplified 2D model could reproduce the complex mechanical response observed in the 3D simulation, especially: (a) stiffness evolution in the elastic and nonlinear ranges; (b) initiation and development of local buckling in compressed linerboard zones; (c) global deformation patterns, including deflection shape and symmetry.

4 RESULTS AND DISCUSSION

4.1 Observations and phenomena

In the full 3D simulation (Model A), local buckling of the bottom linerboard was observed shortly after the linear range, particularly: (i) directly under the point of load application and (ii) near the supports, where the liner was not adequately supported by the fluting.

This is a well-known weakness in corrugated structures and typically leads to nonlinear softening, stress redistribution, and in some cases, delamination or folding.

The 2D shell model (Model B), despite lacking explicit geometric detail, was able to faithfully reproduce the onset of buckling and subsequent stiffness reduction due to the presence of sectional RVEs. This confirms that the method captures not only average stiffness, but also localized failure mechanisms.

4.2 Force-displacement comparison

Figure 4 presents the force-displacement curves obtained from both numerical models, the full 3D model and the simplified 2D shell model with sectional RVE evaluation. The horizontal axis represents the vertical displacement at the midspan of the sample, while the vertical axis corresponds to the loading reaction force. Both curves exhibit similar initial stiffness, indicating that the homogenized elastic parameters used in the 2D model were correctly extracted. A slight divergence appears after the initiation of local buckling, as the 3D model begins to experience geometric softening.

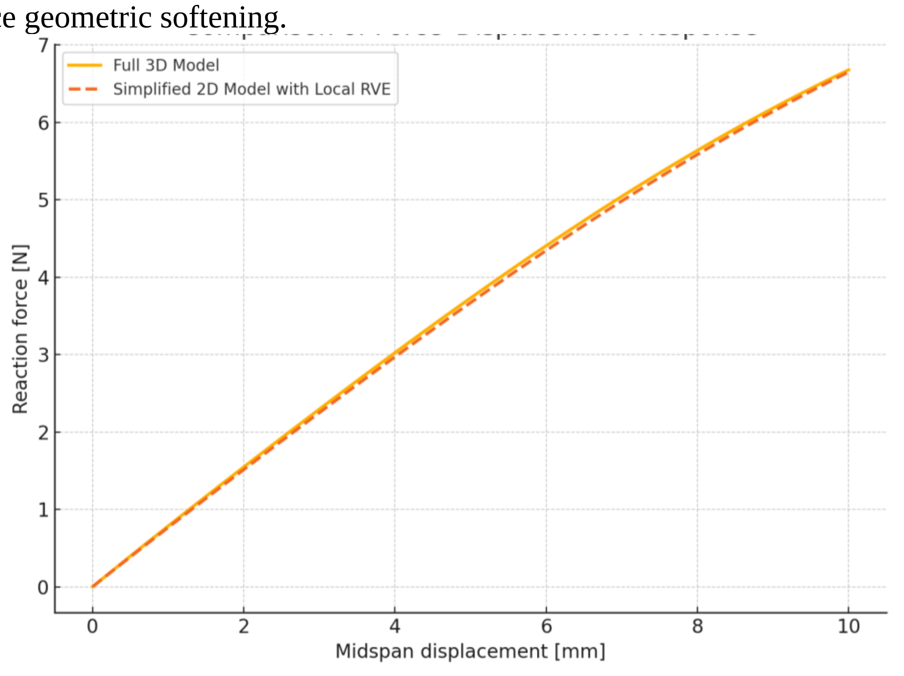


Figure 4. Force-displacement response comparison for the full 3D model and the simplified 2D shell model with local RVEs under three-point bending.

Nevertheless, the 2D model closely reproduces the nonlinear response, including the post-buckling stiffness degradation. The ability of the simplified model to capture this behavior confirms that the calibrated sectional RVEs effectively encode the complex local instabilities observed in the real geometry.

4.3 Stress distribution and local effects

To better understand how each model captures local failure mechanisms, Figure 5 presents the stress field distribution (component σ_{xx}) in the bottom liner at selected load levels in both the 3D and 2D simulations.

In the 3D model (Figure 5a), clear stress concentrations are visible: (i) beneath the point load, where the liner is unsupported by the fluting, (2) and near the supports, due to bending-induced compression.

These zones coincide with regions of local buckling, observable as out-of-plane deformations in the compressed linerboard. The material, in these areas, exhibits a sudden drop in stiffness, initiating nonlinear behavior and global softening.

In the 2D shell model (Figure 5b), although the geometry does not explicitly model the fluting and liner layers, the same critical zones show reduced stress-carrying capacity due to the RVE-based stiffness degradation. This confirms that the proposed method not only reproduces global response, but also identifies local failure-prone regions, even with significantly reduced model complexity.

(a) (b)

Figure 5. Stress distribution (σ_{xx}) and local effects at supports and midspan: (a) half of the 3D model showing concentration zones and buckling in unsupported liners, (b) half of the 2D model showing analogous softening via calibrated RVE behavior.

5 CONCLUSIONS

This study introduced a novel, two-step, sectional homogenization framework for modeling the nonlinear mechanical behavior of corrugated cardboard structures. The method combines the efficiency of simplified 2D shell models with the accuracy and realism of local 3D behavior, as captured through sectional Representative Volume Elements. Its foundation lies in dividing the geometrically periodic unit cell into mechanically distinct regions, each of which undergoes separate homogenization and nonlinear calibration. This allows for accurate modeling of local phenomena, such as liner buckling, stiffness degradation, and stress redistribution, phenomena often neglected or overly smoothed in classic homogenization approaches.

The main conclusions of this work are as follows.

1. The proposed method successfully integrates detailed local behavior into global simulations, allowing simplified shell models to capture highly nonlinear and localized effects typical of corrugated cardboard under compressive and bending loads.
2. Sectional homogenization offers a mechanically meaningful partitioning of the Representative Volume Element, enabling finer resolution of structurally critical areas, such as unsupported liner segments or regions near concentrated loads and supports.
3. The inverse use of RVEs, where strain fields from the 2D model are imposed on pre-calibrated 3D sections, allows recovery of accurate stress fields without relying

on conventional constitutive laws at the macroscale. This increases the predictive power of the simulation, particularly in the post-elastic regime.

4. The method was validated through a three-point bending test simulation. The 2D model, although lacking explicit geometry, replicated the behavior of the full 3D model with impressive accuracy. Both global and local responses, including force-displacement curves, buckling zones, and stress concentrations, were effectively reproduced.
5. The approach is computationally efficient, enabling large-scale structural simulations with realistic post-elastic effects, while drastically reducing model complexity, computational burden and solving time.

The methodology presented here lays a robust foundation for further extensions in both research and industrial practice. Ongoing and future developments will focus on:

- automatic generation and classification of sectional RVEs for arbitrary geometries and boundary conditions, based on machine learning or pattern recognition approaches (see, e.g., [14]);
- application to multi-layered corrugated systems, such as double-wall and triple-wall boards, and to more complex load states, including shear-dominated and biaxial scenarios;
- extension of the RVE calibration to time-dependent effects, including creep, fatigue, and environmental degradation due to relative humidity and temperature fluctuations (see, e.g., [15]);
- integration with design and optimization frameworks, enabling shape and thickness optimization, material selection, and durability-driven packaging assessment and design.

The devised framework also gains the potential to be generalized beyond corrugated cardboard, to other engineered materials with heterogeneous, layered, or architected microstructures (see, e.g., [16, 17, 18])—where mechanical behavior is driven by complex local interactions that cannot be captured by homogenized continuum models alone—for an effective assessment and modeling methodology.

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